Penn State Astrostatistics MCMC tutorial

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Bayesian change point model with Gamma hyperpriors: full conditionals

Our goal is to draw samples from the 5-dimensional **posterior** distribution $f(k, \theta, \lambda, b_1, b_2|\mathbf{Y})$ The posterior distribution is

$$\begin{aligned} f(k, \theta, \lambda, b_1, b_2|\mathbf{Y}) \propto & \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \\ & \times \frac{1}{\Gamma(0.5) b_1^{0.5}} \theta^{-0.5} e^{-\theta/b_1} \times \frac{1}{\Gamma(0.5) b_2^{0.5}} \lambda^{-0.5} e^{-\lambda/b_2} \\ & \times \frac{1}{\Gamma(c_1) d_1^{c_1}} b_1^{c_1-1} e^{-b_1/d_1} \frac{1}{\Gamma(c_2) d_2^{c_2}} b_2^{c_2-1} e^{-b_2/d_2} \times \frac{1}{n} \end{aligned} \tag{1}$$

From 1 we can obtain full conditional distributions for each parameter by ignoring all terms that are constant with respect to the parameter.
For θ:

$$f(\theta|k, \lambda, b_1, b_2, \mathbf{Y}) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \times \frac{1}{\Gamma(0.5) b_1^{0.5}} \theta^{-0.5} e^{-\theta/b_1} \tag{2}$$

For λ:

$$f(\lambda|k, \theta, b_1, b_2, \mathbf{Y}) \propto \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \times \frac{1}{\Gamma(0.5) b_2^{0.5}} \lambda^{-0.5} e^{-\lambda/b_2} \tag{3}$$

For k:

$$f(k|\theta, \lambda, b_1, b_2, \mathbf{Y}) \propto \prod_{i=1}^{k} \frac{\theta^{Y_i} e^{-\theta}}{Y_i!} \prod_{i=k+1}^{n} \frac{\lambda^{Y_i} e^{-\lambda}}{Y_i!} \tag{4}$$

For b_1:

$$f(b_1|k, \theta, \lambda, b_2, \mathbf{Y}) \propto \frac{1}{b_1^{0.5}} e^{-\theta/b_1} \times b_1^{c_1-1} e^{-b_1/d_1} \tag{5}$$

For b_2:

$$f(b_2|k, \theta, \lambda, b_1|\mathbf{Y}) \propto \times \frac{1}{b_2^{0.5}} e^{-\lambda/b_2} \times b_2^{c_2-1} e^{-b_2/d_2} \tag{6}$$

$f(b_1|k, \theta, \lambda, b_2, \mathbf{Y})$ and $f(b_2|k, \theta, \lambda, b_1|\mathbf{Y})$ are not well known densities. We can use a Metropolis-Hastings accept-reject step to sample from their full conditionals.