Test of Nonextensive Statistical Mechanics (NSM) by the Solar Sound Speeds Measured in Helioseismology

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1. Introduction

- **Nonextensiv Statistical Mechanics**

• The Nonextensive Entropy(Tsallis, *J.Stat.Phys.*1988)

$$S_q = \frac{k}{1-q}\left(\sum_i p_i^q - 1\right) \qquad (1)$$

where q is the nonextensive parameter,

k is Boltzmann constant,

p_i is the probability in the i-th configuration.

- The Power-law Distribution
  (Curado and Tsallis, *J.Phys. A* 1991)

$$p_i \sim \left[1-(1-q)\beta\varepsilon_i\right]^{1/(1-q)} \qquad (2)$$

where $\beta = 1/kT$ is the Lagrange parameter associated with the internal energy.

If $q = 1$,

$S_q\big|_{q=1} = S_B$, recovering Boltzmann entropy,

p_i , recovering Boltzmann distribution.

- The Generalized Maxwell-Boltzmann Distribution

(Silva, et al, *Phys.Lett.A* 1998; Du, *Europhys.Lett.* 2004)

$$f_q(\mathbf{r},\mathbf{v}) = nB_q\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\left[1-(1-q)\frac{m\,\mathbf{v}^2}{2kT}\right]^{\frac{1}{1-q}} \qquad (3)$$

where m is the mass of each particle; T , the temperature;

n is the particle number density;

B_q is the q-dependent normalized constant.

$f_q\big|_{q=1}$ =Maxwell-Boltzmann distribution.

The Applications of Nonextensive Statisical Mechanics (NSM) to Astrophysics

- Self-gravitating Systems:

  Du, *Europhys.Lett.*2004.

  Taruya, et al, *Phys.Rev.Lett.*2003;

  Chavanis, *A&A*. 2003; 2002;

- Antonov Problem:

  Taruya, et al, *MNRAS* 2005.

- Solar Wind Intermittency:

  Leubner, et al, *ApJ*. 2004.

- Solar Neutrino Problem:

  Quarati et al, *Phys.Lett.A* 2005;
  Physica A 2004; 2003.
  Haubold et al, *Physica A* 2004.

- Jeans Gravitational Instability:

  Lima et al, *A&A* 2002;

  Du, *Phys.Lett. A* 2004, *Physica A* 2004;

  Shaikh et al, *Z.Naturforsch A* 2006.

- Non-Maxwell Distribution in Plasma:

  Silva and Lima et al, *PRE* 2000, *Physica A* 2005;

  Du, *Phys.Lett. A* 2004;

  Leubner, *ApJ*. 2004.

- Galaxy Clusters:

  Quarati et al, *Astrophys. Lett. Comm.* 1998;
  Hansen, *New Astronomy* 2005.

- Dark Matter and Dark Energy:

  Leubner, *ApJ*. 2005;
  Beck, *Physica A* 2004;
  Hansen, et al, *New Astronomy* 2005;
  Matos, et al, *Gen. Relativ. Gravit.* 2005.

- The Sun and Stars:

  Du, *Europhys.Lett.* 2006;
  New Astronomy 2006.

- Black Hole:

  Oliveira,et al, *Phys.Rev.D* 2005.

What does the Nonextensive Parameter $q \neq 1$ Stand for ?

- The Formula Expression (Du, *Europhys.Lett.*2004; *Phys.Lett. A* 2004):

$$k\nabla T + (1-q)m\nabla\varphi = 0 \qquad (4)$$

where T is temperature

φ is gravitational potential

m is mass of the particle.

For the interior of a star, equivalently,

(Du, *ApSS* 2006; *New Astronomy* 2006),

$$1 - q = -\frac{k}{\mu\, m_{\mathrm{H}}} \frac{dT}{dr} \Big/ \frac{GM(r)}{r^2} \qquad (5)$$

where μ is the mean molecular weight

m_{H} is the mass of the hydrogen atom

G is the gravitational constant

$M(r)$ is the mass interior to a sphere of radius r.

- The Physical Meaning of the Statistics

  (Du, *Europhys.Lett.* 2004, 2006; *Phys.Lett. A* 2004)

$$k\nabla T + (1-q)m\nabla\varphi = 0 \qquad (4)$$

- $q \neq 1$ if and only if $\nabla T \neq 0$.
- The statistics can be reasonably applied to describe the thermodynamic properties of the system being in an external field and in *the non-equilibrium stationary state*.
- At the base of *equilibrium of the non-extensive system* in an external potential field.

