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Probability distribution functions to represent the solar surface magnetic fields

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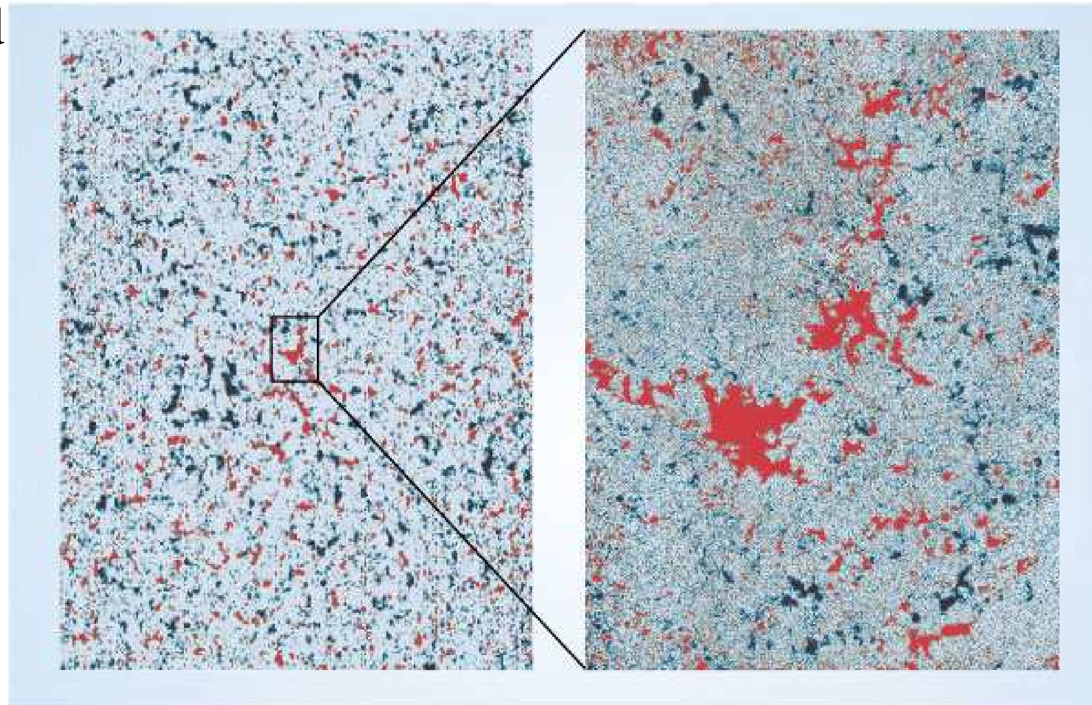
Observational motivation for the Prob. Dist^n Fn^s (PDFs)

★ The magnetic fields in the so called "voids" are shown to be actually turbulent using modeling of Hanle effect (Trujillo Bueno et al. 2004)

★ Turbulent eddies are shown to be formed by magneto-convection (see eg., Stein & Nordlund 2006), through numerical simulations.



Magnetogram showing opposite polarity (red and blue patches) regions separated by grey voids ("turbulent" fields). Courtesy : Stenflo (2004)

★ The size distribution of the eddies are $\ll$ spatial resolution scale of the present day instruments (0.2"). $\therefore$ Indirect proof for the "existence of eddies" is necessary.

★ Observed Stokes profiles are 'always' averaged quantities over space, time and magnetic field.

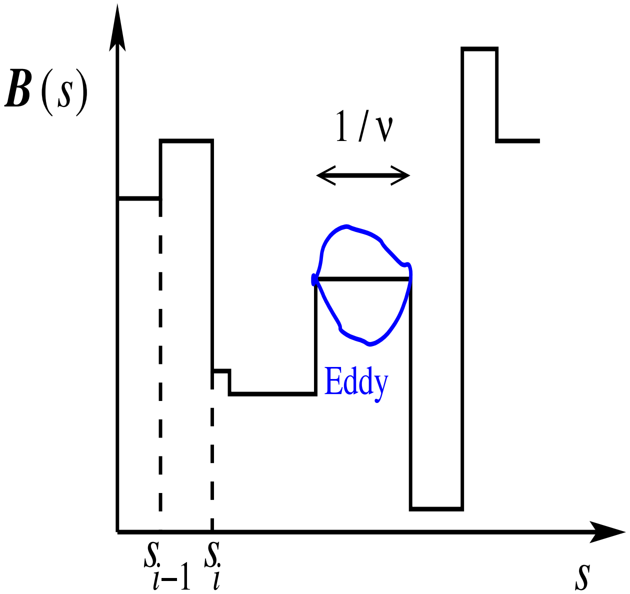
★ The purpose of modeling or simulation is to "extract the scale of turbulence" (size distribution of eddies).

