Indian Institute of Astrophysics, Bangalore, Seminar Feb. 20, 2008

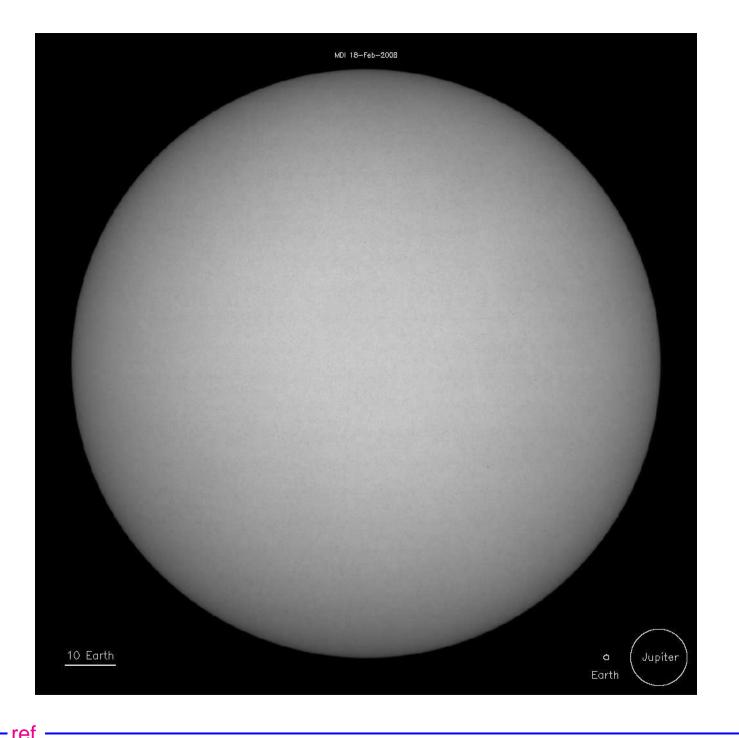
# On the structure of the solar internetwork magnetic field: Numerical simulations in comparison to observations with Hinode

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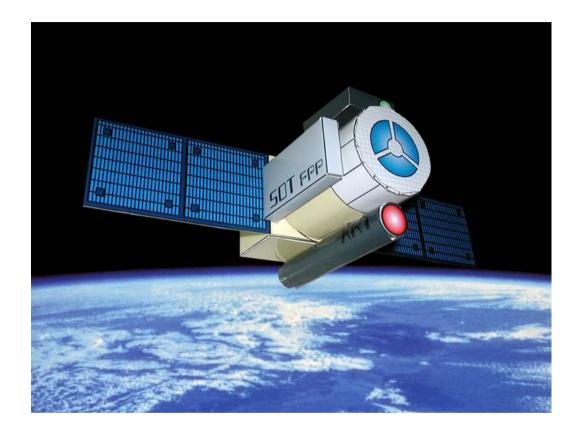


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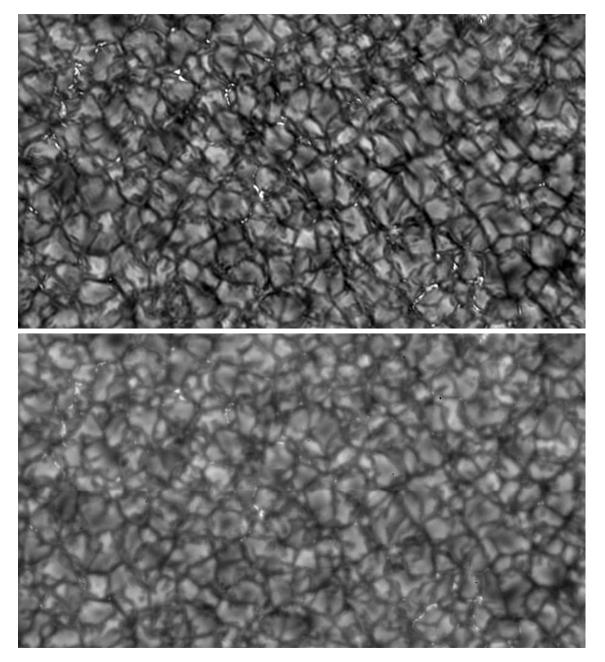


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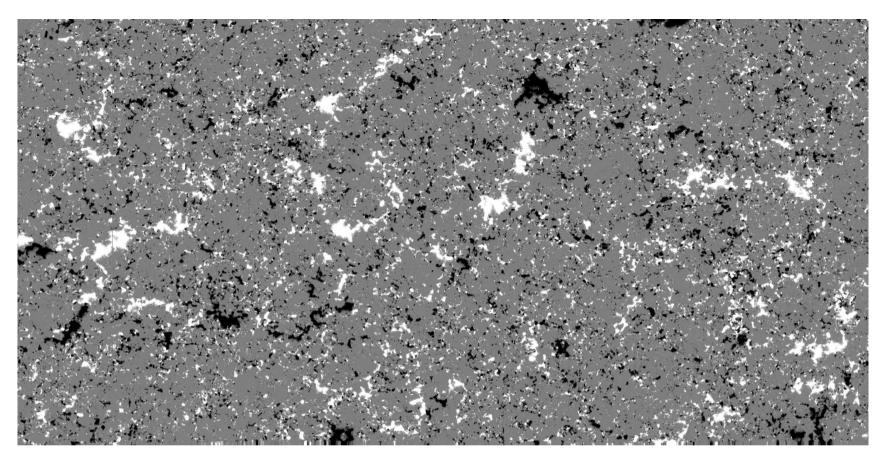
## **1. Observations with Hinode**



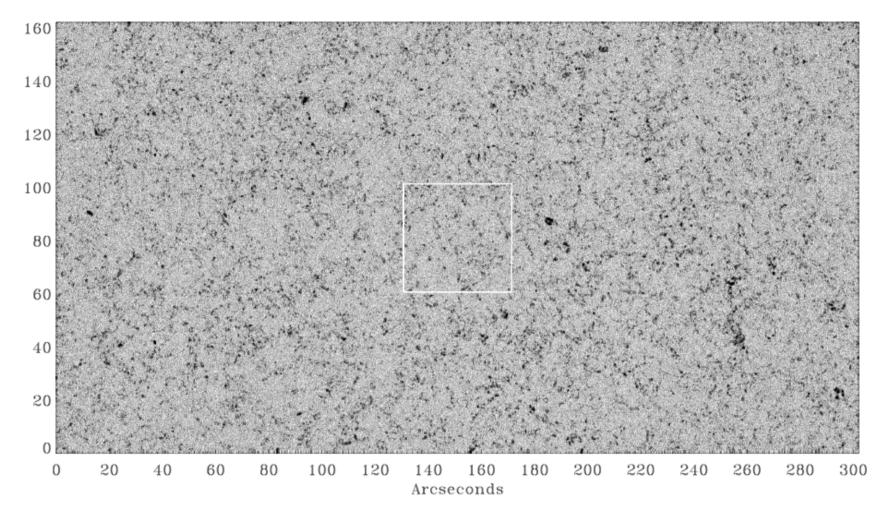
Hinode (Solar-B) launched on 22 September 2006. The Solar Optical Telescope (SOT) has an aperture of 50 cm featuring an image stabilization system consisting of a piezo-driven tip-tilt mirror. The Spectral-polarimeter (SP) generates Stokes IQUV spectral images. http://solar-b.nao.ac.jp



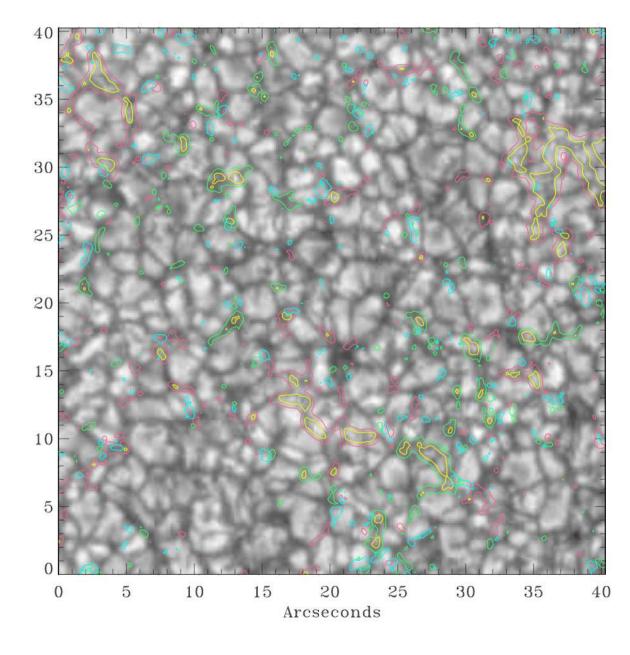
Comparison of G-band images from a ground based solar observatory (the DST) and the Hinode/SOT. Courtesy *Friedrich Wöger and Kevin Reardon*, Sacramento Peak Observatory.



