Short Scale Magnetic Turbulence In The Solar wind

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Study of solar wind turbulence

Direct observations

Collisionless plasma, wave-wave interactions

Propagation of cosmic rays

Scattering of particles on fluctuations

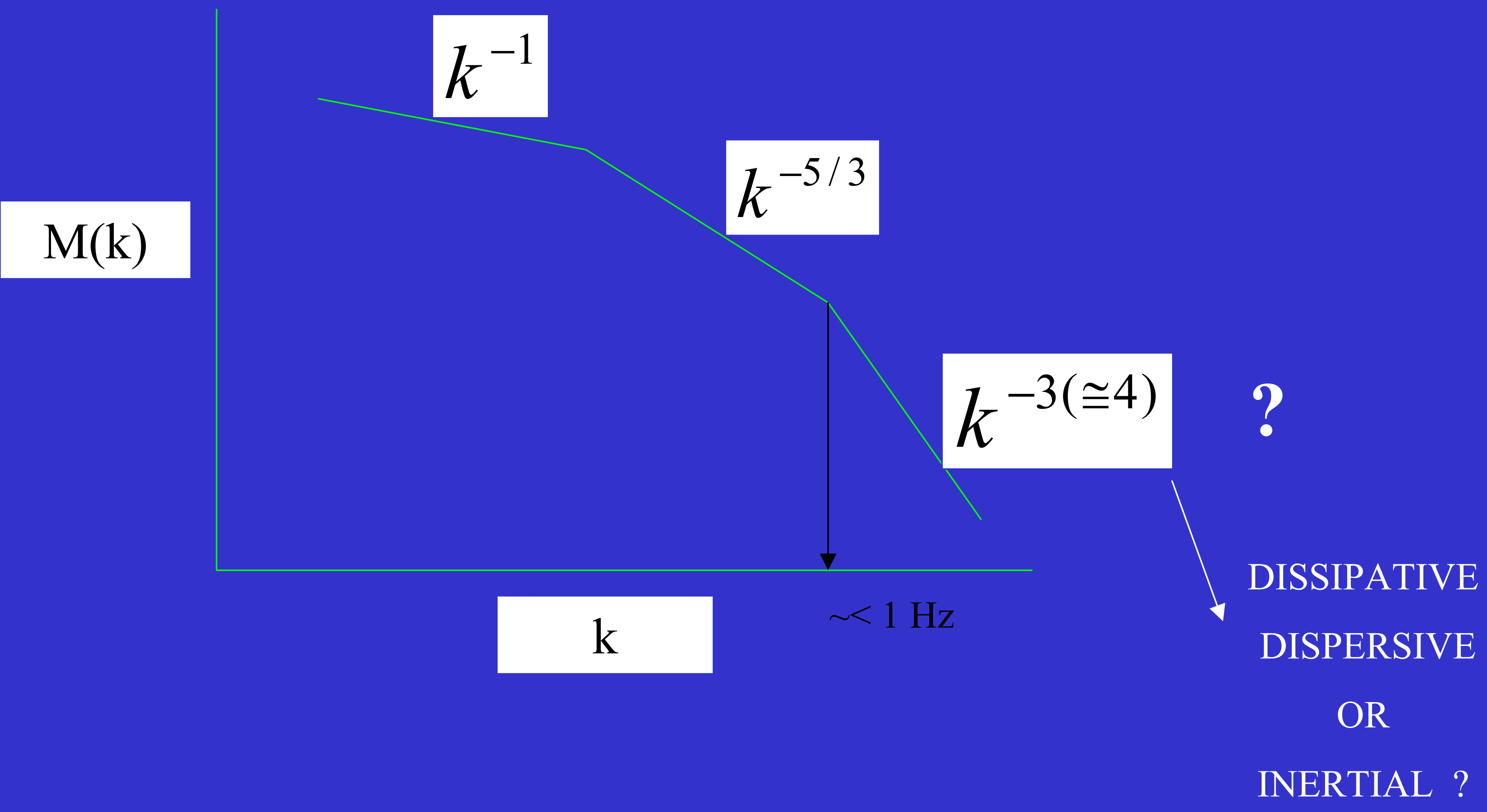
Solar –terrestrial relation

Coupling of SW with earth’s magnetosphere,
anomalous viscosity

Several exponents on the short spatial scales

Lessons for other astrophysical systems

SCHEMATIC MAGNETIC SPECTRA IN THE SOLAR WIND



the steepening cannot be accounted for by dissipation

Since the damping of Alfven waves by ion cyclotron resonanace has an exponential

and

the damping of the Magnetosonic waves by Landau resonance has much stronger power law dependence on k!