Magnetic turbulence/ historical background

- ★ The effect of micro-turbulent magnetic field on Zeeman absorption coefficients was considered long back by Dolginov & Pavolv (1972), and Domke & Pavlov (1979).
- ★ The *first formulation* of Zeeman line radiative transfer in the micro, and also macro-turbulent limits was due to Stenflo (1971, 1973).
- ★ MISMA hypothesis of Sánchez Almeida et al. (1996) is based on the idea of 'micro-turbulence'. MISMA is a useful tool in magnetic field diagnostics.
- ★ A theory of Rad. Transf. in meso-turbulent regime :- Landi Degl'Innocenti (1994).
- ★ A more general theory of Zeeman turbulence and Rad. Transf. is developed in Frisch, Sampoorna, & Nagendra (2006); also Sampoorna et al. (2007, 2008a, 2008b).
- ★ Recently Carroll & Staude (2003, 2005, 2006), and Carroll & Kopf (2007), have taken an approach similar to ours, and tried to model photospheric turbulent fields.
- ★ It is well known that "Probability Distribution Functions (PDFs)" can be used to mathematically represent the randomness of the field variables (B, θ_B, ϕ_B).
- ★ In this talk I highlight the importance of PDFs in Zeeman line profile computations.

A model for the random fields

★ We express fluctuations through a Kubo-Anderson process (KAP). It describes the magnetic atmosphere in terms of "eddies".

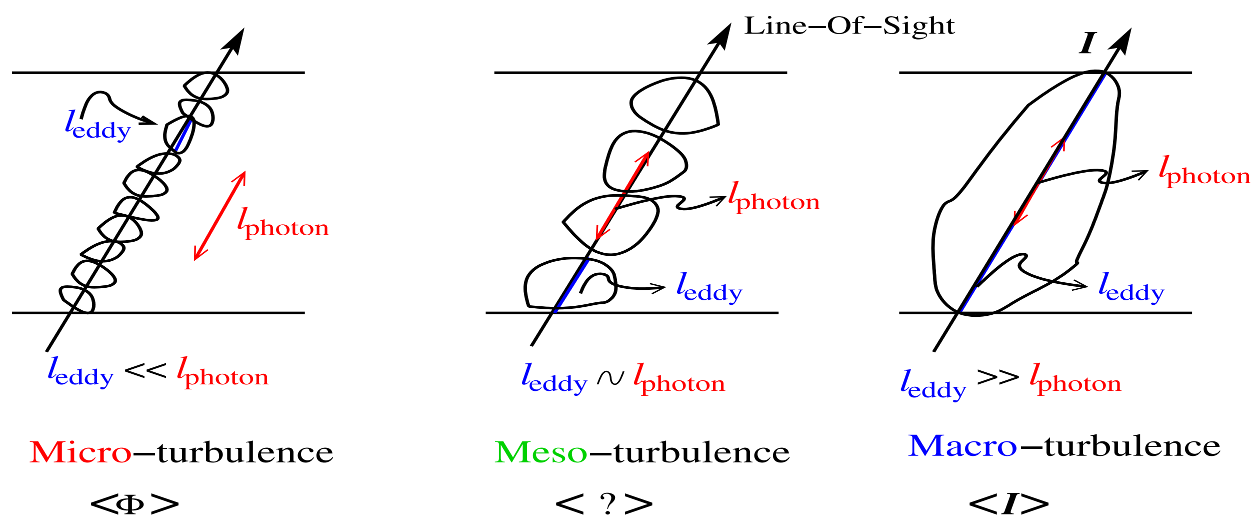


- In each eddy, $\boldsymbol{B}$ remains constant & takes "random values" according to a PDF.
- Further, $\boldsymbol{B}$ "jumps" at the boundaries of the eddies.
- s_i are jumping points distributed according to Poisson law.
- If ν = the number of jumps/unit optical depth, then "correlation length" $\simeq$ mean eddy size $(l_{\text{eddy}}) = 1/\nu$.

2 length scales are involved

1. Rad. transfer length scale
$\simeq$ photon mean free path

2. Turb. length scale
$\simeq$ mean eddy size itself



- $\nu \to 0$: macro ; $\nu \to \infty$: micro ; $\langle\ \rangle$ denote the "mean"

Mean residual emergent Stokes parameters

★ We consider Residual emergent Stokes parameters :

$$\boldsymbol{r}(\tau_c = 0) = \frac{1}{C_1}[\boldsymbol{I}_c(0) - \boldsymbol{I}(0)]; \quad \boldsymbol{I}_c(0) = (C_0 + C_1)\boldsymbol{U}.$$

★ Using KAP in a ME model, we compute the mean values in case of meso-turbulence

$$\langle \boldsymbol{r}(0)\rangle_{\mathrm{KAP}} = (1+\nu)\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\mathbf{\Phi}\right)\left[\mathbf{E} + \nu\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\mathbf{\Phi}\right)\right]^{-1}\boldsymbol{U}\,,$$

$\beta = k_{\mathrm{L}}/k_{\mathrm{c}} =$ line strength, $\mathbf{\Phi} =$ absorption matrix, $(1/\nu) =$ correlation length, and

$$\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\mathbf{\Phi}\right) = \left\langle \frac{\beta}{1+\nu}\mathbf{\Phi}\left[\mathbf{E} + \frac{\beta}{1+\nu}\mathbf{\Phi}\right]^{-1}\right\rangle_{\mathrm{PDF}},$$

★ Rad. Transf. in turbulent media is computationally difficult because of $\langle\ \rangle_{\mathrm{PDF}}$!

