Kinematic Dynamo Models of the Solar Cycle

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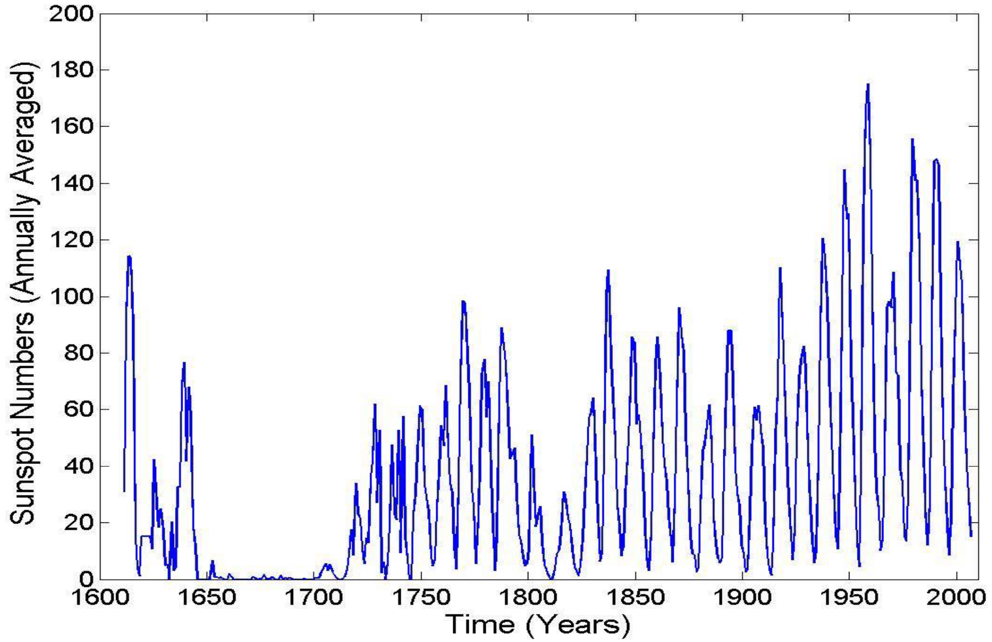
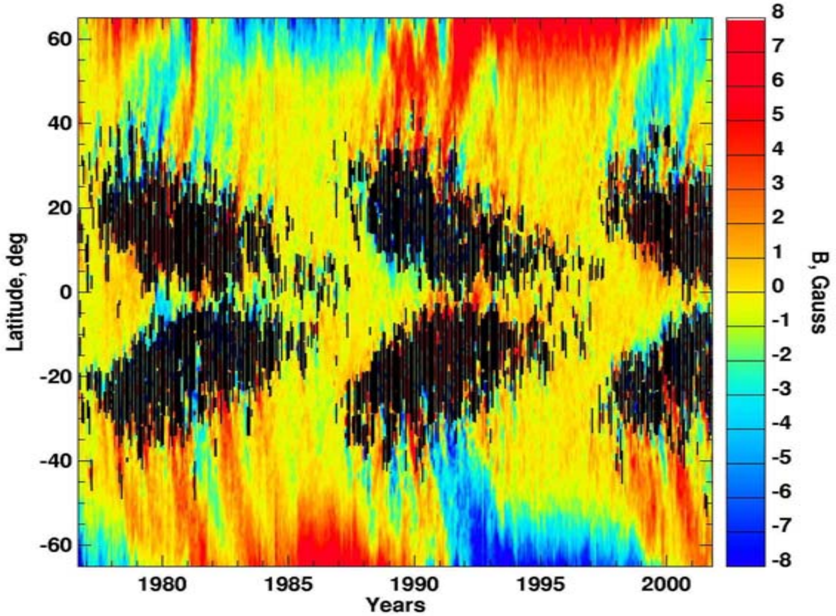
Outline:

- Solar cycle: Observational characteristics
- MHD: Some basic concepts
- Historical development of ideas
- A typical kinematic $\alpha\Omega$ dynamo model
- Outstanding issues…..

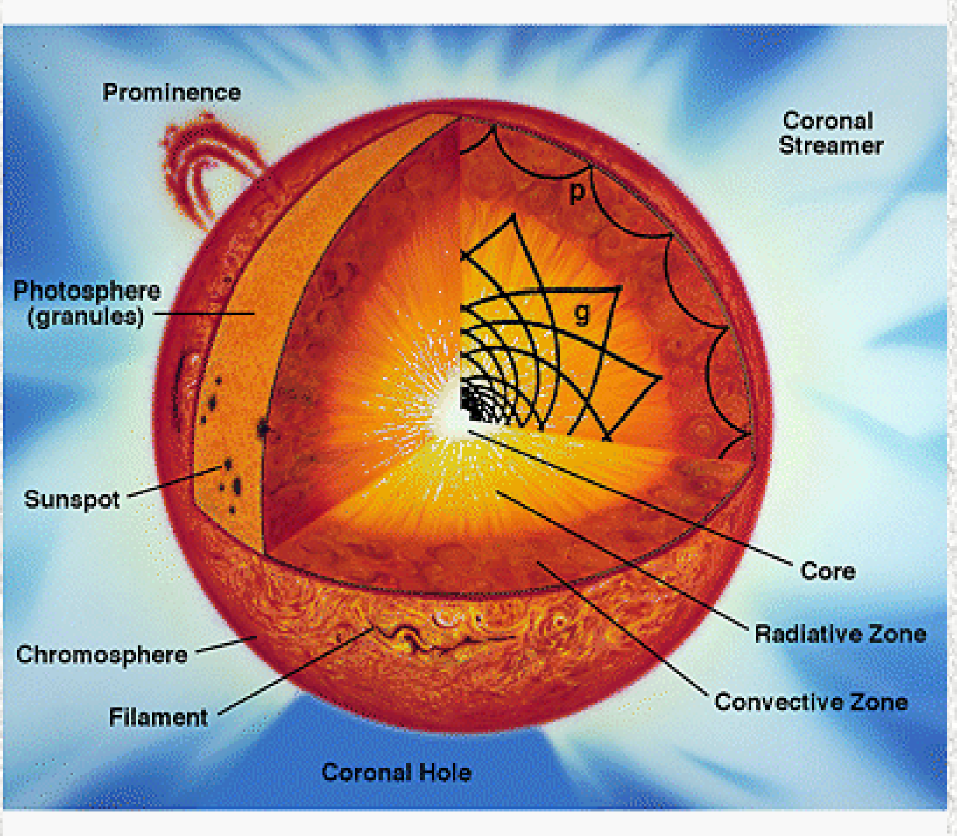
Magnetic Field as Tracers of the Solar Cycle





- Number of sunspots observed on the Sun varies cyclically
- However, there are large fluctuations in the amplitude
- Equatorward migration of sunspots
- Poleward migration of surface radial field
- Polar field reversal at time of sunspot maximum
- Both have an average periodicity of 11 years

Window to the Solar Interior: Plasma Motions



- Interior temperature exceeds a million degrees
- Matter exists in the plasma state (highly ionized)
- Convection zone has both small-scale turbulent motions and large-scale structured flows
- We are dealing with the dynamics of magnetized plasmas…..

Some Issues in MHD: The Induction Equation and Flux Freezing

- Governing equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

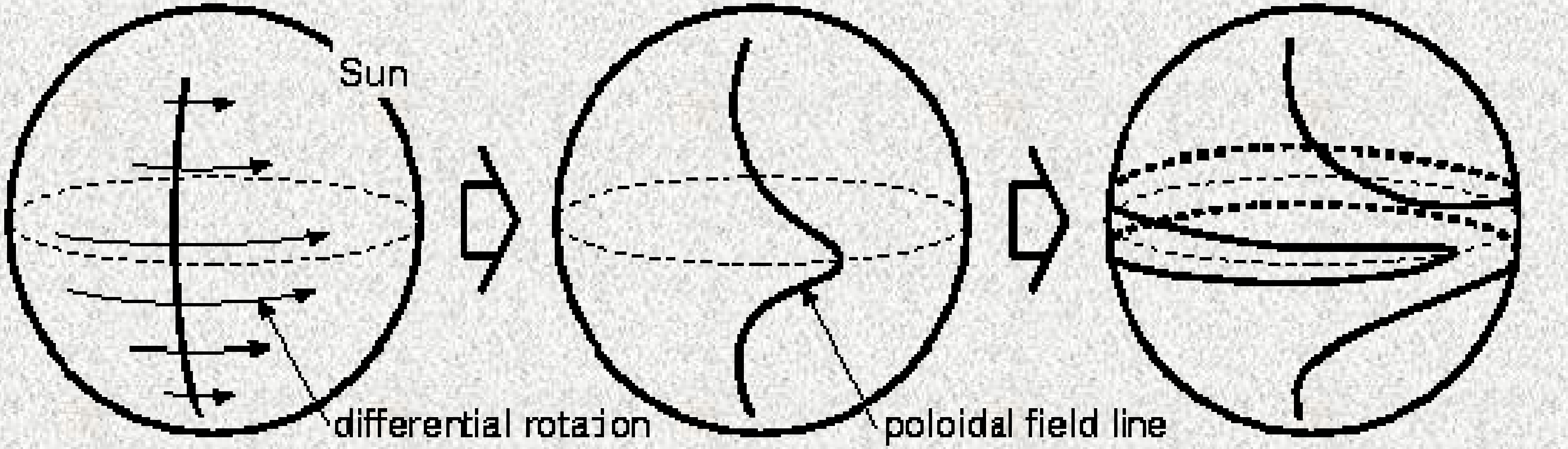
- Magnetic Reynolds Number:

$$R_m = \frac{VB / L}{\eta B / L^2} = \frac{VL}{\eta}$$

- In Astrophysical systems, R_M usually high, magnetic fields move with plasma – flux is frozen (Alfven, 1942)