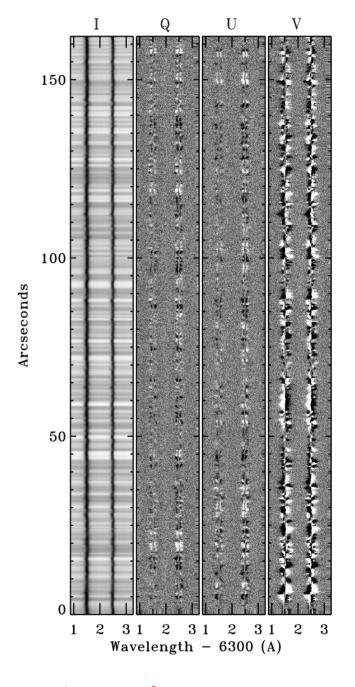
Apparent vertical magnetic flux density,  $B_{\rm app}^{\rm L}$ . of the quiet Sun over a field of view of  $302'' \times 162''$ . The grey scale saturates at  $\pm 50 \,\text{Mx}\,\text{cm}^{-2}$ . 2048 steps to 5 s.  $\langle |B_{\rm app}^{\rm L}| \rangle = 11.7 \,\text{Mx}\,\text{cm}^{-2}$ . From Lites et. al. 2008, ApJ 672, 1237



Apparent horizontal magnetic flux density,  $B_{\rm app}^{\rm T}$ , of the quiet Sun over a field of view of  $302'' \times 162''$ . The grey scale saturates at  $\pm 200 \, \text{Mx} \, \text{cm}^{-2}$ . 2048 steps to 5 s.  $\langle B_{\rm app}^{\rm T} \rangle = 60.0 \, \text{Mx} \, \text{cm}^{-2}$ . From *Lites et. al.* 08

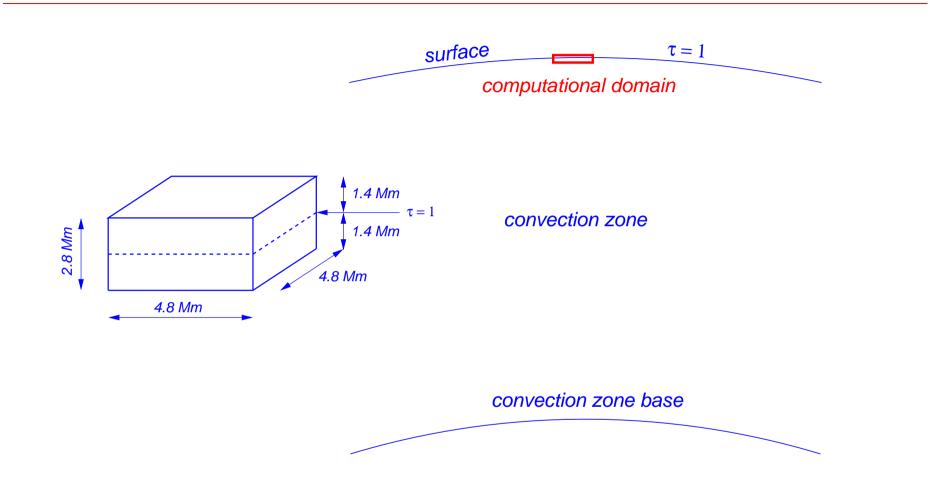


*Red* and *green*: contours of  $B_{\rm app}^{\rm L} = 24 \ {\rm Mx \, cm}^{-2}$ , respectively positive and negative. Yellow: contours of  $B_{\rm app}^{\rm L} = 100 \,\,{\rm Mx}\,{\rm cm}^{-2}$ . Blue contours correspond to  $B_{\rm app}^{\rm T} = 122 \,\,{\rm Mx}\,{\rm cm}^{-2}$ . Horizontal flux preferentially occurs at locations between lanes and granule centers. From Lites et. al. 08



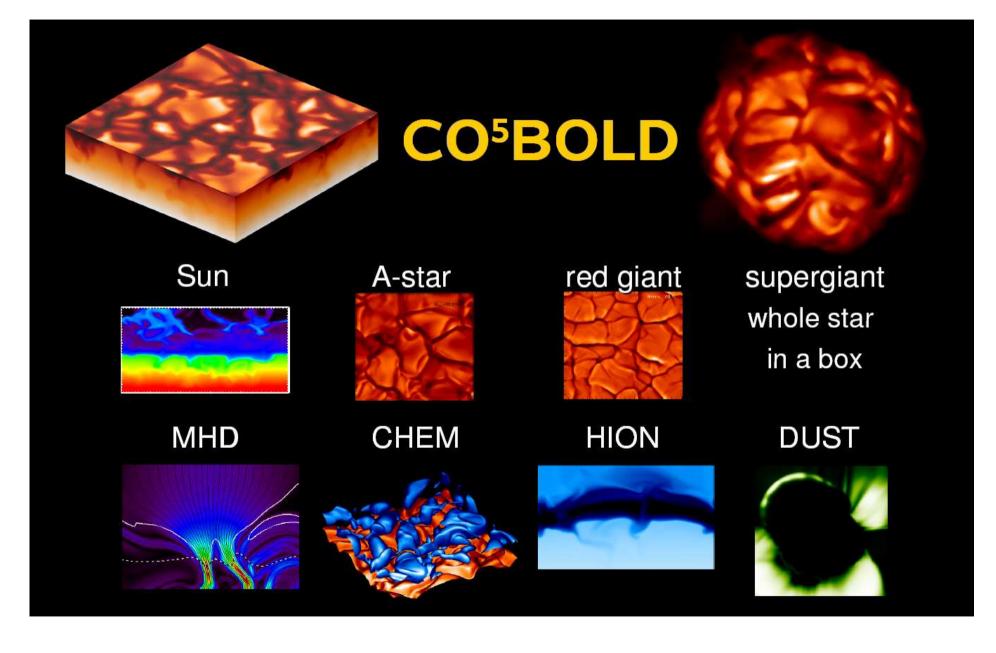
Deep mode Stokes spectra with an integration time of 67.2 s. rms polarization in the continuum of  $3 \times 10^{-4}$ . From a 2-hour time series Lites et al. obtain  $\langle B_{\rm app}^{\rm L} \rangle = 11.0 \, \text{Mx} \, \text{cm}^{-2}$ .  $\langle B_{\rm app}^{\rm T} \rangle = 55.3 \, \text{Mx} \, \text{cm}^{-2}$ . From Lites et. al. 08