★ Analytic simplification of the problem is possible only in micro-turbulent limit.

$$\langle \boldsymbol{r}(0)\rangle_{\mathrm{micro}} = \beta\,\underbrace{\langle\mathbf{\Phi}\rangle}[\mathbf{E} + \beta\langle\mathbf{\Phi}\rangle]^{-1}\boldsymbol{U}\,, \quad \langle \boldsymbol{r}(0)\rangle_{\mathrm{macro}} = \underbrace{\langle\beta\mathbf{\Phi}[\mathbf{E} + \beta\mathbf{\Phi}]^{-1}\rangle}\boldsymbol{U}\,.$$

Discussion on PDFs in this Talk

I. Scalar PDFs $\Rightarrow$ only field strength fluctuates
(eg., Voigt, and stretched exponential)

$\longrightarrow$ Symmetric PDFs (zero NET flux)

$\longrightarrow$ Asymmetric PDFs (non-zero net flux)

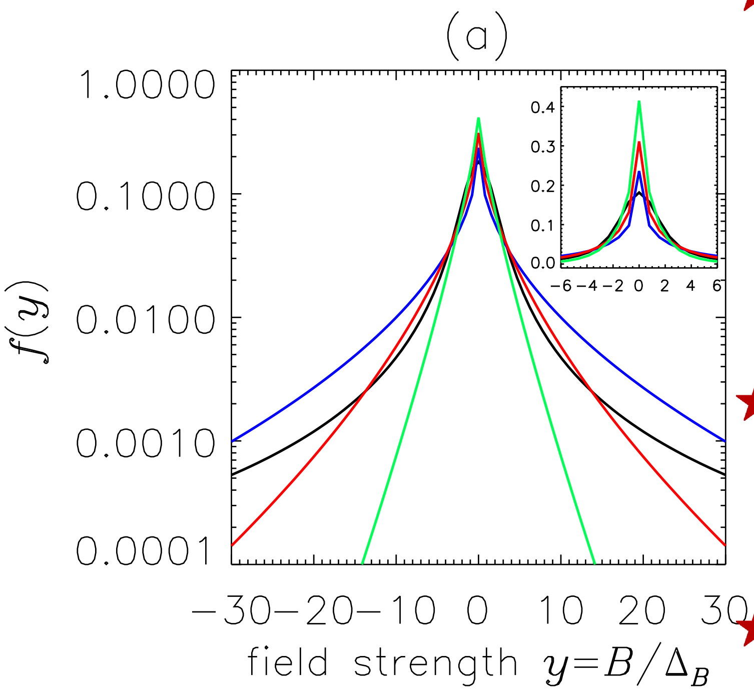
II. Angular PDFs $\Rightarrow$ Fluctuations of orientation only
(eg., Power law)

III. Vector PDFs $\Rightarrow$ All the variables (B, θ_B, ϕ_B) fluctuate

$\longrightarrow$ combination of Voigt and Power law

$\longrightarrow$ combination of Stretched Exp. and Power law

Symmetric scalar PDFs (zero net flux)



Voigt and Stretched exponential PDFs

Black line: $f(y) = P_{\rm Voigt}(y, a_B = 1.5)$

Blue line: $f(y) = P_{\rm SE}(y)$ with $k = 0.5$

Red line: $f(y) = P_{\rm SE}(y)$ with $k = 0.6$

Green line: $P_{\rm SE}(y)$ with $k = 0.8$

Inset shows a magnified view of the core region of the PDF.

★ Stenflo & Holzreuter (2003) deduce a Voigt like PDF from high-resolution La Palma & MDI magnetograms :

$$P_{\rm Voigt}\left(\frac{B}{\Delta_B}, a_B\right) = \frac{a_B}{\pi^{3/2}} \int_{-\infty}^{+\infty} \frac{e^{-(B_1/\Delta_B)^2}}{[(B-B_1)/\Delta_B]^2 + a_B^2} \frac{dB_1}{\Delta_B},$$

$\Delta_B \rightarrow$ magnetic width, $a_B \rightarrow$ magnetic damping parameter.