- Magnetoconvection (Chandrasekhar 1952, Weiss 1981) – convective region gets separated into non-magnetic and magnetic space – the latter constitutes flux tubes

Historical Development – Toroidal Field Generation (Omega Effect)



Poloidal field

Toroidal Field

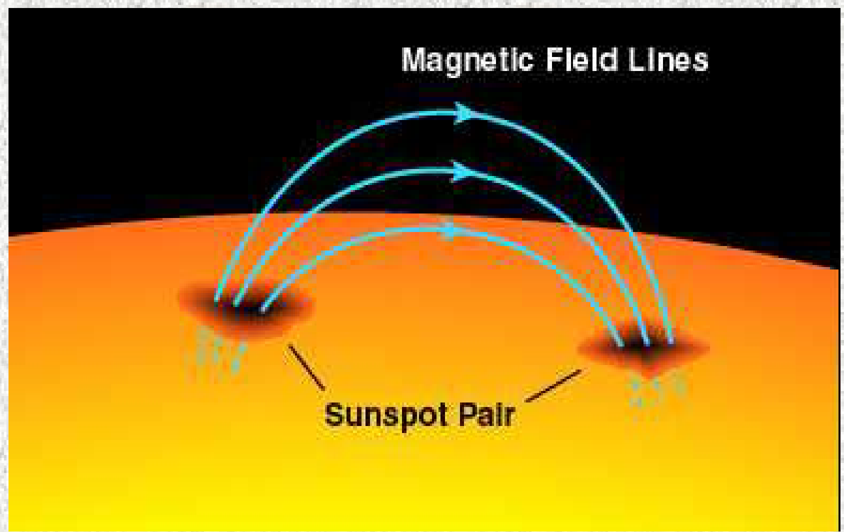
- Differential rotation will stretch a pre-existing poloidal field in the direction of rotation – creating a toroidal component (Parker 1955)

Historical Development – Magnetic Buoyancy and Sunspot Formation

- Stability of Toroidal Flux Tubes – Magnetic Buoyancy (Parker 1955)

$$P_E = P_I + \frac{B^2}{8\pi}$$

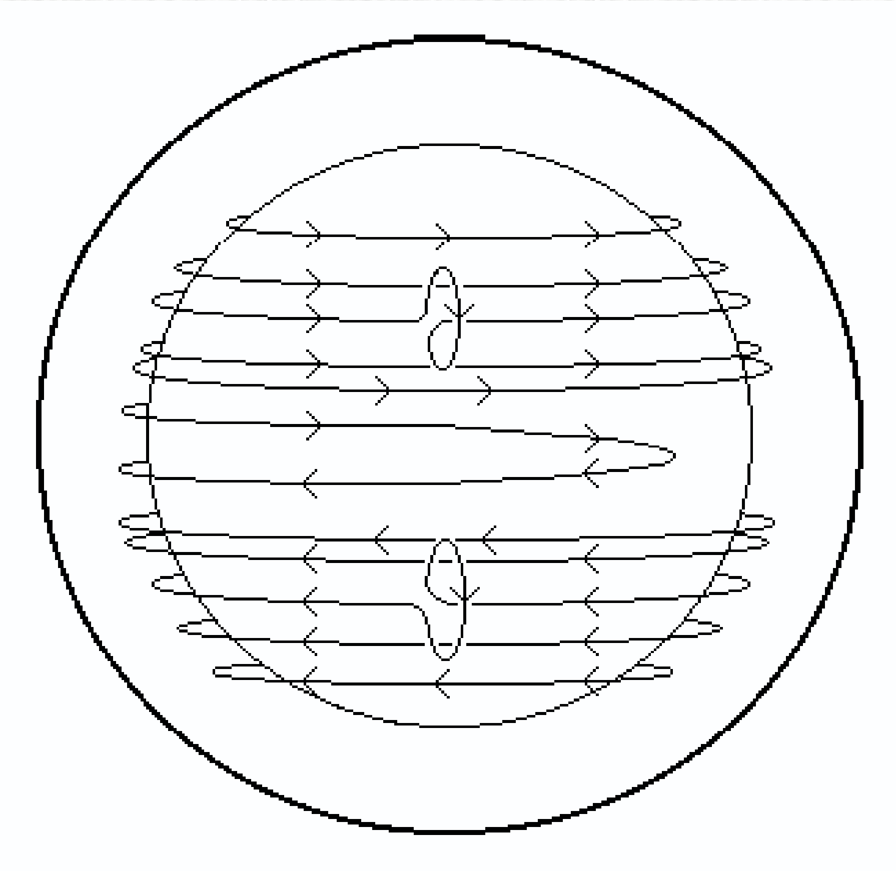
$$\rho_{\text{Internal}} < \rho_{\text{External}}$$



- Buoyant eruption, Coriolis force imparts tilts

- Where is the toroidal field stored and amplified ?
  - Convection zone susceptible to buoyancy, ruled out (Parker 1975)
  - In the overshoot layer, at base of convection zone (Spiegel & Weiss 1980; van Ballegooijen 1982)

Historical Development – Poloidal Field Generation – The MF α-effect



- Small scale helical convection – Mean-Field α-effect (Parker 1955)
- Buoyantly rising toroidal field is twisted by helical turbulent convection, creating loops in the poloidal plane
- The small-scale loops diffuse to generate a large-scale poloidal field

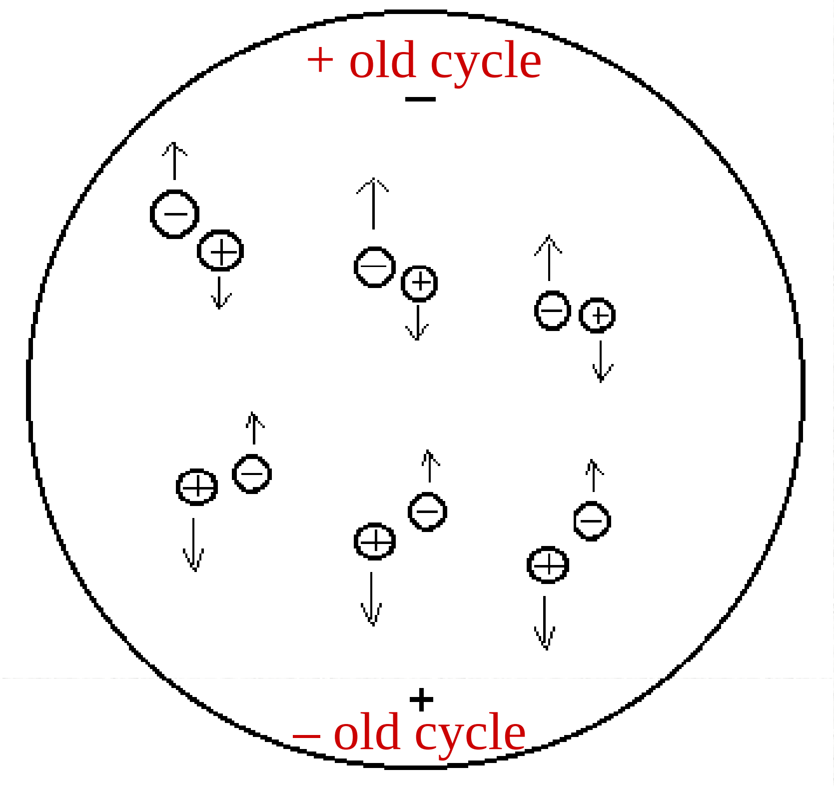
Last Two Decade – Flux Tube Dynamics and a Crisis in Dynamo Theory