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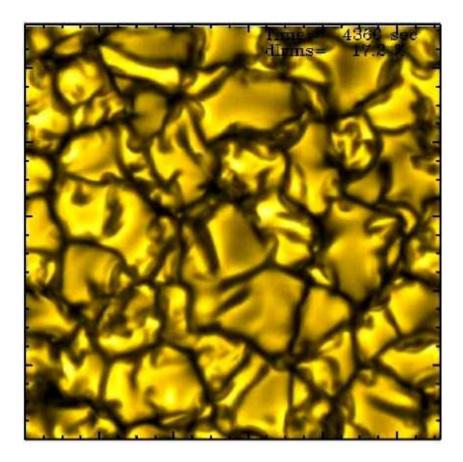


Typical size of a three-dimensional computational box (left) on scale with the convection zone boundaries (right)

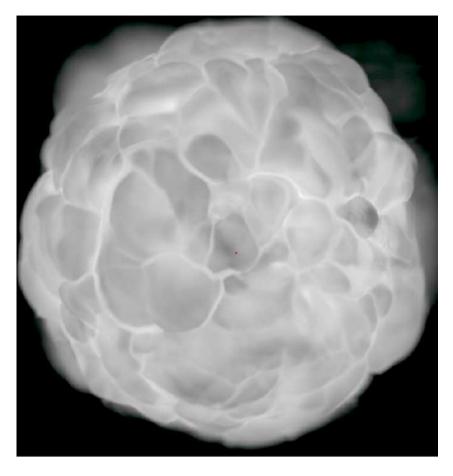
## Application examples of CO<sup>5</sup>BOLD (Courtesy Sven Wedemeyer-Böhm)



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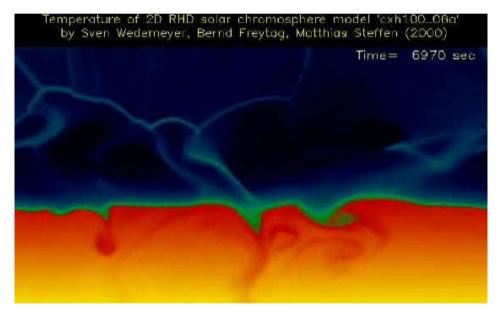


Simulation of solar granulation with  $CO^5BOLD.$   $400 \times 400 \times 165$  grid cells,  $11.2 \times 11.2$  Mm, Mean contrast at  $\lambda \approx 620$  nm is 16.65%. Courtesy *M. Steffen, AIP* 



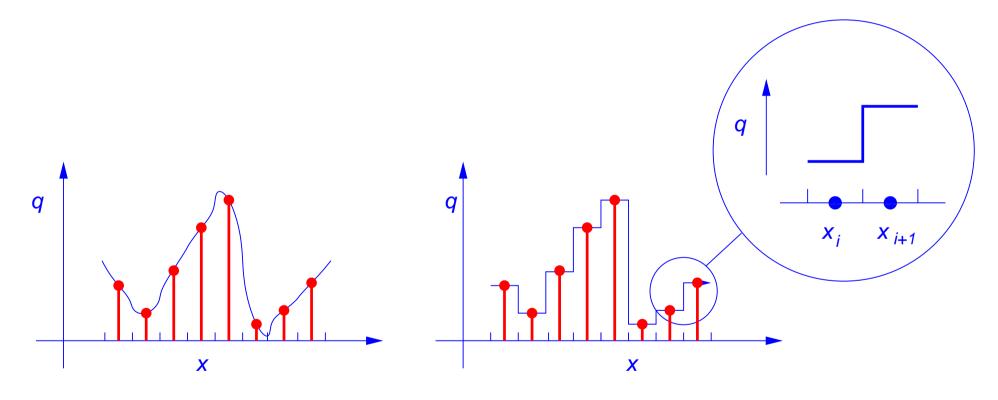
Simulation of a red supergiant with  $CO^5BOLD. 235^3$  grid cells,  $m_{\rm star} = 12m_{\odot}, T_{\rm eff} = 3436$  K,  $R_{\rm star} = 875R_{\odot}$ Courtesy Bernd Freytag

Two-dimensional radiation-hydrodynamic simulation of surface convection including the chromospheric layer. The dimensions of the computational domain are: Width, 5600 km; Height above the surface of  $\tau = 1$ , 1700 km; Depth below this surface level: 1400 km.

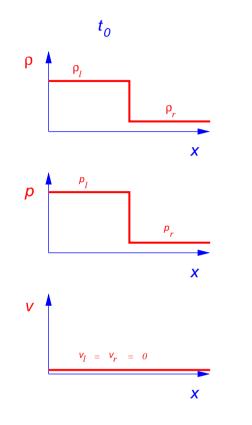


S. Wedemeyer et al. 2004, A&A 414, 1121

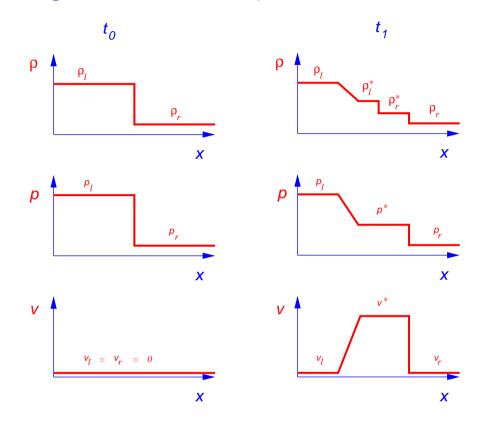
For a basic example of a shock capturing numerical scheme, consider a piecewise constant reconstruction with discontinuities at cell interfaces (S.K. Godunov, 1959).



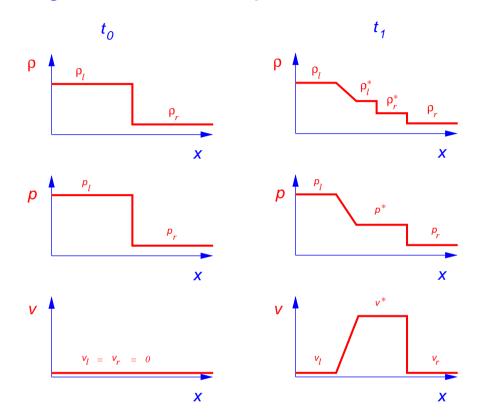
Solving the shock-tube problem for obtaining the fluxes at cell interfaces



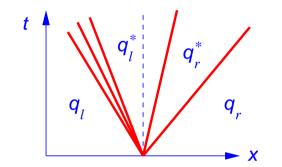
Solving the shock-tube problem for obtaining the fluxes at cell interfaces



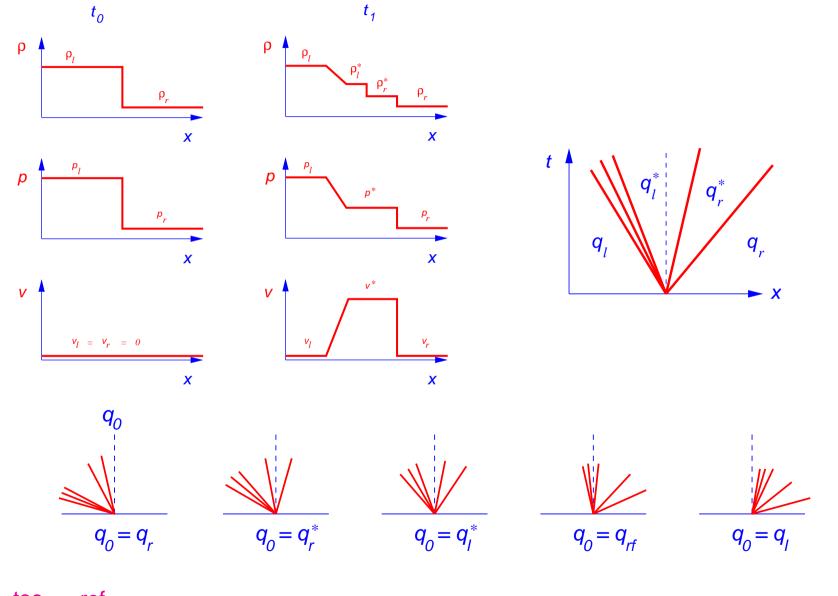
Solving the shock-tube problem for obtaining the fluxes at cell interfaces



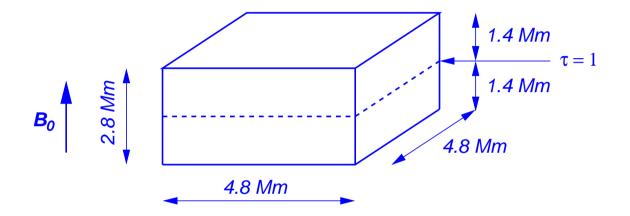
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Solving the shock-tube problem for obtaining the fluxes at cell interfaces



The three-dimensional computational domain encompasses the integral layers from the upper convection zone to the middle chromosphere.