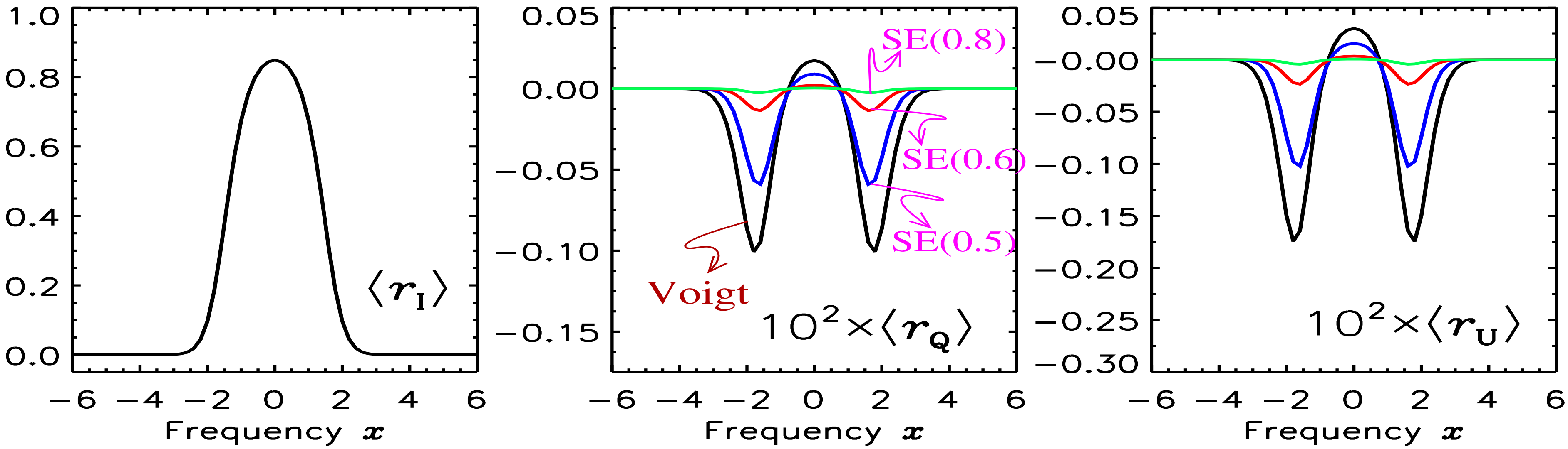
★ They show that such a PDF, with $\Delta_B = 6\,$G and $a_B = 1.5$ best fits the La Palma magnetogram data.

★ Stein & Nordlund (2006) propose a Stretched Exponential PDF, derived from magneto-convection simulations :

$$P_{\rm SE}\left(\frac{B}{\Delta_B}\right) = \frac{k}{2\Gamma(1/k)}\, e^{-|B/\Delta_B|^k},$$

$k \rightarrow$ is the stretching parameter : $0 \leq k \leq 1$.

Mean solutions computed using Symmetric PDFs

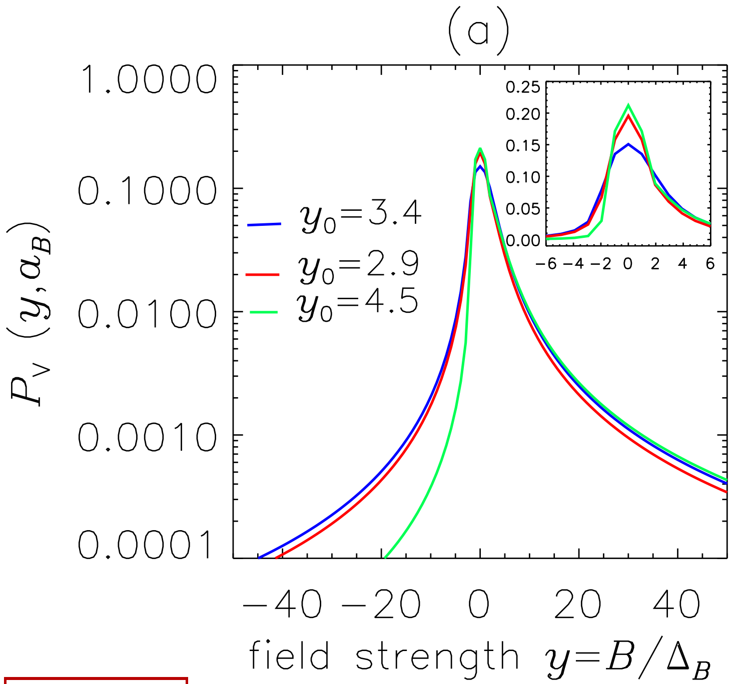


Model $\Delta_B/B_{\rm D} = 0.0056$, where $1/B_{\rm D} = ge/(4\pi mc\Delta\nu_{\rm D})$; damping parameter $a = 0$; line strength $\beta = 10$; field orientation $(\theta, \phi) = (60^\circ, 30^\circ)$; correlation length $(1/\nu = 1/5)$.

Results

1. $\langle r_{\rm I} \rangle$ is insensitive to the type of PDF, because rms fluctuations $= \Delta_B/B_{\rm D} \ll 1$.
2. $\langle r_{\rm Q,U} \rangle$ are very sensitive to the type of PDF $\Rightarrow \therefore$ diagnostically more useful
3. Voigt PDF :- give largest $\langle r_{\rm Q,U} \rangle$, $\because$ strong field tails of $P_{\rm Voigt}$ are large.
4. $P_{\rm SE}(y)$:- As k increases, $\langle r_{\rm Q,U} \rangle$ decreases $\because$ strong field tails of $P_{\rm SE}(y) \to 0$.