Inclusion of the Hall-Effect in the generally accepted Alfvenic Turbulence

Why Hall ?

Steepening observed near the Ion Inertial scale

$$\lambda_i = \frac{c}{\omega_{pi}}$$

Towards short scales

Dispersion caused by the Hall effect modifies the V B relation around these scales

Departures from Alfvenicity also observed at short scales

Different V B spectra

THE HALL-MHD

The two –Fluid Model

ELECTRON EQ.

$$m_e n_e[\partial V_e / \partial t + (V_e.\nabla)V_e] = -\nabla p_e - en_e[E + V_e \times B / c]$$

For Inertialess electrons (m_e -> 0) , Electric field is found to be

$$E = -c^{-1} V_e \times B - (ne)^{-1} \nabla (p_e)$$

The Ion Eq.

$$m_i n_i[\partial V_i / \partial t + (V_i.\nabla)V_i] = -\nabla p_i + e n_i[E + V_i \times B / c]$$

Substitute for E from the inertialess electron eq.

$$m_i n_i[\partial V_i / \partial t + (V_i.\nabla)V_i] = -\nabla(p_i + p_e) + J \times B / c$$

with

$$J = ne(V_i - V_e)$$

$$n = n_e = n_i$$

The Induction Eq.

$$\partial B / \partial t = -c\nabla \times E = \nabla \times (V_e \times B)$$

B is frozen to electrons

OR

$$\partial B / \partial t = -c\nabla \times E = \nabla \times (V_i - J / en) \times B$$

B is not frozen to ions

Hall Velocity

Hall term dominates for

$$(nec)^{-1} J \times B \geq V_i \times B / c \text{ or } L \leq M_A\, c / \omega_{pi} \text{ and } T \geq \omega_{ci}^{-1}$$

The Hall term decouples electron and ion motion on ion inertial length scales and ion cyclotron times

Waves in Hall-MHD

$$\vec{v} - \varepsilon \nabla \times \vec{b} = -\frac{\omega}{k}\vec{b}$$

$$\nabla \times \vec{b} = -\frac{\omega}{k}\nabla \times \vec{v}$$

$$B_k = \alpha(k) V_k$$

$$and \quad \alpha(k) \equiv (-\frac{\omega}{k}) = [-k/2 \pm (k^2/4+1)^{1/2}]$$

NOTICE DEPARTURES FROM THE ALFVEN STATE FOR WHICH V= pm B

THE THREE INVARIANTS OF THE HALL-MHD

Total Energy $E = \frac{1}{2}\int (V^2 + B^2) d^3x = \frac{1}{2}\sum_k |V_k|^2 + |B_k|^2$

Magnetic Helicity $H_M = \frac{1}{2}\int \boldsymbol{A}\cdot\boldsymbol{B} d^3\boldsymbol{x} = \frac{1}{2}\sum_k \frac{i}{k^2}(\boldsymbol{k}\times\boldsymbol{B}_k)\cdot\boldsymbol{B}_{-k}$

Generalized Helicity $H_G = \frac{1}{2}\int (\boldsymbol{A}+\boldsymbol{V})\cdot(\boldsymbol{B}+\boldsymbol{\nabla}\times\boldsymbol{V}) d^3\boldsymbol{x}$

$$= \frac{1}{2}\sum_k \left[\frac{i\boldsymbol{k}\times\boldsymbol{B}_k}{k^2} + \boldsymbol{V}_k\right]\cdot\left[\boldsymbol{B}_{-k} - i\boldsymbol{k}\times\boldsymbol{V}_{-k}\right]$$