With  $120^3$  grid cells, the spatial resolution in the horizontal direction is 40 km, while in the vertical direction it is 20 km throughout the photosphere and chromosphere increasing to 50 km through the convection-zone layer.

The ideal MHD-equations can be written in conservative form as:

$$rac{\partial oldsymbol{U}}{\partial t} + oldsymbol{
abla} \cdot oldsymbol{\mathcal{F}} = oldsymbol{S} \ ,$$

where the vector of conserved variables  $m{U}$ , the source term  $m{S}$  due to gravity and radiation, and the flux tensor  $m{\mathcal{F}}$  are

 $\boldsymbol{U} = (\rho, \rho \boldsymbol{v}, \boldsymbol{B}, E) , \qquad \boldsymbol{S} = (0, \rho \boldsymbol{g}, 0, \rho \boldsymbol{g} \cdot \boldsymbol{v} + q_{\text{rad}}) ,$ 

$$\boldsymbol{\mathcal{F}} = \begin{pmatrix} \rho \boldsymbol{v} \\ \rho \boldsymbol{v} \boldsymbol{v} + \left( p + \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{8\pi} \right) \boldsymbol{I} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi} \\ \boldsymbol{v} \boldsymbol{B} - \boldsymbol{B} \boldsymbol{v} \\ \left( \boldsymbol{E} + p + \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{8\pi} \right) \boldsymbol{v} - \frac{1}{4\pi} \left( \boldsymbol{v} \cdot \boldsymbol{B} \right) \boldsymbol{B} \end{pmatrix}$$

The tensor product of two vectors  $\boldsymbol{a}$  and  $\boldsymbol{b}$  is the tensor  $\boldsymbol{ab} = \boldsymbol{C}$  with elements  $c_{mn} = a_m b_n$ .

The total energy E is given by

$$E = \rho \epsilon + \rho \frac{\boldsymbol{v} \cdot \boldsymbol{v}}{2} + \frac{\boldsymbol{B} \cdot \boldsymbol{B}}{8\pi},$$

where  $\epsilon$  is the thermal energy per unit mass. The additional solenoidality constraint,

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0,$$

must also be fulfilled. The MHD equations must be closed by an equation of state which gives the gas pressure as a function of the density and the thermal energy per unit mass

$$p = p(\rho, \epsilon)$$
,

usually available to the program in tabulated form. The radiative source term is given by

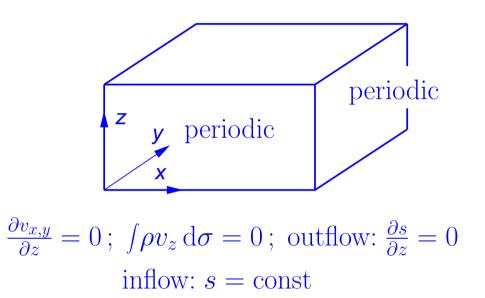
$$q_{\rm rad} = 4\pi\rho \int \kappa_{\nu} (J_{\nu} - B_{\nu}) \mathrm{d}\nu \,,$$

$$J_{\nu}(\boldsymbol{r}) = \frac{1}{4\pi} \oint I_{\nu}(\boldsymbol{r}, \boldsymbol{n}) \mathrm{d}\Omega, \qquad I(\boldsymbol{r}, \boldsymbol{n}) = I_0 e^{-\tau_0} + \int_0^{\tau_0} \left(\frac{\sigma}{\pi} T^4(\tau)\right) e^{-\tau} \mathrm{d}\tau$$

—— toc — ref –

Common boundary conditions for the thermal variables and velocities

$$\frac{\partial v_{x,y}}{\partial z} = 0; \ v_z = 0; \ \frac{\partial \epsilon}{\partial z} = 0 \quad \text{or} \quad \frac{\partial^2 \epsilon}{\partial z^2} = 0$$



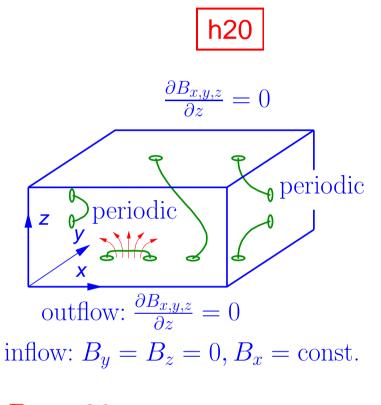
Both runs have *periodic lateral boundary conditions* in all variables. *Open bottom boundary* in the sense that the fluid can freely flow in and out of the computational domain under the condition of vanishing total mass flux.

Reflecting (closed) top boundary in run v10. Open top boundary in run h20.

Different boundary conditions for the magnetic field

**v10**  $B_{x,y} = 0; \quad \frac{\partial B_z}{\partial z} = 0$  **v** periodic **v** periodic

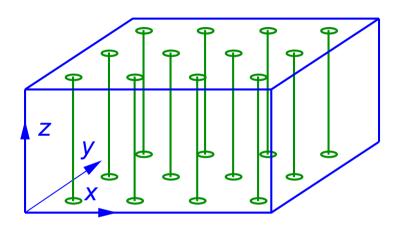
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 $B_x = 20 \text{ G}$ 

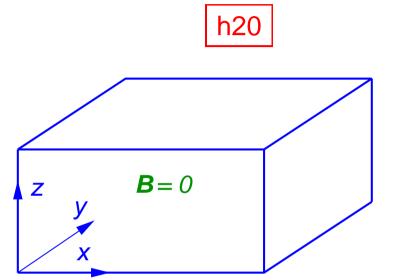
Different initial states for the magnetic field

v10

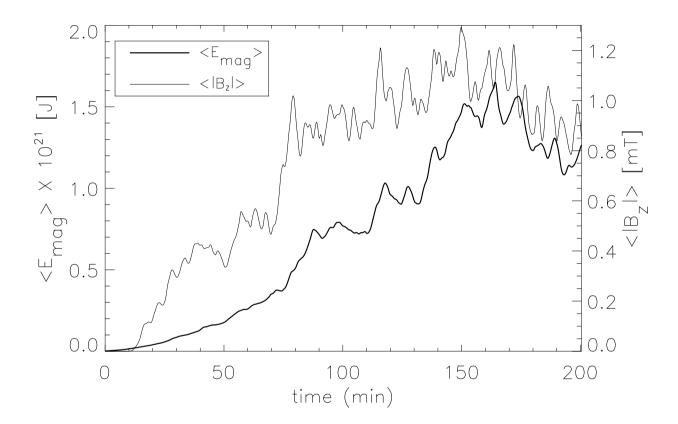


Initial homogeneous, vertical, unipolar **B**-field of 10 G.

toc — ref



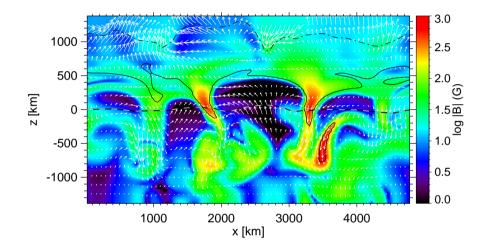
Fluid that enters the simulation domain from below carries horizontal magnetic field of a uniform flux density of 20 G and of uniform direction parallel to the x-axis