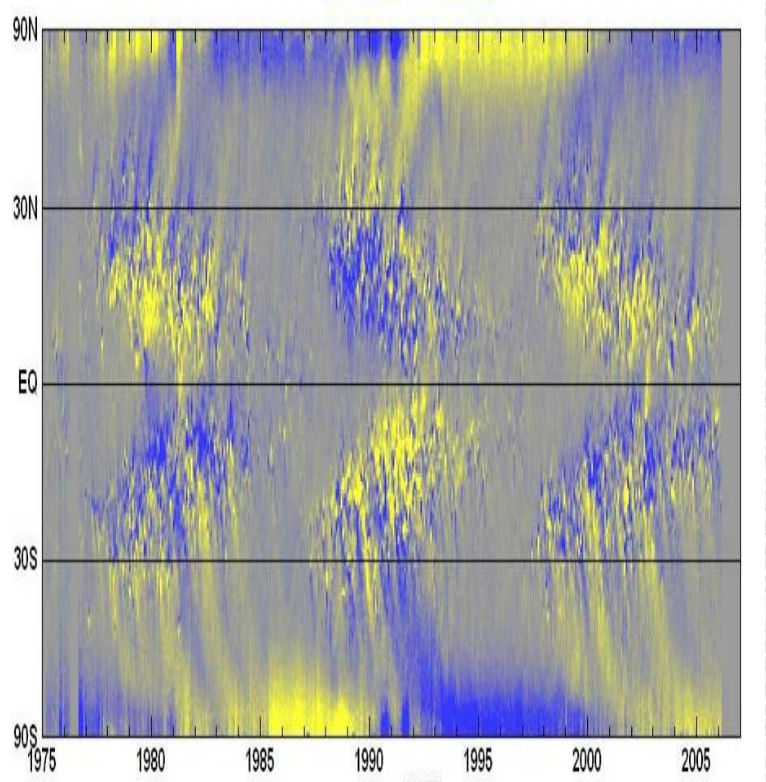
- Simulations of flux tube dynamics (Choudhuri & Gilman 1987; D'Silva & Choudhuri, 1993; Fan, Fisher & DeLuca 1993) and flux storage (Moreno-Insertis, Schüssler & Ferriz-Mas 1992) pointed out flux tube strength at base of SCZ must be **$\approx 10^5$ G**

- Equipartition field strength in convection zone **$\approx 10^4$ G**

- Small-scale helical convection will get quenched – alternative ideas for poloidal field generation necessary

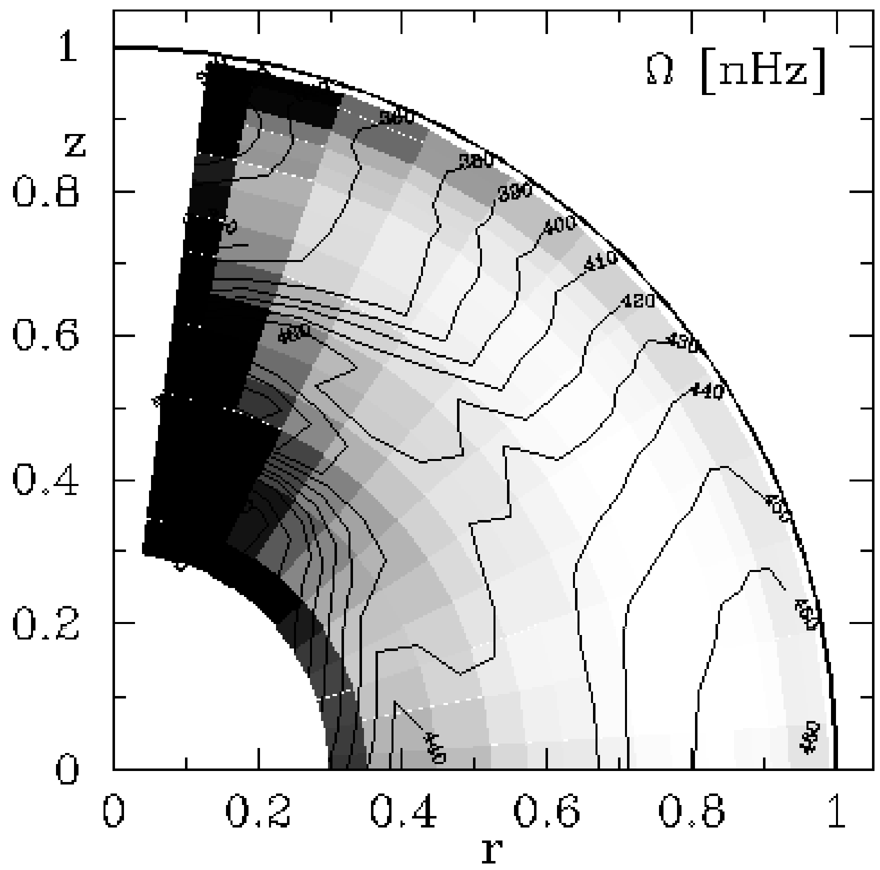
<u>The Modern Era: Revival of the Babcock-Leighton Idea</u>

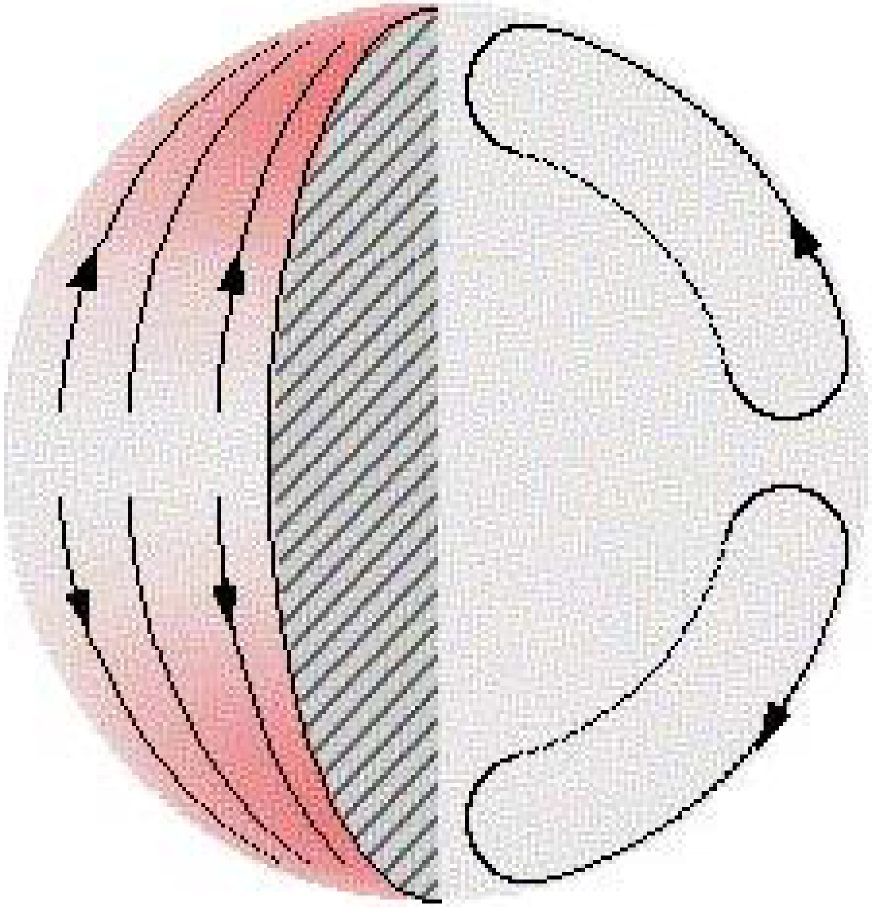




- Babcock (1961) & Leighton (1969) idea – decay of tilted bipolar sunspots – distinct from the MF α-effect – <u>and is observed</u>
- Numerous models have been constructed based on the BL idea (Choudhuri et al. 1995, Durney 1997, Dikpati & Charbonneau 1999, Nandy & Choudhuri 2001, 2002, Chatterjee et al. 2004…)

The Modern Era: Large Scale Internal Flows from Helioseismology





- Differential rotation in the interior determined from helioseismology, strongest rotational shear in tachocline at SCZ base
- Poleward meridional circulation observed in the outer 15%, mass conservation requires counterflow – possibly near SCZ base

Building an Axisymmetric Kinematic αΩ Dynamo Model

- Axisymmetric Magnetic Fields:

$$\boldsymbol{B} = B\boldsymbol{e}_{\phi} + \nabla \times (A\boldsymbol{e}_{\phi})$$

- Axisymmetric Velocity Fields:

$$\boldsymbol{v} = \boldsymbol{v}_p + r \sin \theta \Omega \boldsymbol{e}_{\phi}$$

- Plug these into the Induction Equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

to obtain…..