CASCADES

In order to derive the spectral energy distributions

we resort to the Kolmogorov **HYPOTHESES**

the spectral cascades proceed at a constant rate governed

By the Eddy Turn Over Time $(kV_k)^{-1}$

CASCADING OF THE TOTAL ENERGY GIVES

$$(kV_k)[1+\alpha^2]V_k^2/2=\varepsilon_E$$

$$B_k=\alpha(k)V_k$$

The omnidirectional spectral distribution function $W_E(k)$ (kinetic energy per gram per

unit wave vector V_k^2/k), then, takes the form

$$W_E(k)=(2\varepsilon_E)^{2/3}[1+\alpha^2]^{-2/3}k^{-5/3}$$

AND THE CORRESPONDING MAGNETIC SPECTRUM

$$M_E(k)=\alpha^2 W_E(k)$$

The cascading of the magnetic helicity H_M

$$(kV_k)(0.5B_k^2/k) = \varepsilon_H$$

resulting in a different spectral energy distribution

$$W_H(k) = (2\varepsilon_H)^{2/3}\alpha^{-4/3}k^{-1}$$

with its corresponding

$$M_H(k) = \alpha^2 W_H(k)$$

Finally, the cascading of the generalized helicity

$$(kV_k)[0.5g(k)V_k^2) = \varepsilon_G$$

WITH

$$g(k) = (\alpha + k)^2 k^{-1}$$

GIVES

$$W_G(k) = (2\varepsilon)^{2/3}[g(k)]^{-2/3} k^{-5/3}$$

AND

$$M_G(k) = \alpha^2 W_G(k)$$

SPECTRA IN THE HALL STATE

From The Hall relation

$$V_k = [\alpha(k)]^{-1} B_k$$

For k>>1,

$$\alpha(k) \approx k^{-1} \textit{ and } \alpha(k) \approx k \textit{ are the two roots}$$

AND THE SPECTRA FOR $\alpha(k) \approx k^{-1}$ ***ARE***

$$W_{E_1}(k) = (2\varepsilon_E)^{2/3} k^{-5/3}, \quad M_{E_1}(k) = (2\varepsilon)^{2/3} k^{-11/3}$$

$$W_{H_1}(k) = (2\varepsilon_H)^{2/3} k^{1/3}, \quad M_{H_1}(k) = (2\varepsilon_H)^{2/3} k^{-5/3}$$

$$W_{G_1}(k) = (2\varepsilon_G)^{2/3} k^{-7/3}, \quad M_{G_1}(k) = (2\varepsilon_G)^{2/3} k^{-13/3}$$

$$For\ the\ \sec ond\ root\ \alpha \approx k$$

$$The\ Spectra\ are:$$

$$W_{E_w}(k) = (2\varepsilon_E)^{2/3} k^{-3}, \quad M_{E_w}(k) = (2\varepsilon_E)^{2/3} k^{-1}$$

$$W_{H_w}(k) = (2\varepsilon_H)^{2/3} k^{-7/3}, \quad M_{H_w}(k) = (2\varepsilon_H)^{2/3} k^{-1/3}$$

$$W_{G_w}(k) = (2\varepsilon_G)^{2/3} k^{-7/3}, \quad M_{G_w}(k) = (2\varepsilon_G)^{2/3} k^{-1/3}$$