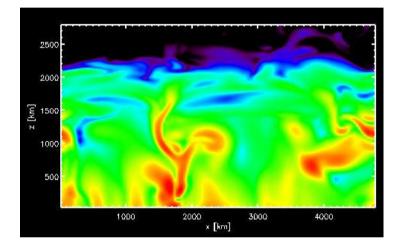
*Growth of magnetic energy* in the computational box of *run h20* and mean absolute vertical magnetic field strength at a fixed geometrical height corresponding to the mean optical depth unity,  $\langle |B_z| \rangle (\langle \tau_{500 \text{ nm}} \rangle = 1)$ . — toc — ref

Vertical cross sections through 3-D simulation domain







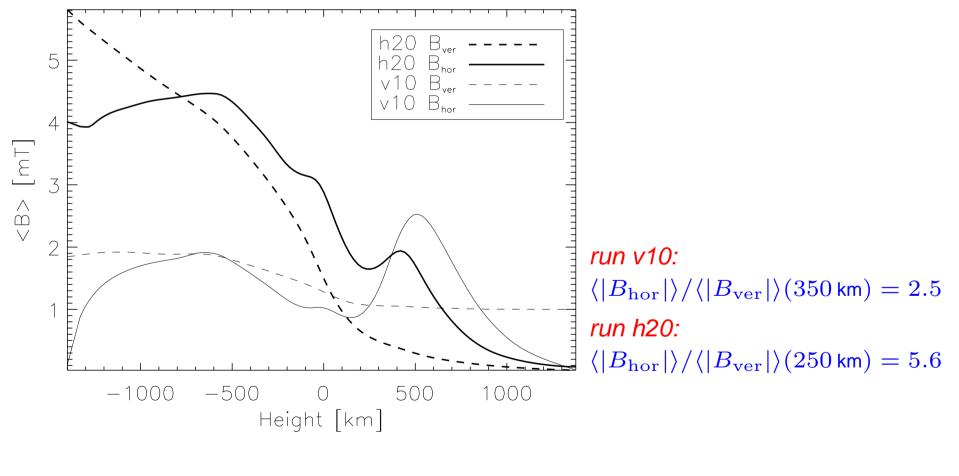


Colors indicate  $0 \le \log |B| \le 3.0$ 

Colors indicate  $0.5 \leq \log |B| \leq 2.5$ 

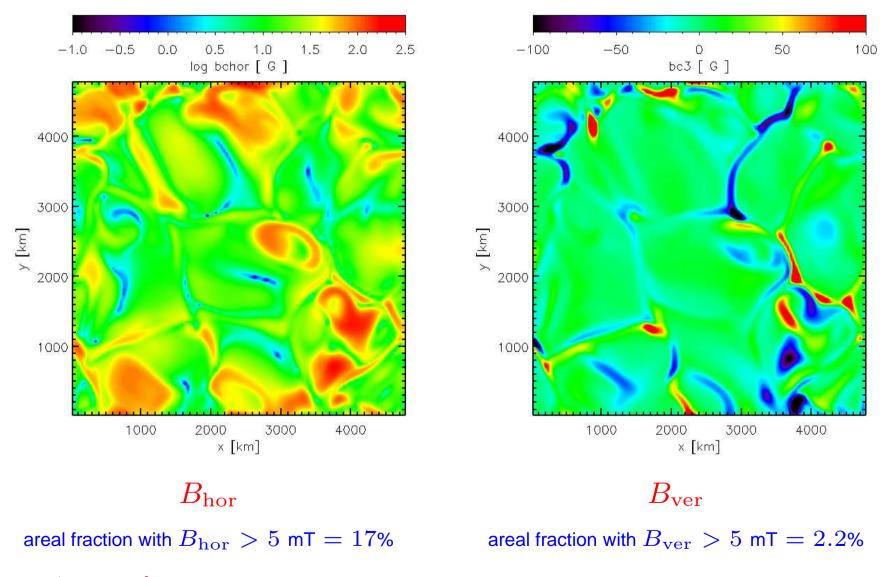
### 3. Structure and development of the horizontal magnetic field

Horizontally and temporally averaged absolute vertical and horizontal magnetic flux density as a function of height for both runs.



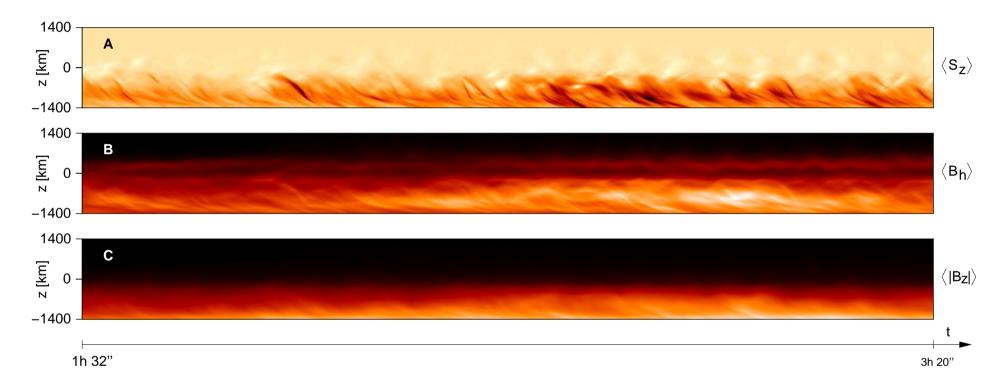
#### Structure and development of the horizontal magnetic field (cont.)

Snapshot of  $B_{\rm hor}$  and  $B_{\rm ver}$  of *run h20* in the horizontal section of  $\langle \tau_{500 \, \rm nm} \rangle = 1$ .



#### Structure and development of the horizontal magnetic field (cont.)

Vertically directed Poynting flux,  $\langle S_z \rangle$ ,  $\langle B_{hor} \rangle$ , and  $\langle |B_z| \rangle$  as a function of time and height in the atmosphere.

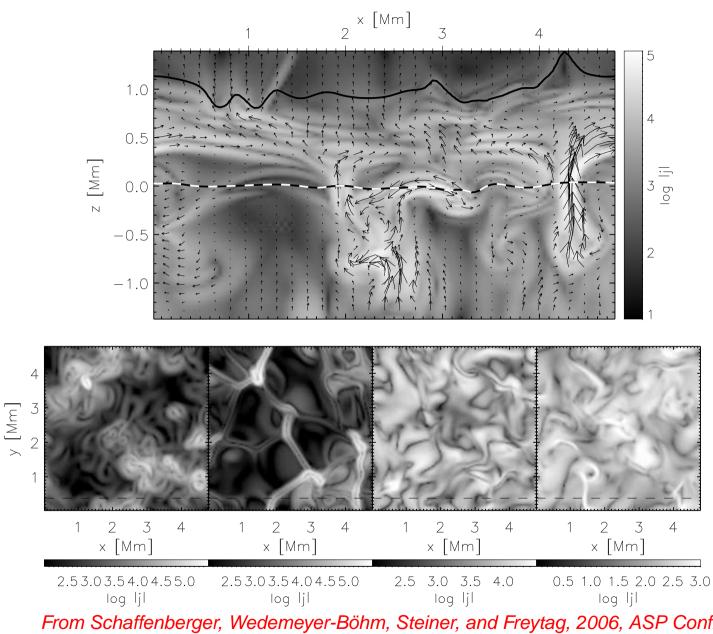


The surface of optical depth unity acts as a separatrix for the vertically directed Poynting

flux.

$$\mathbf{S} = \frac{1}{4\pi} (\mathbf{B} \times (\mathbf{v} \times \mathbf{B}))$$

#### Structure and development of the horizontal magnetic field (cont.)



Logarithmic current density,  $\log |j|$ , in a vertical cross section (top panel) and in four horizontal cross sections in a depth of 1180 km below, and at heights of 90 km, 610 km, and 1310 km above the average height of optical depth unity from left to right, respectively. The arrows in the top panel indicate the magnetic field strength and direction.