Asymmetric Voigt PDF (non-zero NET flux)



★ Unbalanced NET fields can be generated only by Asymmetric PDFs. Symmetric PDFs have zero net flux.

★ Asymmetric PDFs are constructed as follows : use different a_B values for $+$ve and $-$ve polarities ; and keep the Gaussian core symmetric.

★ Asymmetric PDFs have a non-zero mean field y_0.

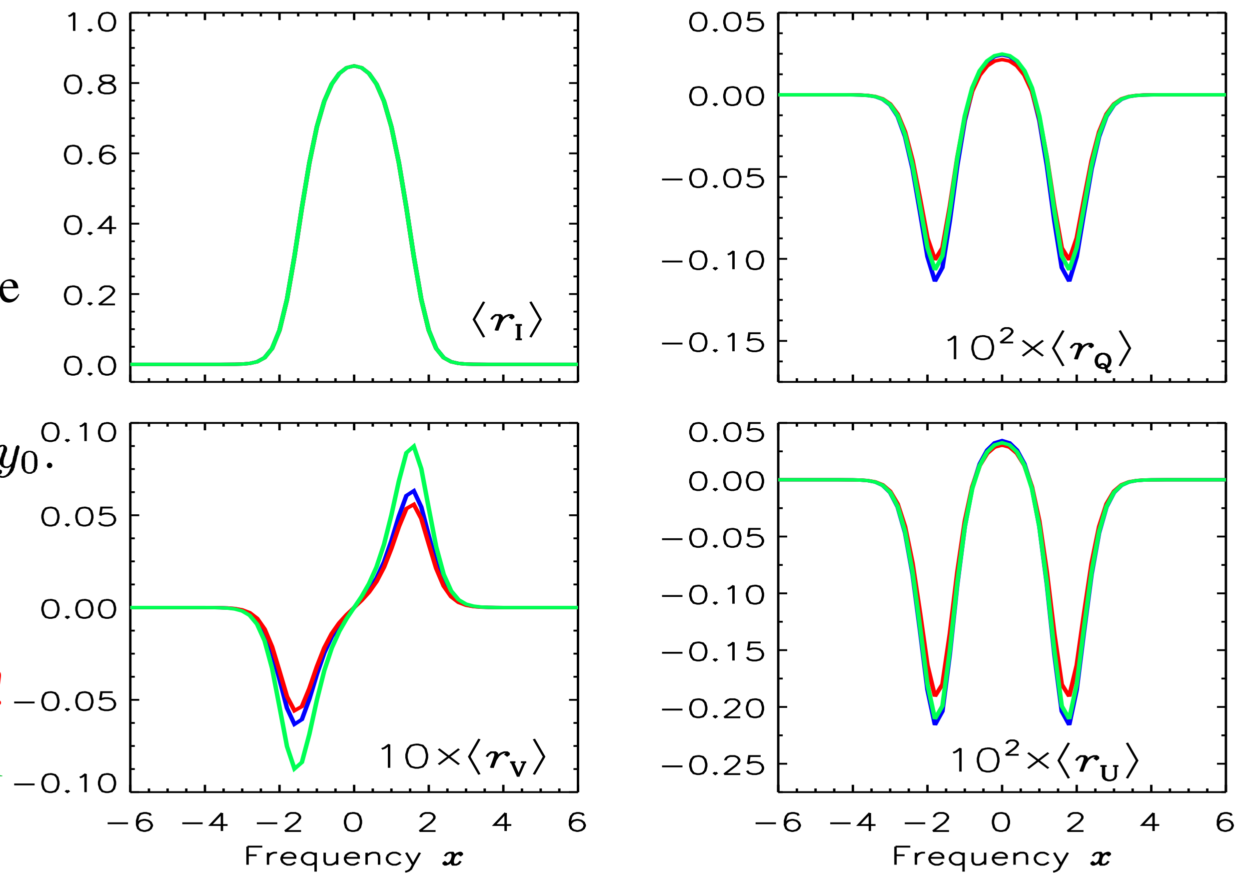
Results

1. $\langle r_{Q,U}\rangle$ are nearly insensitive to the asymmetry of the PDFs.

2. $\langle r_V\rangle$ peaks at same $x\approx 1.5$ for all y_0.

3. The amplitude of $\langle r_V\rangle$ peaks increases with the mean field, $\because$ it is generated by the NET flux!

$\langle r_V\rangle$ has larger diagnostic potential



Angular PDFs

★ Quiet solar atmosphere is filled with mixed polarity fields.

★ Inter-granular lanes contain fields directed either upward or downward.

★ Such a scenario can be represented by an angular PDF that has a 'constant field strength' B but 'random orientations' (θ_B, ϕ_B) :

Ex : power law (see Stenflo 1987) : $P_{\rm pl}(\mu_B) = [(p+1)/(4\pi)]\, |\mu_B|^p, \quad -1 \le \mu_B \le +1,$

where $\mu_B = \cos\theta_B$, with θ_B the field orientation with respect to the atmospheric normal.