SPECTRA IN THE ALFVEN STATE

$$\alpha = 1$$

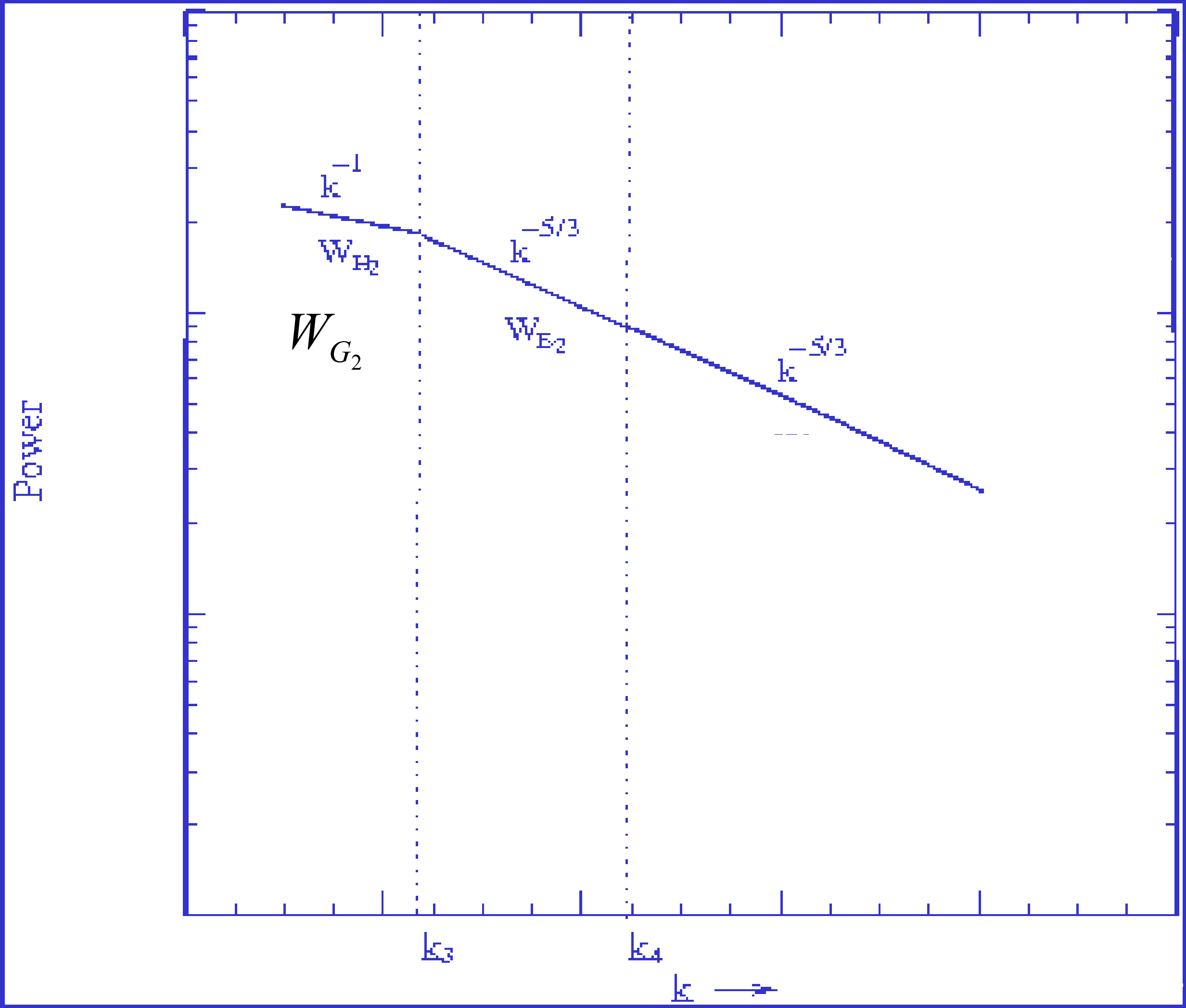
the corresponding spectra (denoted by suffix E_2 etc.):