From Schaffenberger, Wedemeyer-Böhm, Steiner, and Freytag, 2006, ASP Conf. Ser., Vol. 354, p. 345

## 3.1. Comparison with polarimetry from Hinode

Lites et al. (2007) found from the deep mode series

$$\frac{\langle B_{\rm app}^{\rm T} \rangle}{\langle B_{\rm app}^{\rm L} \rangle} = \frac{55.3\,{\rm Mx\,cm^{-2}}}{11.0\,{\rm Mx\,cm^{-2}}} \approx 5~. \label{eq:Bapping}$$

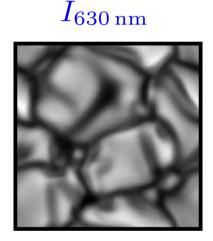
From the simulation we find

$$\frac{\langle B_{\rm hor} \rangle}{\langle |B_{\rm ver}| \rangle} = \begin{cases} 25 \,\mathrm{G}/10.2 \,\mathrm{G} = 2.5 & \text{for run v10} \\ 19.5 \,\mathrm{G}/3.8 \,\mathrm{G} = 5.6 & \text{for run h20} \end{cases}$$

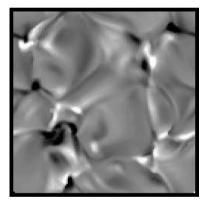
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But remember the difference between apparent and true magnetic flux density!

From the two simulation runs, we synthesized the Stokes profiles of both 630 nm Fe I spectral lines observed by the Hinode SP. Profiles were computed with the radiative transfer code SIR along vertical lines of sight ( disk center) with a spectral sampling of 2 pm.



 $V_{\rm tot\,630\,nm}$ 



For a faithful comparison with the results of Lites et al. we subject the synthetic profiles to exactly the same procedure for conversion to apparent flux density as was done with the real data.

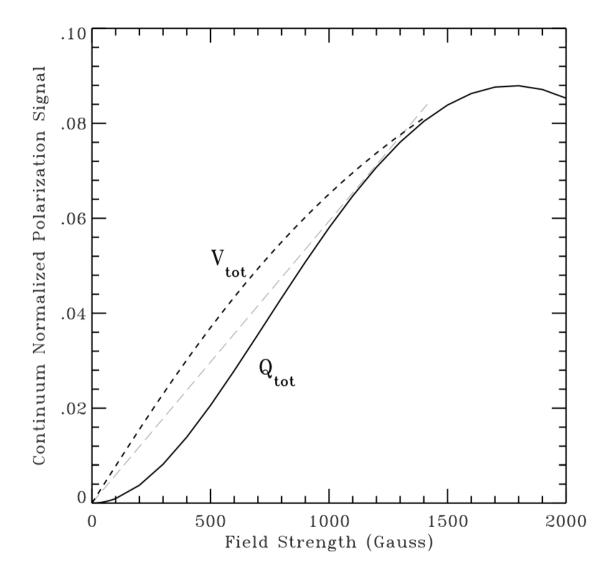
The conversion to apparent flux densities proceeds by *first* computing

$$V_{\rm tot} = \operatorname{sign}(V_{\rm blue}) \frac{\left|\int_0^{\lambda_0} V(\lambda) d\lambda\right| + \left|\int_{\lambda_0}^{\infty} V(\lambda) d\lambda\right|}{I_c \int d\lambda} ,$$

and

$$Q_{\text{tot}} = \frac{\int Q(\lambda) Q_{\text{mask}}(\lambda) d\lambda}{I_c \int Q_{\text{mask}}(\lambda) d\lambda} ,$$

where Q is mesured relative to the "preferred-frame azimuth"  $\phi_r$ , viz, relative to the projection of the magnetic field vector on the plane of sky.  $Q_{mask}$  is a weightening factor that is the normalized sum of all the Q-profiles in the preferred-frame azimuth. *Second*, we compute  $V_{tot}$  and  $Q_{tot}$  for a standard atmosphere with a given (known) set of homogeneous magnetic fields for establishing a *calibration curve*.



Calibration curve from Lites et al. 07 derived from a Milne-Eddington atmosphere with a homogeneous horizontal magnetic field for  $Q_{tot}$  and a magnetic field inclined by  $45^{\circ}$ for  $V_{tot}$ .

Lites et al. (2007) found from the deep mode series

$$\frac{\langle B_{\rm app}^{\rm T}\rangle}{\langle B_{\rm app}^{\rm L}\rangle} = \frac{55.3\,{\rm Mx\,cm^{-2}}}{11.0\,{\rm Mx\,cm^{-2}}}\approx 5~. \label{eq:Bapping}$$

From the simulation we find

$$\frac{\langle B_{\rm hor} \rangle}{\langle |B_{\rm ver}| \rangle} = \begin{cases} 25 \,\mathrm{G}/10.2 \,\mathrm{G} = 2.5 & \text{for run v10} \\ 19.5 \,\mathrm{G}/3.8 \,\mathrm{G} = 5.6 & \text{for run h20} \end{cases}$$

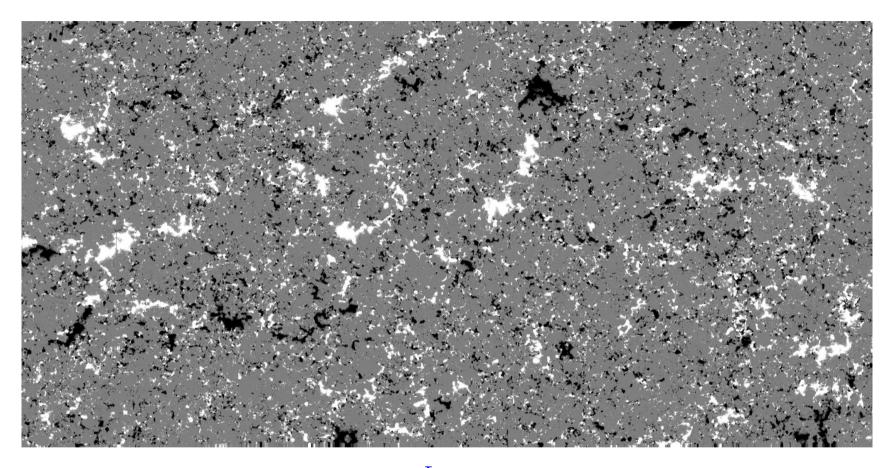
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From the synthesized Fe I 630 nm Stokes profile pair we find

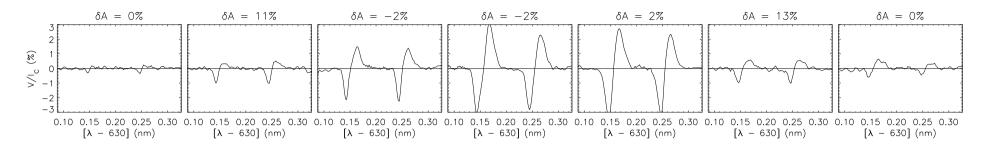
$$\frac{\langle B_{\rm app}^{\rm T} \rangle}{\langle B_{\rm app}^{\rm L} \rangle} = \begin{cases} 11.5 \,\text{G}/7.5 \,\text{G} = 1.5 & \text{for run } \textit{v10} \\ 24.8 \,\text{G}/8.8 \,\text{G} = 2.8 & \text{for run } \textit{h20} \end{cases}$$

### 4. The structure of internetwork magnetic elements

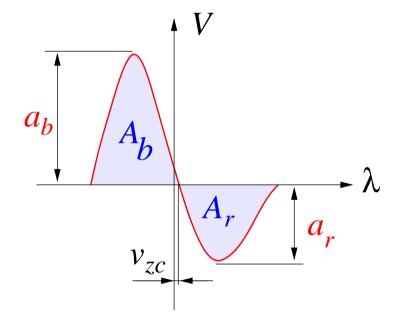


Apparent vertical magnetic flux density  $B_{\rm app}^{\rm L}$  of the quiet Sun over a field of view of  $302'' \times 162''$  observed from the Hinode space observatory. The grey scale saturates at  $\pm 50 \,\text{Mx}\,\text{cm}^{-2}$ . 2048 steps to 5 s.