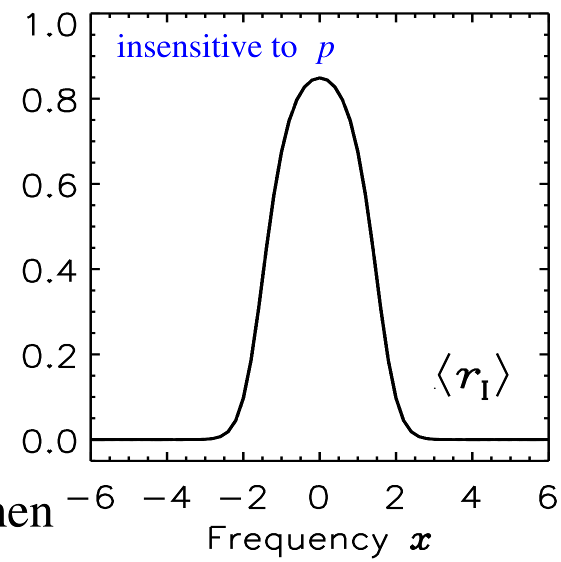
★ When the power law index $p = 0$ the distribution is isotropic (photospheric like).

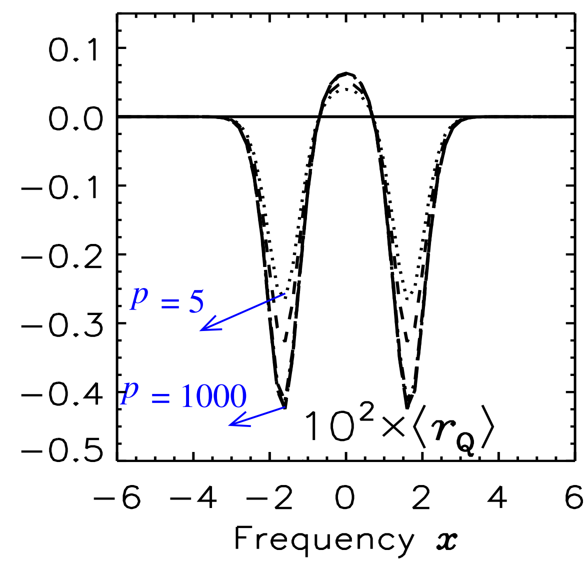
★ With increasing p the PDF becomes more and more vertically peaked (flux tube like).

Model ν=5 (meso-turbulent); $B/B_{\rm D}$=0.1; β=10; a=0; μ=0.1 and p : $0 \rightarrow 1000$.

Results

1. $\langle r_{\rm Q}\rangle \ll \langle r_{\rm I}\rangle$ ∵ the field is weak.





2. $\langle r_{\rm Q}\rangle$ first increases with p and then saturates (when $p \simeq 100$) to a deterministic solution.

Composite PDFs that may mimic solar surface fields

★ Observation and theory together point to the following facts about the random fields :

1. Angular variation of the random field should not be the same for all field strength.
2. PDF should become vertically peaked in the strong field regime (flux tube like).
3. The "Same" PDF should become isotropic in the weak field regime (photospheric like).
4. Transition from "isotropic $\longrightarrow$ peaked" distribution should be gradual ($B_t \approx 50$ G).

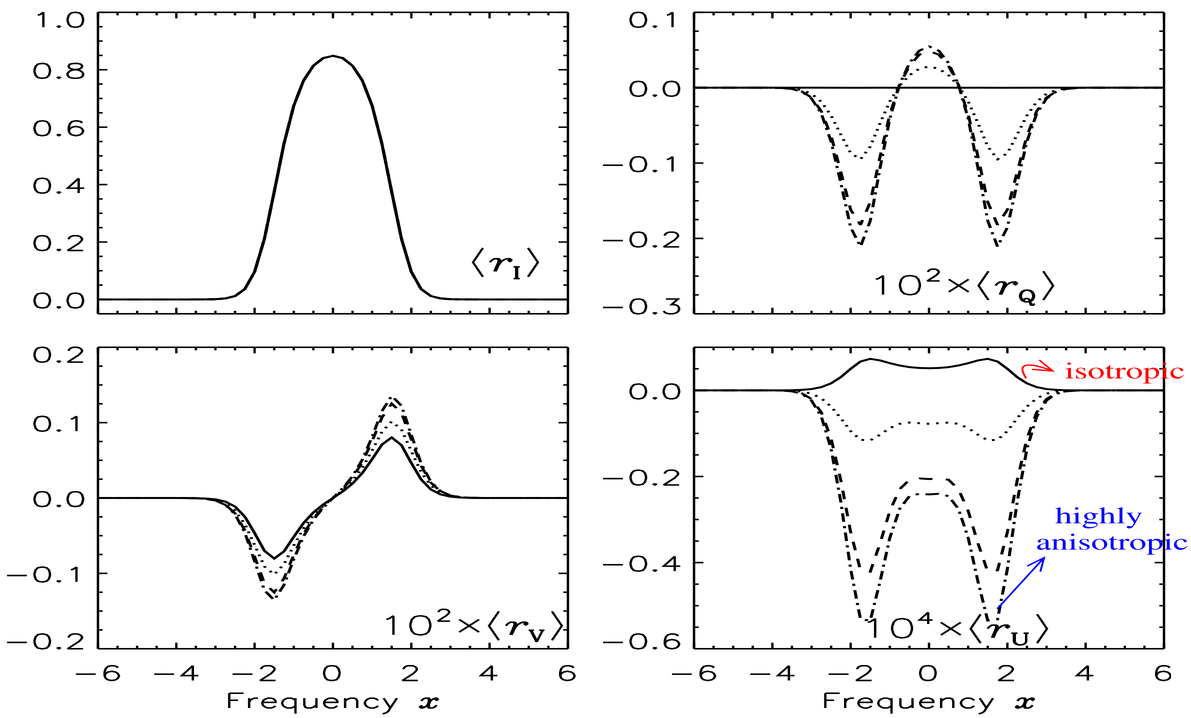
★ Based on these facts, we propose 2 composite PDFs (Sampoorna et al. 2008b) :