2.Test of Nonextensive Statistical Mechanics by Solar Sound Speeds

- **The Theory** (Du, *Europhys.Lett.* 2006)

We introduce the sound speed inside the Sun that is defined by

$$v_s = \sqrt{\Gamma_1 \frac{kT}{\mu\, m_H}} \qquad (6)$$

$\Gamma_1 \approx 5/3$ is the adiabatic index.

So, Eq.(5) can be written as

$$1-q=\left[-\frac{2v_s}{\Gamma_1}\frac{dv_s}{dr}-\frac{kT}{\mu^2 m_H}\frac{d\mu}{dr}\right]\Bigg/\frac{GM(r)}{r^2} \qquad (7)$$

By eliminating the second term in Eq.(7), we obtain the inequality (8),

$$1-q\geq-\frac{2v_s}{\Gamma_1}\frac{dv_s}{dr}\Bigg/\frac{GM(r)}{r^2} \qquad (8)$$

This inequality provides a lower limit to (1-*q*) for the solar interior.

- The Inequality Providing a Lower Limit to (1-q) for the Solar Interior:

$$1 - q \geq (1 - q)^{*} \qquad (9)$$

The minimum nonextensive degree parameter, (1-q)*, can be uniquely determined by the solar sound speeds measured in helioseismology:

- The Minimum Nonextensive Degree Parameter

Inside the Sun

$$(1-q)^* = -\frac{6}{5}\, v_s \frac{dv_s}{dr} \Big/ \frac{GM(r)}{r^2} \qquad (10)$$

It can be uniquely determined by the solar sound speeds measured in helioseismology.

The Calculations for (1- *q*)*** inside the Sun**

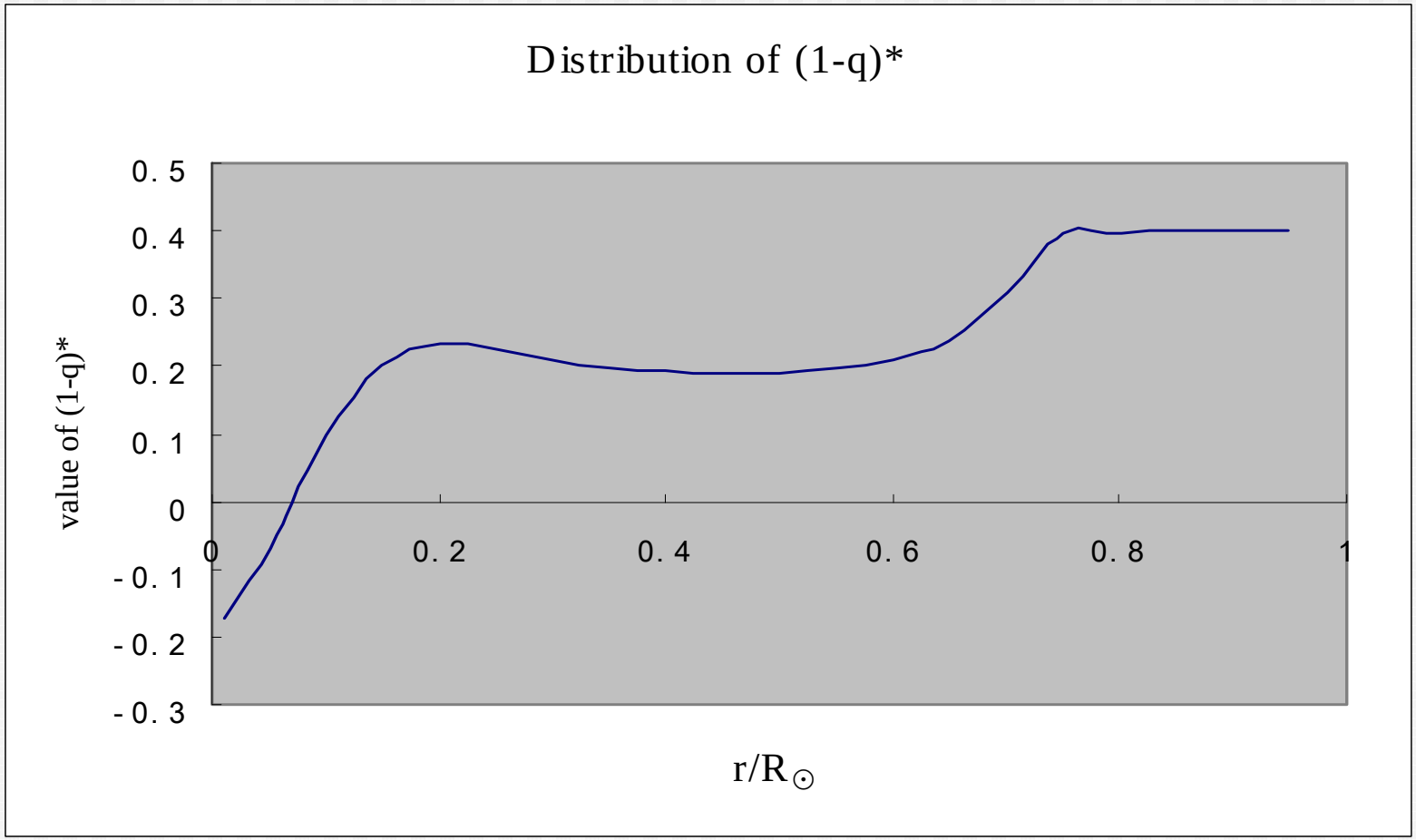
- The BP2000 Sound Speeds (Bahcall et al, *ApJ*. 2001)