#### The structure of internetwork magnetic elements (cont.)

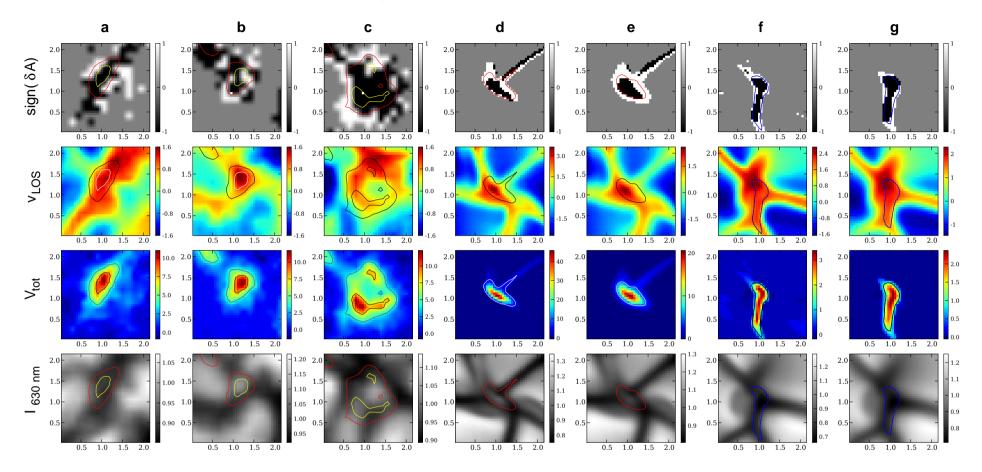


Stokes-V profiles across a magnetic element of the internetwork from the Hinode data.



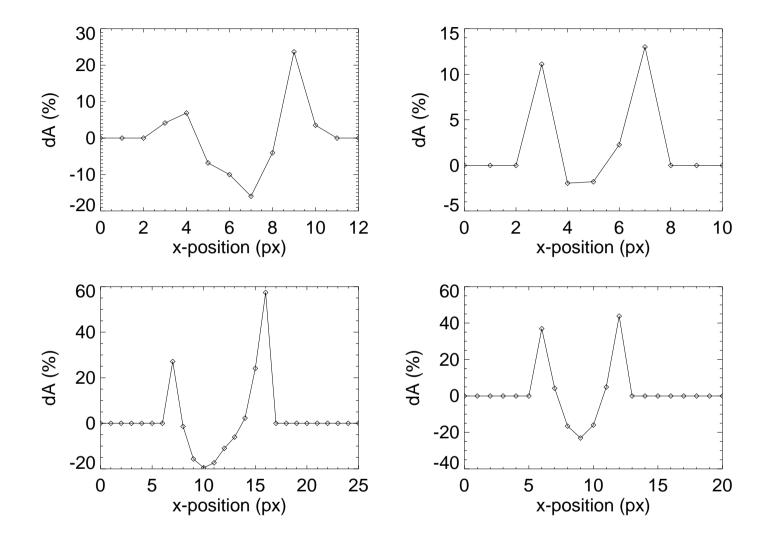
$$\delta A := \frac{A_b - A_r}{A_b + A_r}$$
sign( $\delta A$ ) = -sign( $\frac{d|B|}{d\tau} \cdot \frac{dv(\tau)}{d\tau}$ )
Solanki & Pahlke, 1988; Sanchez Almeida
et al., 1989

#### The structure of internetwork magnetic elements (cont.)



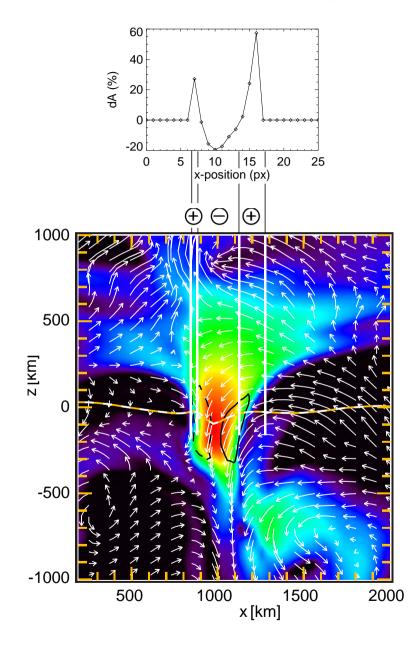
Columns a-c: observational data obtained with the spectro-polarimeter of Hinode/SOT. Columns d and f: synthetic data from the 3-D MHD simulation. Columns e and g: same as d and f but after application of the SOT-PSF to the synthetic intensity maps. Distance between tick marks is 0.5''. From Rezaei, Steiner, Wedemeyer-Böhm et al. 2007, A&A 476, L33

#### The structure of internetwork magnetic elements (cont.)



Variation in  $\delta A$  across magnetic elements from the Hinode data *(top row)* and the simulation *(bottom row)*.

#### The structure of internetwork magnetic elements (cont.)



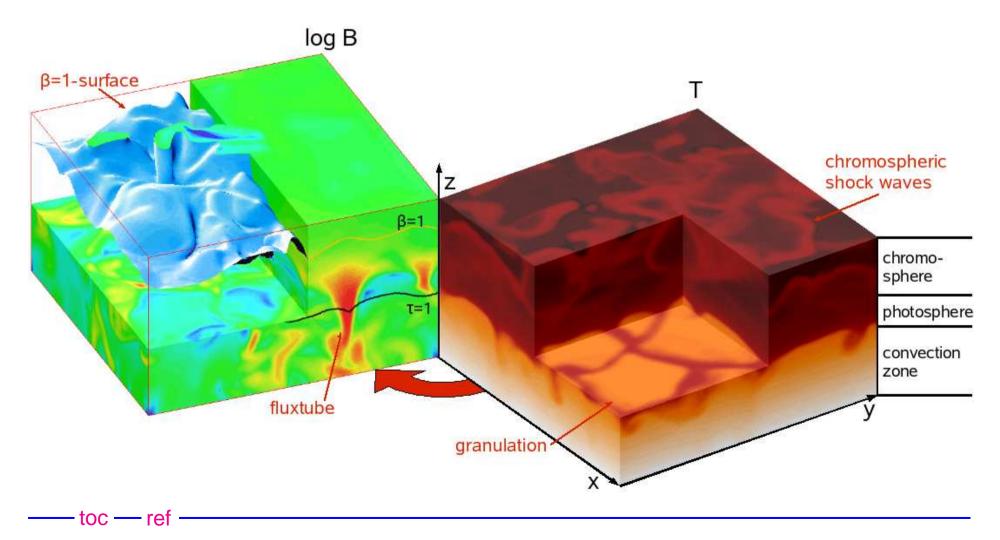
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$$v=0$$

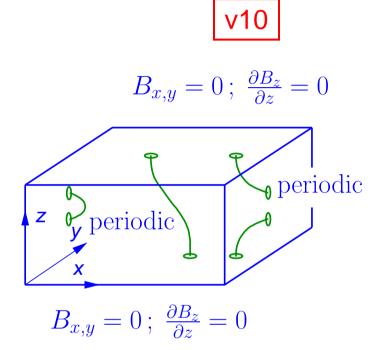
$$\left. \begin{array}{c} \frac{\mathrm{d}|B|}{\mathrm{d}\tau} < 0\\ \frac{\mathrm{d}v(\tau)}{\mathrm{d}\tau} > 0 \end{array} \right\} \Rightarrow \delta A > 0$$

Vertical cross section through the simulation box. *Colour* displays the logarithmic magnetic field strength, *arrows* the velocity field, *black contours* the electric current density normal to the plane. The *white vertical lines* indicate ranges of either positive or negative area asymmetry,  $\delta A$ .