$$P_{\text{composite}}(\boldsymbol{B})d\boldsymbol{B} = \begin{cases} \overbrace{P_{\text{V}}\,(B/\Delta_B, a_B)\,dB/\Delta_B}^{\text{for strength variation}} \;\star\; \overbrace{\mu_B^p\, d\mu_B\, d\phi_B}^{\text{for angular variation}} \,, \\ \underbrace{P_{\text{SE}}\,(B/\Delta_B)\,dB/\Delta_B} \;\star\; \underbrace{\mu_B^p\, d\mu_B\, d\phi_B} \,, \end{cases}$$

$\mu_B = \cos\theta_B$, with $\theta_B =$ orientation w.r.t. the vertical;
power law index $p = |B|/B_t$.
$B_t =$ transition field strength between isotropic and peaked distribution (≈ 50 G).

★ $B_t = \infty \Rightarrow p = 0$ and hence $P_{\text{composite}}$ becomes isotropic distribution for all B.

Mean profiles computed with composite PDFs



Model $(a, \beta, \Delta_B/B_{\rm D})=(0, 10, 0.0056)$; the LOS ≈ parallel to the limb (μ=0.1); asymmetric Voigt PDF with mean field y_0=4.5; meso-turbulence (ν=5).

Line types [Solid $y_t=B_t/\Delta_B=\infty$] (isotropic), [dotted y_t=50], [dashed y_t=10], and [dash-dotted y_t=5].

Results As y_t decreases, the PDF becomes more and more anisotropic. ∴ both $\langle r_{\rm Q,V} \rangle$ increase in magnitude. $\langle r_{\rm U} \rangle$ is created purely by magneto-optical effects.

Conclusions

1. PDFs are well suited to represent in a compact way the randomness of the magnetic field.

2. Asymmetric Voigt PDF is a good choice because the strong field contribution which come from tail regions are correctly represented.

3. We represent a turbulent vector magnetic field by a combination of angular and strength dependent PDFs. Such composite PDFs better describe solar turbulent fields.

4. We formulated and tested few PDFs. They can be used in radiative transfer modeling, simulation, and inversion codes.

The Next Steps · · ·

1. Turbulence in scattering media (Hanle effect) requires more involved theory. Such a theory is recently formulated by Frisch (2006).

2. We have used Frisch (2006) formulation in radiative transfer calculations with Hanle effect (Frisch, Anusha, Sampoorna, & Nagendra, 2008, in preparation).

3. The effect of different PDFs on Hanle scattered mean Stokes profiles is studied in Anusha et al. (2008, see the poster presented in this conference).