Building a Dynamo Model: The αΩ Dynamo Equations

- Toroidal field evolution:

$$\frac{\partial B_\phi}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r v_r B_\phi\right) + \frac{\partial}{\partial \theta}\left(v_\theta B_\phi\right)\right]$$

$$= \eta\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right)B_\phi + r\sin\theta\left(B_P . \nabla\right)\Omega - \nabla\eta \times \left(\nabla \times B_\phi\right)$$

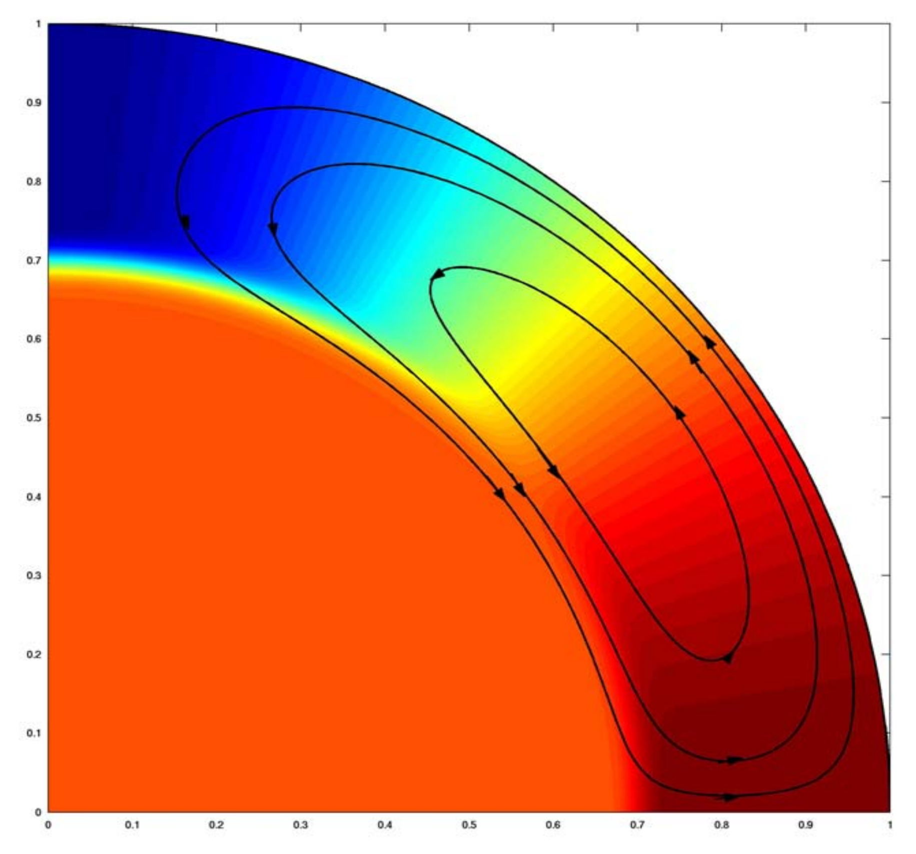
- Poloidal field evolution:

$$\frac{\partial A}{\partial t} + \frac{1}{r\sin\theta}\left(v_P . \nabla\right)\left(r\sin\theta A\right) = \eta\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right)A + S_\alpha$$

- Where the BL alpha effect $S_\alpha = \alpha B_\phi$ acts on erupted toroidal fields

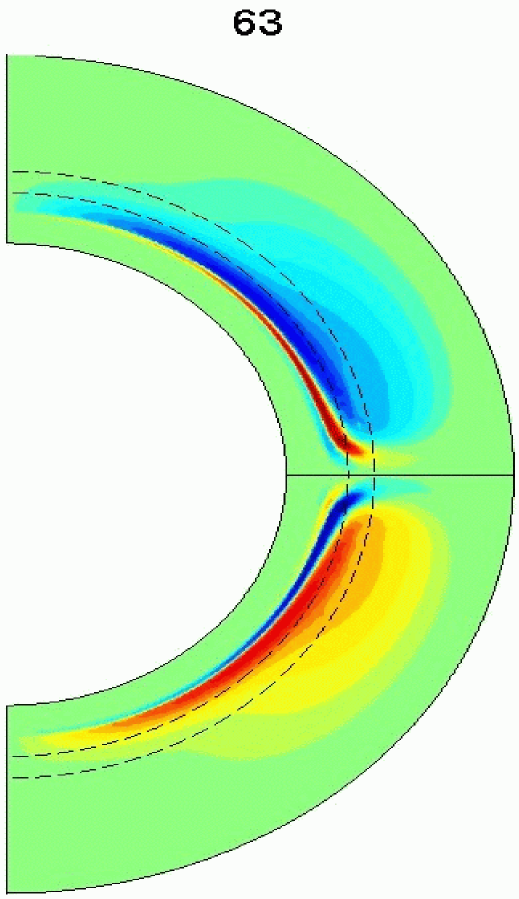
- Often, the alpha-term includes quenching, to limit field amplitude

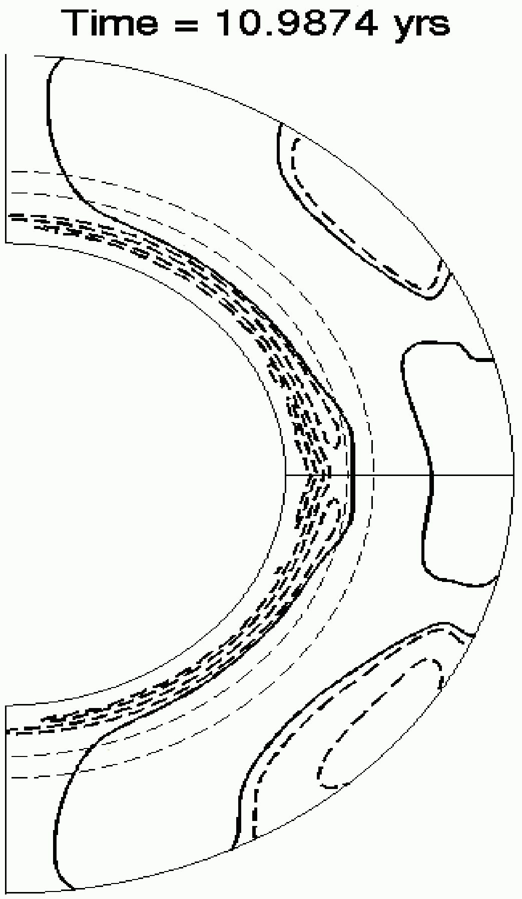
Building a Dynamo Model: Typical Model Inputs



- Analytic fit to helioseismically observed differential rotation
- Single-cell meridional flow that matches near surface observations
- A depth dependent diffusivity profile
- A functional form for the BL α-effect (confined to near surface layers)
- Magnetic buoyancy algorithm (transports fields to surface layers)