The BP2000 sound speeds by Bahcall et al and the helioseismologically measured sound speeds are the excellent agreement everywhere in the Sun

- Solar Sound Speed Gradient and the Minimum Nonextensive Degree Parameter, (1-q)*

r/R	M/M	v_s	μ	$d\,v_s/dr$	(1-q)*
0.01000	0.0001081	5.05656E+07	0.8532	8.4605E-05	-0.1733
0.05034	0.0124165	5.10132E+07	0.7955	1.4879E-04	-0.0678
0.06961	0.0301280	5.11219E+07	0.7578	0.0000E-00	0.0000
0.10006	0.0762478	5.07757E+07	0.7042	-3.3330E-04	0.0973
0.15018	0.1925732	4.86931E+07	0.6494	-8.0941E-04	0.2022
0.20107	0.3393826	4.54218E+07	0.6253	-9.8517E-04	0.2335
0.25089	0.4841900	4.20378E+07	0.6171	-9.4609E-04	0.2265
0.30016	0.6099028	3.89736E+07	0.6143	-8.3665E-04	0.2110
0.35075	0.7132949	3.62308E+07	0.6124	-7.2889E-04	0.1995
0.40067	0.7907148	3.38604E+07	0.6114	-6.4093E-04	0.1932
0.45055	0.8480440	3.17676E+07	0.6106	-5.7143E-04	0.1902
0.50063	0.8902432	2.98790E+07	0.6100	-5.1831E-04	0.1910
0.55083	0.9211915	2.81372E+07	0.6094	-4.8374E-04	0.1964
0.60081	0.9438189	2.64813E+07	0.6090	-4.7327E-04	0.2099
0.64986	0.9602980	2.48312E+07	0.6084	-4.9740E-04	0.2379
0.70042	0.9730081	2.29284E+07	0.6021	-6.1235E-04	0.3101
0.71377	0.9758090	2.23201E+07	0.5989	-8.4817E-04	0.4330
0.75033	0.9826089	2.03346E+07	0.5989	-7.8089E-04	0.3985
0.80103	0.9900343	1.75530E+07	0.5989	-7.9843E-04	0.3979
0.85023	0.9950969	1.47421E+07	0.5989	-8.4980E-04	0.3986
0.90020	0.9982619	1.16042E+07	0.5989	-9.7051E-04	0.4004
0.94896	0.9997083	7.87052E+06	0.5989	-1.2871E-03	0.3997

- Distribution of (1-*q*)* in the Sun
  (Du, *Europhys.Lett.* 2006)



The Conclusions

- $(1\text{-}q)^* \geq 0.1902$ for all Solar Radii between $0.15R_\odot$ and $0.95R_\odot$.
- $(1\text{-}q)= 0.4$ for the out layers, $0.75R_\odot \leq r \leq 0.95R_\odot$.
- The Nonextensive Parameter q is Significantly Different from Unity in the Sun has Received the Support by the Experiment Measurements for the Solar Sound Speeds in the Helioseismology.

Thank You Very Much!