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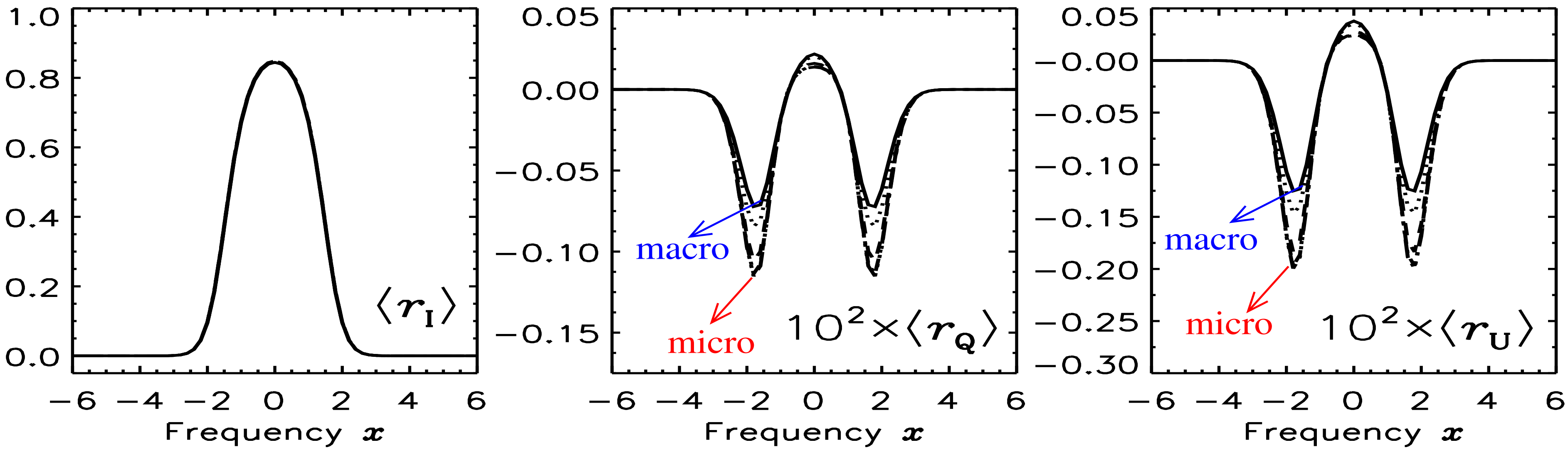
Effect of eddy size ($\approx 1/\nu$) on mean Stokes profiles



Model Voigt PDF with a_B=1.5; Δ_B/B_D=0.0056; a=0; β=10; (θ,ϕ)=$(60^\circ, 30^\circ)$.

Line types solid: macro-turbulent (ν=0); dotted (ν=1); dashed (ν=10); dot-dashed (ν=100); and dash triple-dotted: micro-turbulent (ν=200).

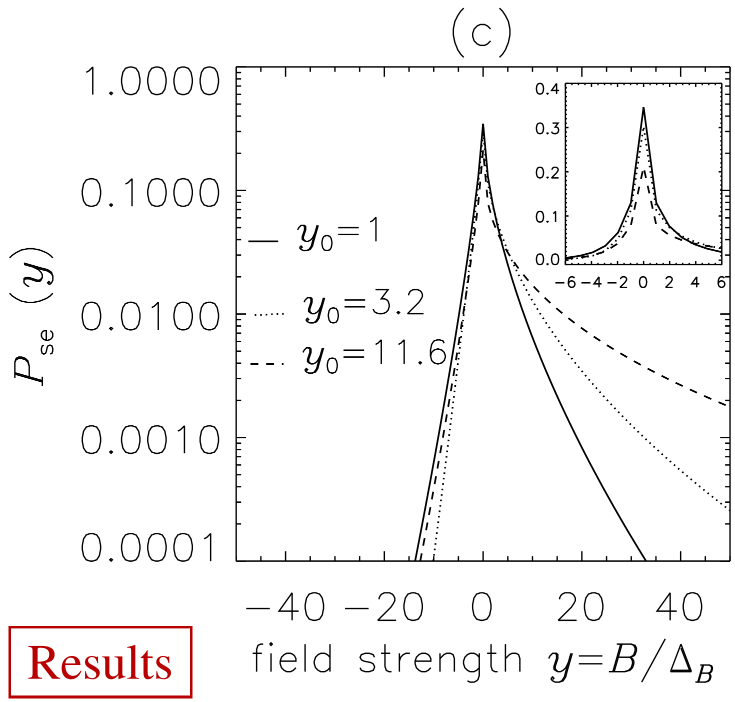
Result I :- Since $\Delta_B/B_\mathrm{D} \ll 1$, $\langle r_\mathrm{I} \rangle$ profiles are insensitive to the eddy size.

Result II :- Meso-turbulent solutions lie between the macro-turbulent ($\nu = 0$; solid) and micro-turbulent ($\nu = 200$; dash triple-dotted) limits.

Result III :- Significant differences are observed in the wing peaks of $\langle r_\mathrm{Q,U} \rangle$.

Result IV :- $\langle r_\mathrm{Q,U} \rangle$ is larger in micro-turbulent case than in macro-turbulent ones.

Asymmetric Stretched Exp. PDF (non-zero NET flux)



★ In this case asymmetric PDFs with non-zero y_0 are constructed by choosing different k for $+$ve & $-$ve polarities.

★ Line types: Solid, dotted, and dashed lines refer to meso-turbulence. Heavy dot-dashed line refers to macro-turbulent limit (with y_0=11.6), for comparison.

Results

1. Peak values of $\langle r_{\mathrm{Q,U,V}} \rangle$ increase with y_0. Peak positions are however insensitive to the mean field y_0.

2. In $\langle r_{\mathrm{Q,U}} \rangle$ the differences between meso and macro-turbulent profiles are due to dominance of the $+$ve tail of the PDF compared to the $-$ve.

3. $\langle r_{\mathrm{V}} \rangle$ is relatively insensitive to ν.