Solar Cycle Simulations

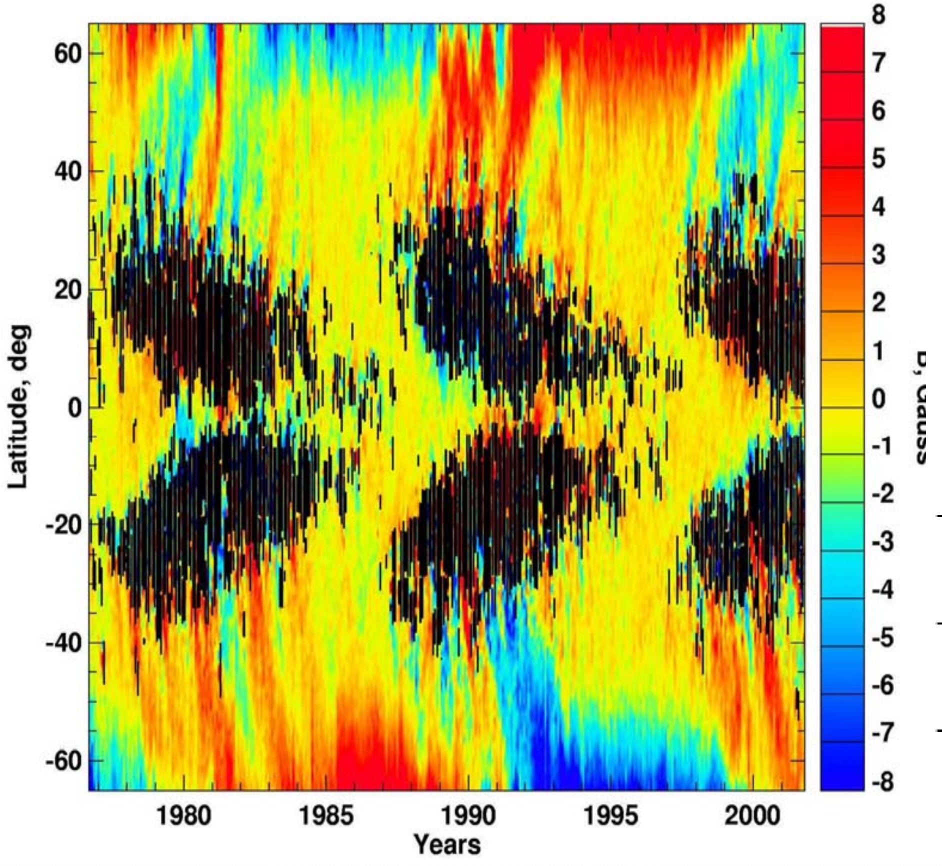




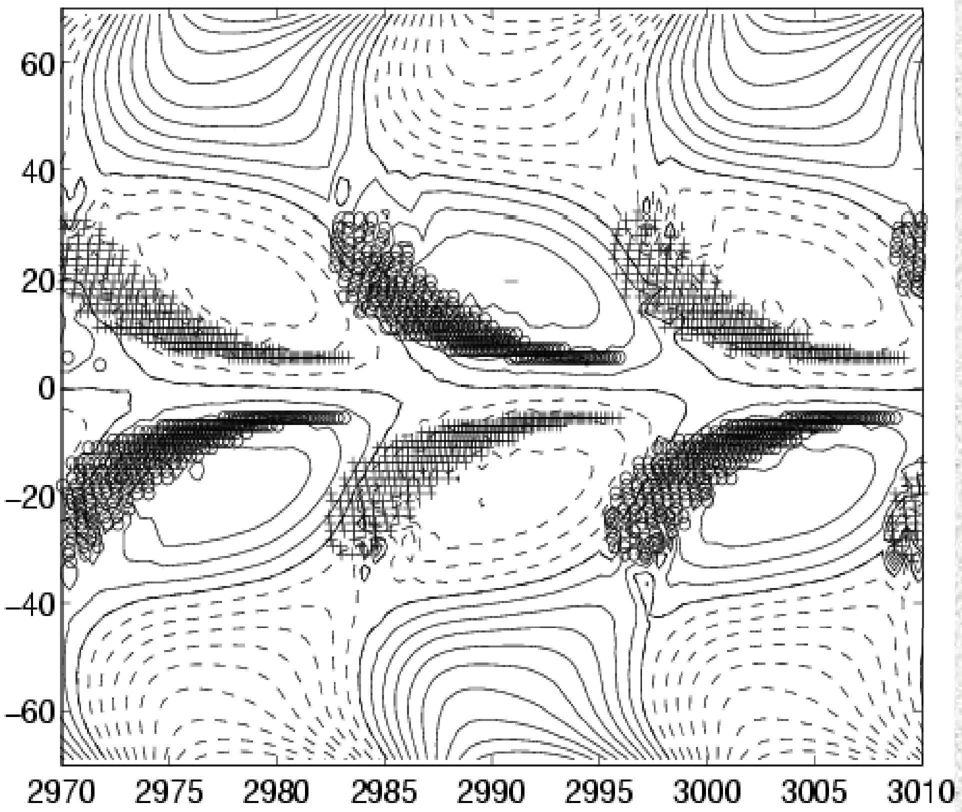
Toroidal Field Evolution

Poloidal Field Evolution

Solar Cycle Simulations

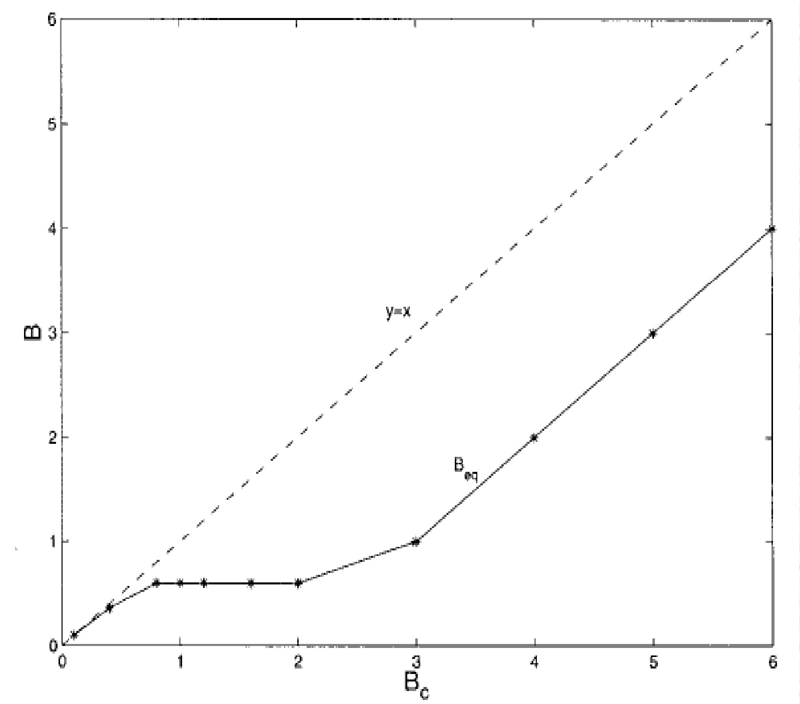


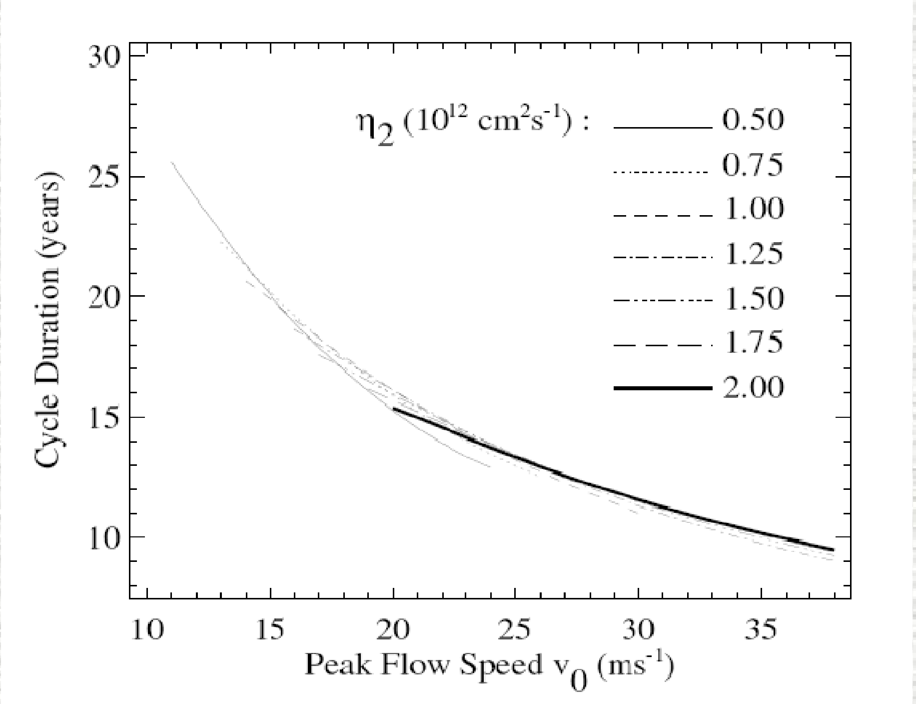
Observations



Simulations

What Determines Dynamo Amplitude and Period in these Models?





Amplitude (If α-effect source term and quenching field is fixed):

- Primary constraint: Critical threshold for buoyancy (B_c)
- Therefore peak toroidal field at base of SCZ ~ B_c ~ 10^5 G

Period:

- The speed of the meridional circulation sets the dynamo cycle period
- Note: Period is governed by slowest process in the dynamo chain