$$W_{E_2} = (2\varepsilon_E)^{2/3} k^{-5/3}$$

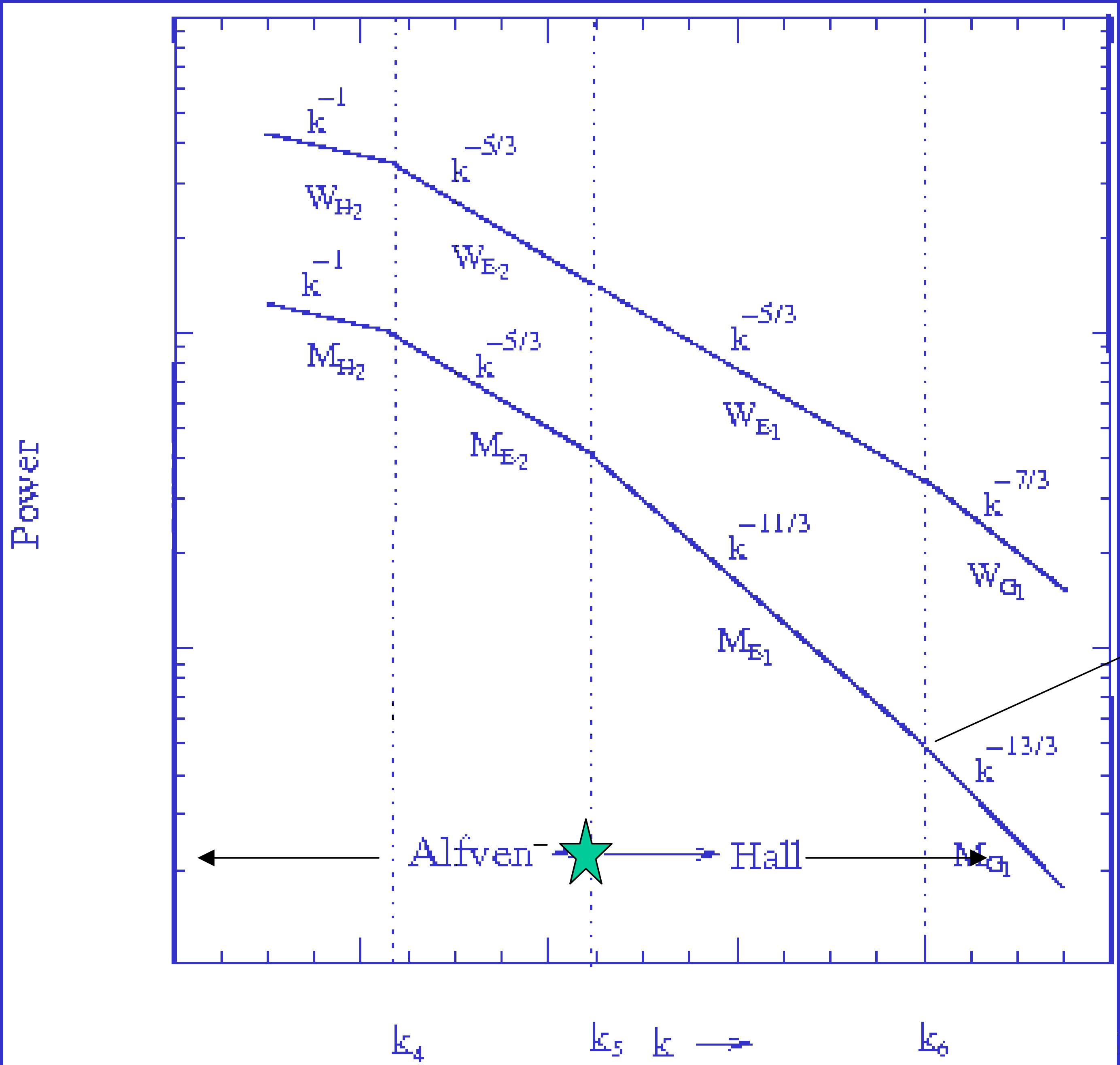
$$W_{H_2} = (2\varepsilon_H)^{2/3} k^{-1}$$

$$W_{G_2} = (2\varepsilon_G)^{2/3} k^{-1} \quad and \quad M(k) = W(k)$$

SPECTRUM IN THE ALFVEN STATE $\alpha = 1$



Modeled solar wind spectra,



Change of invariant and energy injection scale

Summarizing : In the Hall –MHD, the steepened part of the solar wind magnetic fluctuations arises in the inertial range and is a consequence of the V~ k B relation such that the kinetic energy Kolmogorov spectrum

$$k^{-5/3}$$

Corresponds to the magnetic spectrum

$$k^{-11/3}$$

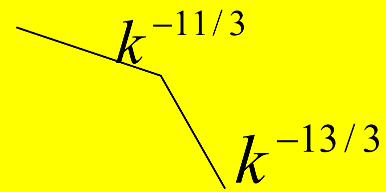
A steeper spectral branch at higher k also exists as

$$k^{-13/3}$$

All in the inertial range !

STEEPENED MAGNETIC SPECTRUM $k^{-(3->4)}$

TIMESCALE	RANGE
1..DAMPING OF ALFVEN AND MAGNETOSONIC MODES	DISSIPATIVE Exponential or high power of k
2. HIGHER DISPERSION OF MAGNETOSONIC WAVES ? $\tau \approx \omega^{-1} \propto (k + k^2)^{-1}$	DISPERSION k^{-3}
3. HALL $\tau = (kV_H)^{-1} \cong (k^2 B_k)^{-1}$	INERTIAL



Finite Plasma Beta

$$-\omega v = (k \times b) \times e_z - \beta (k.v) k$$

$$-\omega b = [k_z v - (k.v) e_z] - i\varepsilon k_z (k \times b)$$

ISSUES

IDENTIFICATIONS OF FLUCTUATIONS

SPECTRAL ANISOTROPY

VARIANCE ANISOTROPY

ANISOTROPIC CASCADE RATES

EVOLUTION

+

+

+