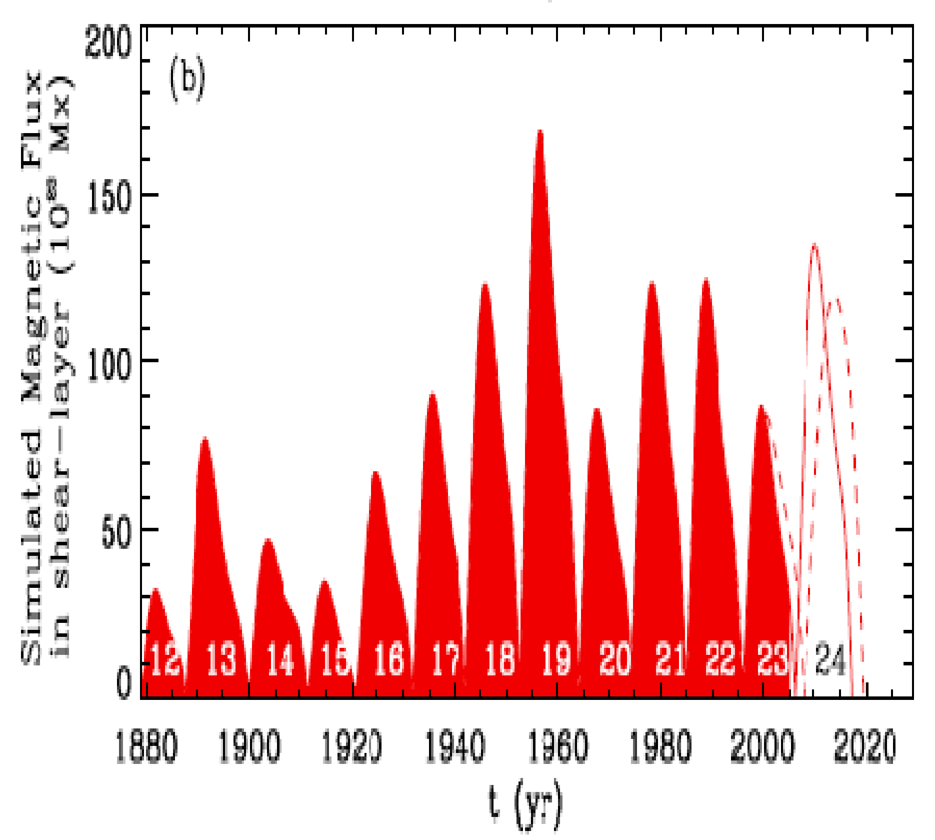
And the Rosy Picture is…

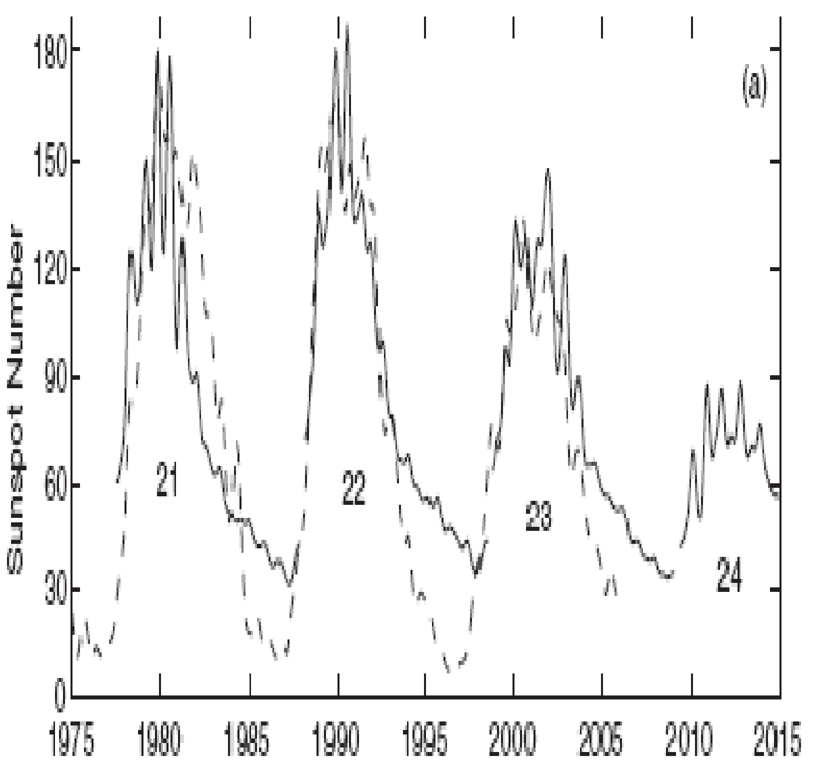
- Using observed large-scale flows (kinematic regime), we can reproduce the observed large-scale magnetic field evolution reasonably well

- Then perhaps we are getting some aspects of the physics right???

- So lets make some predictions for the next cycle…

Fluctuations, Memory & Solar Cycle Predictions

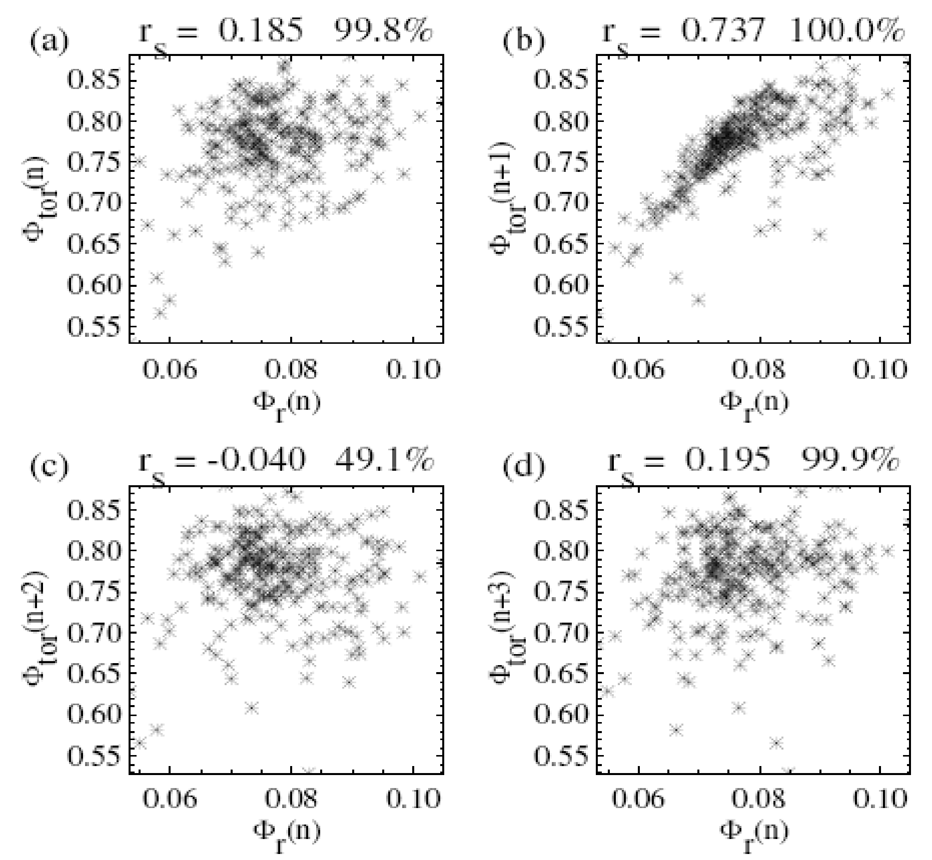




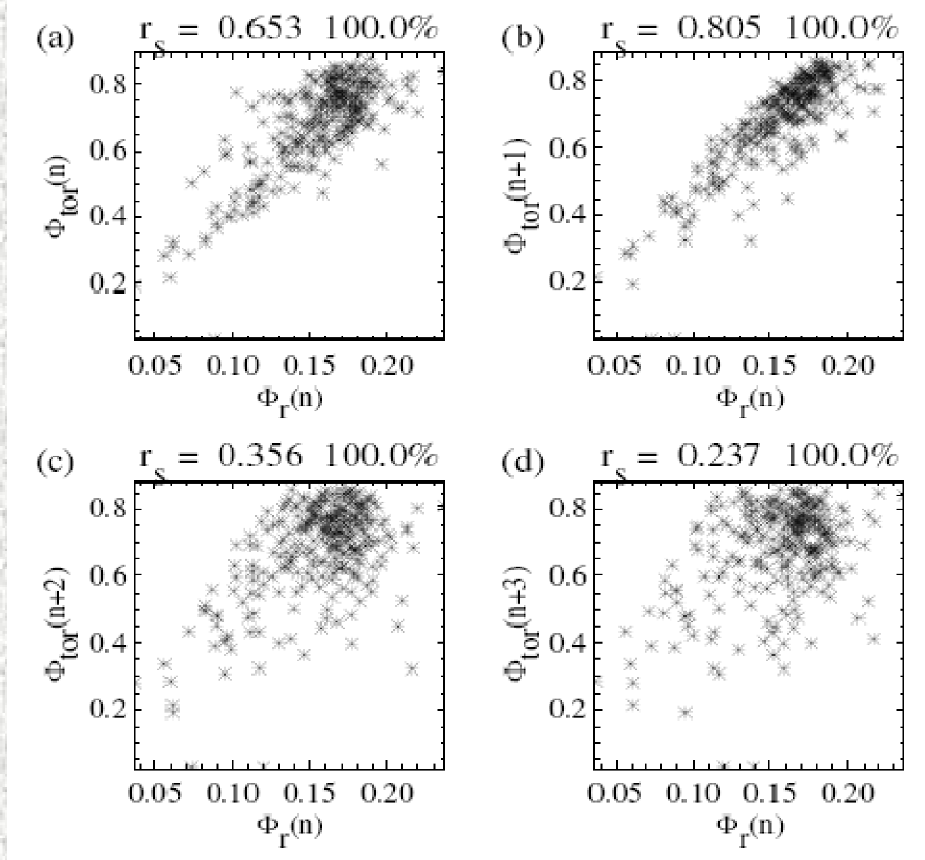
- Flux transport takes finite time = time-delay = memory mechanism (Charbonneau & Dikpati 2000; Wilmot-Smith et al. 2006)
- Dikpati et al. (2006) predict a very strong solar cycle 24, Choudhuri et al. (2007) predict a much weaker solar cycle 24!

Solar Cycle Predictions: What Leads to Different Predictions?

(Stochastically Forced Model: Yeates, Nandy & Mackay 2008)



(Diffusion Dominated Flux Transport)

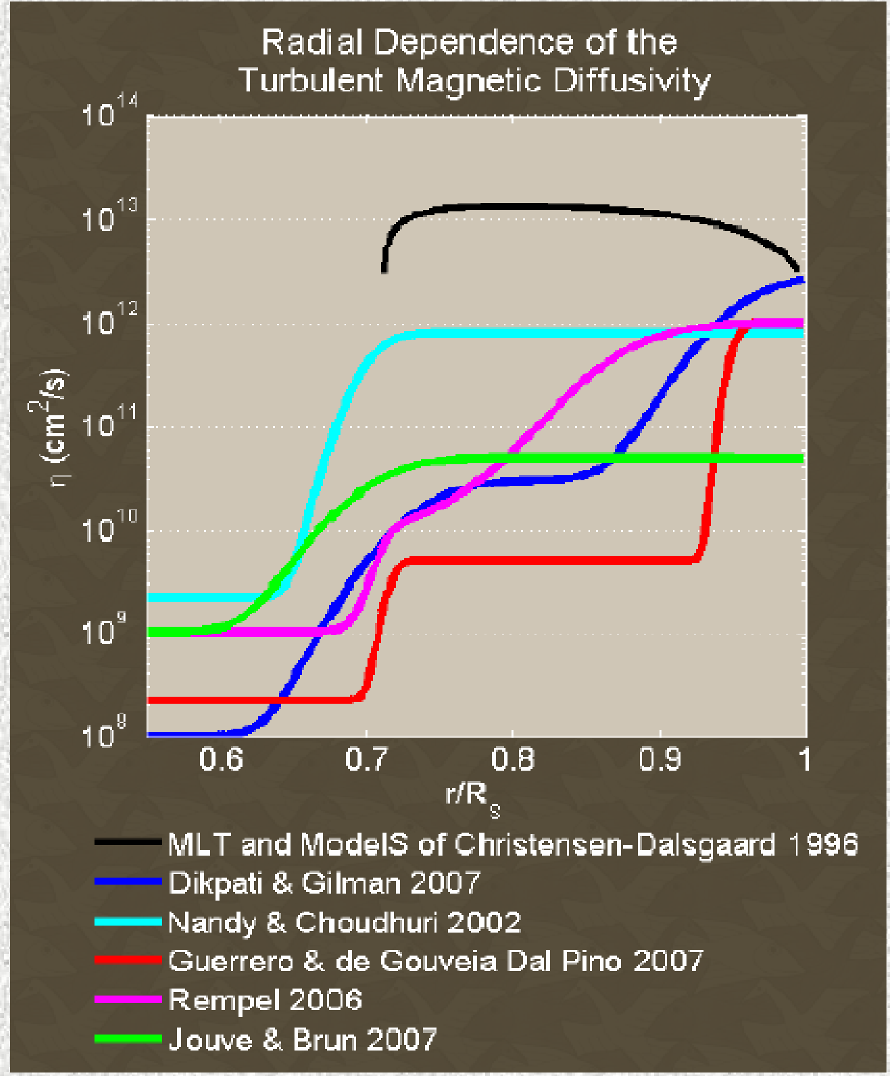


(Advection Dominated Flux Transport)

- Memory of fluctuations different in diffusive and advective regimes
- Diffusive flux transport short-circuits advective flux transport
- Differing memory leads to different predictions for the next cycle

Outstanding Issues: Parameterization of Turbulent Diffusivity

- Mixing-length theory suggests much higher turbulent diffusivity values (10^{12-13}cm^2/s) than currently used in the so-called "flux-transport" solar dynamo models (10^{10-11}cm^2/s). Such high values will invariably make the SCZ diffusion dominated



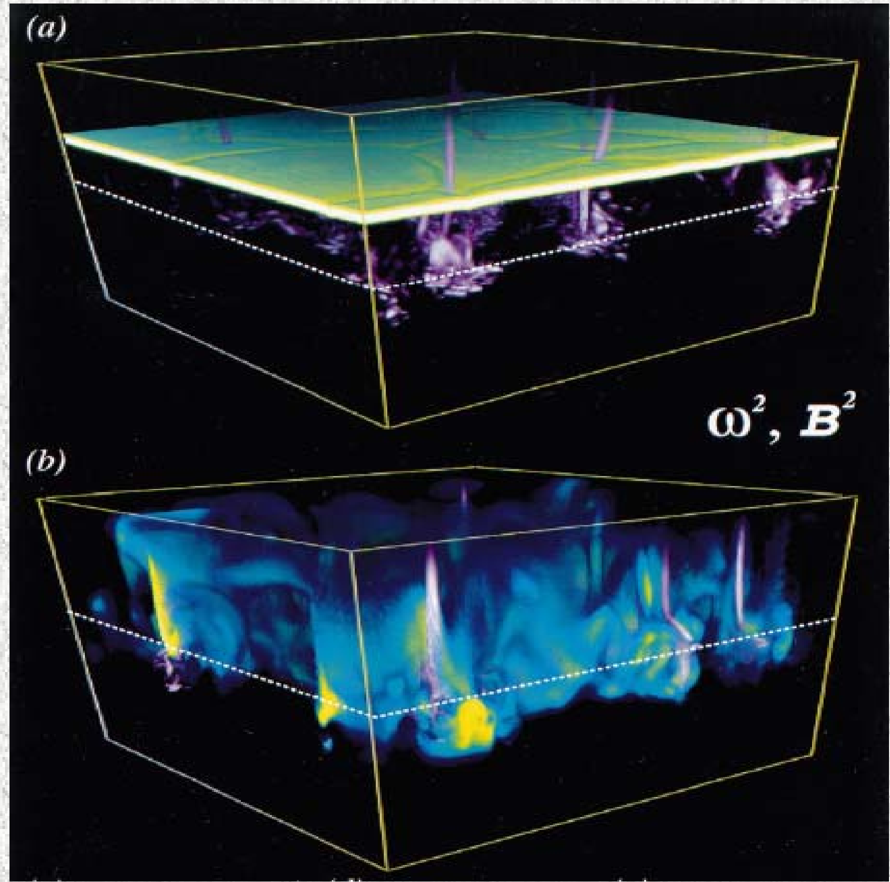
High Diffusivity bad for FT Dynamos

- Short-circuits meridional flow
- Reduces cycle period
- Shortens cycle memory
- Difficult for flux storage

Possible Resolutions

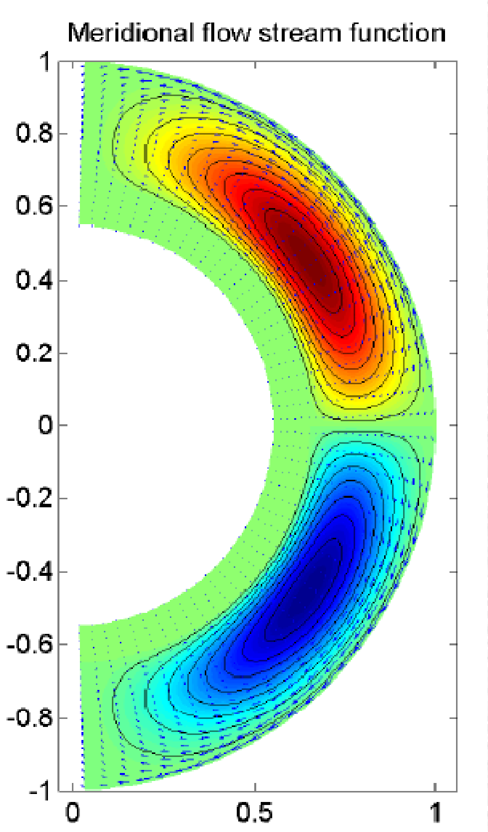
- Quenching of turbulent diffusivity
- Downward flux pumping

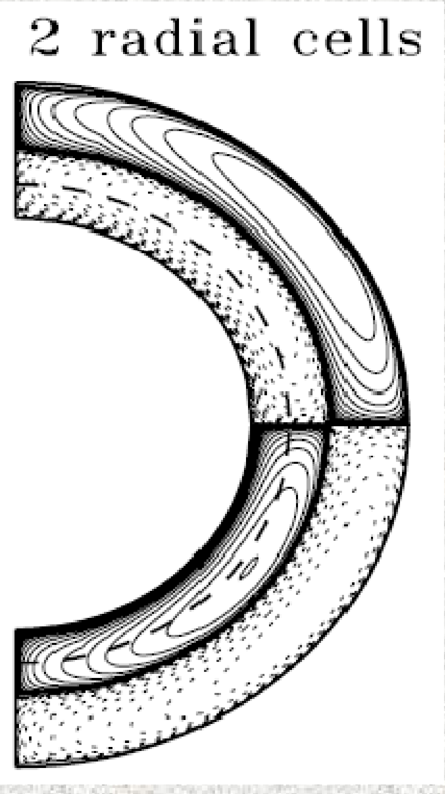
Outstanding Issues: Inclusion of Turbulent Flux Pumping

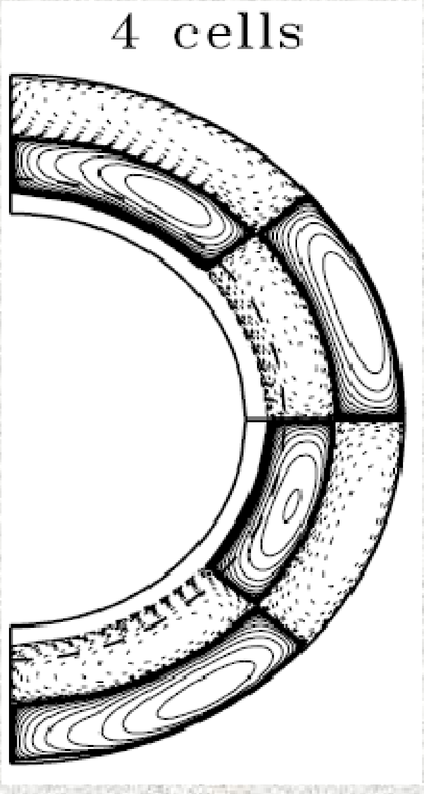


- Preferential downward pumping of magnetic flux, in the presence of rotating, stratified convection – usually ignored in kinematic dynamos
- Suggests typical downward velocity ~ 10 m/s (Tobias et al. 2001)
- Will affect flux transport, flux-storage, cycle-period (Guererro & Dal Pino 2008) and plausibly solar cycle memory

Outstanding Issues: Meridional Circulation Profile

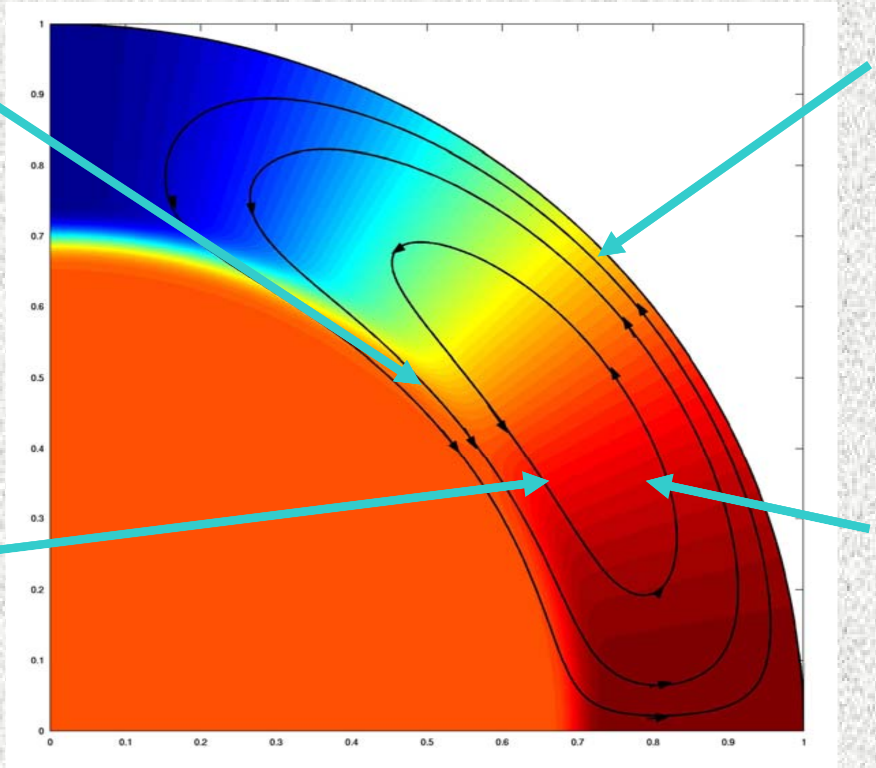






- Meridional Circulation: One cell? Multi-cellular? Intermittent?
- Full MHD numerical simulations often generate multi-cellular and variable flow profiles (Miesch et al. 2000, Browning et al. 2006)
- Multi-cellular flows profoundly alter magnetic butterfly diagrams and dynamo-periods (Jouve & Brun 2007); will affect flux transport

Outstanding Issues: Which α-effect and Where?



Differential rotation
instability (tachocline)
[Dikpati & Gilman 2001]

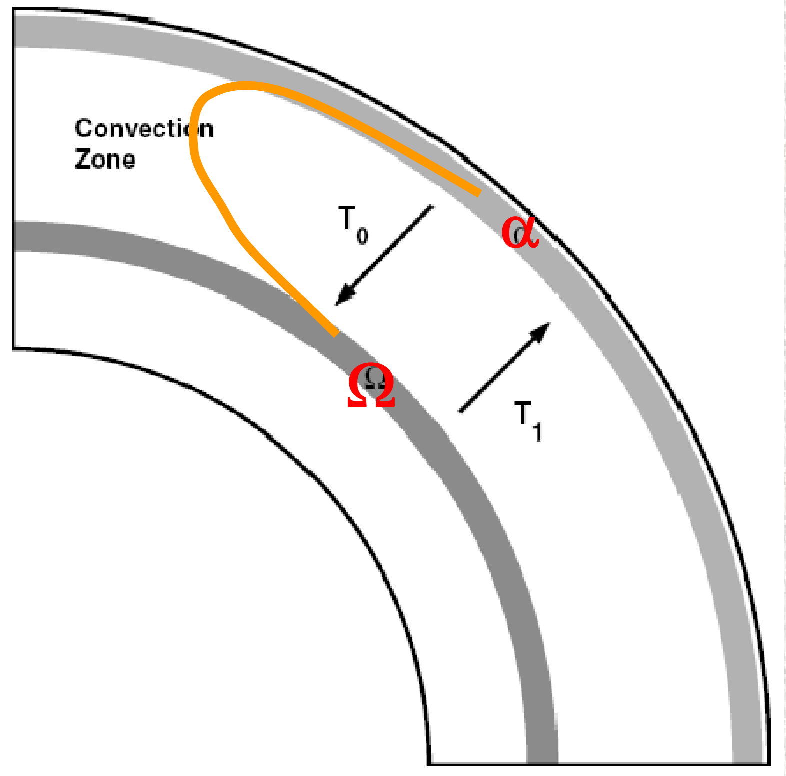
Babcock-Leighton
(Near-surface)
[Babcock 1961,
Leighton 1968]

Buoyancy Instability
(Base of SCZ)
[Ferriz-Mas et al 1994]

Mean-Field α-effect
(SCZ)
[Parker 1955]

- Is a combination of α-effects working together?
- If yes, which is dominant?
- The fact that the solar cycle recovered from the Maunder minimum requires the presence of an α-effect that can work on weak fields

The Bottom-line: A Story of (Communication) Timescales



Flux Transport Timescales

- Meridional Flow (20 m/s)
  τ_v = 10 yrs
- Turbulent Diffusion (5 x10^{12} cm^2/s)
  τ_η = 2.8 yrs
- Turbulent Pumping (v =10 m/s)
  τ_{pumping} = 0.67 yrs

- Relative locations of the two source-layers (Ω and α-effects)?
  - depends on what kind of α-effect is the main poloidal field source
- Which physical process defines T_0?
  - Flux-transport dynamics, cycle-period, memory (and by extension any predictions) will depend on that
- Kinematic dynamos have to confront these issues

Conclusions

- We have learnt much in the last 50 years since Eugene Parker first presented a kinematic dynamo model of the solar cycle in 1955

- However, we are also realizing that there is much more that we do not know, specifically about the interplay between individual physical processes that together constitute the dynamo mechanism

- But we are beginning to understand and address those deficiencies….