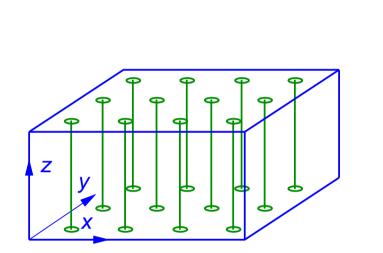
Synoptic summary of magnetoconvective simulation from the convection zone to the chromosphere.



Case v10: boundary and initial conditions for the magnetic field

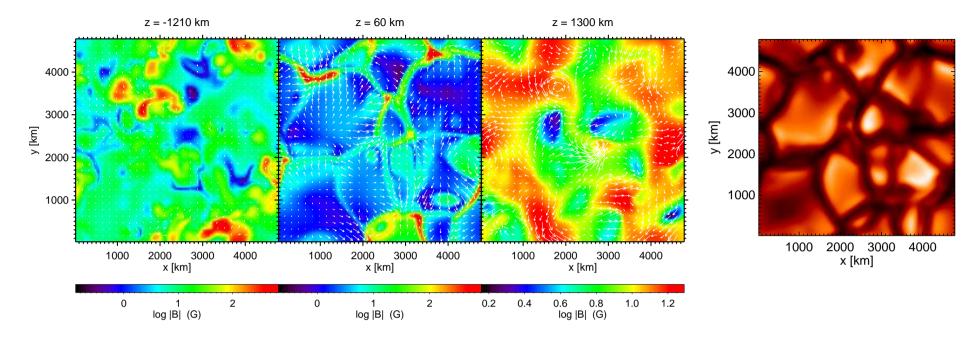


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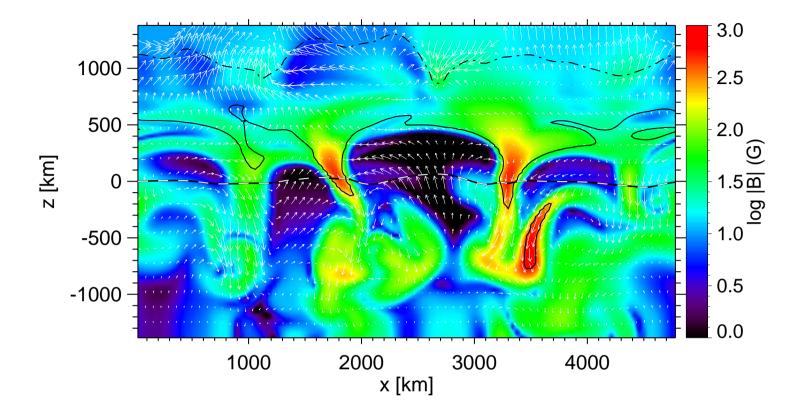


Initial homogeneous, vertical, unipolar **B**-field of 10 G.

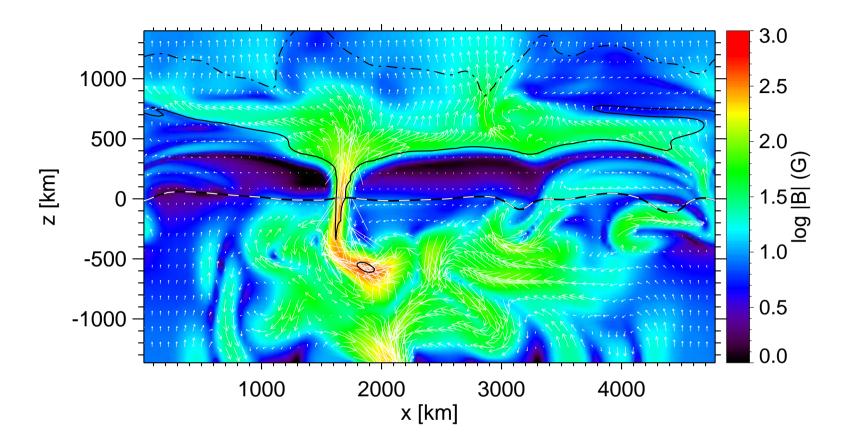
v10



Horizontal sections through 3-D computational domain. Color coding displays  $\log |B|$  with individual scaling for each panel. Left: Bottom layer at a depth of 1210 km. Middle: Layer 60 km above optical depth  $\tau_c = 1$ . Right: Top, chromospheric layer in a height of 1300 km. White arrows indicate the horizontal *velocity* on a common scaling. Longest arrows in the panels from left to right correspond to 4.5, 8.8, and 25.2 km/s, respectively. Rightmost: Emergent *intensity*.



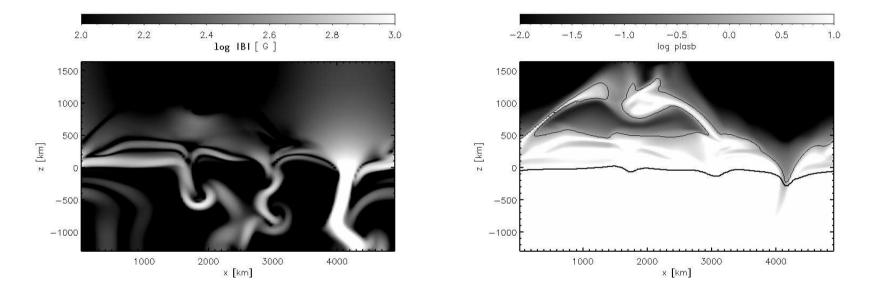
Snapshot of a vertical section showing  $\log |B|$  (color coded) and *velocity vectors* projected on the vertical plane (white arrows). The b/w dashed curve shows optical depth unity and the dot-dashed and solid black contours  $\beta = 1$  and 100, respectively. movie with  $\beta = 1$  surface Schaffenberger, Wedemeyer-Böhm, Steiner & Freytag, 2005, in *Chromospheric and Coronal Magnetic Fields*, Innes, Lagg, Solanki, & Danesy (eds.), ESA Publication SP-596



Snapshot of a vertical section showing  $\log |B|$  (color coded) and **B** projected on the vertical plane (white arrows). The b/w dashed curve shows optical depth unity and the dot-dashed and solid black contours  $\beta = 1$  and 100, respectively. Schaffenberger, Wedemeyer-Böhm, Steiner & Freytag, 2005, in *Chromospheric and Coronal Magnetic Fields*, ESA Publication SP-596

# 5.1. Wave propagation in a magnetically structured atmosphere

Time sequence of a two-dimensional simulation of magnetoconvection starting with an initial homogeneous vertical magnetic field of 100 G.



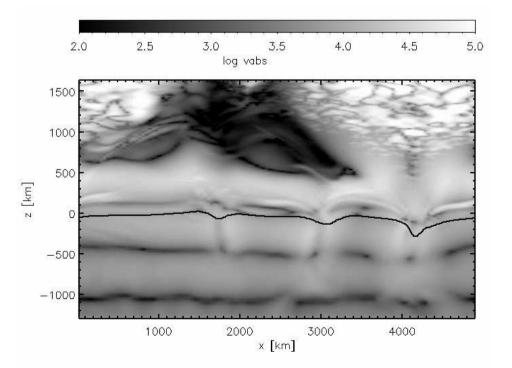
*Left:* Logarithmic magnetic field strength 1368 s after starting with an initial homogeneous vertical field of 100 G. *Right:* Logarithm of thermal to magnetic energy density (plasma- $\beta$ ) together with the conour of  $\beta = 1$ .

Wave propagation in a magnetically structured atmosphere (cont.)

Residual velocity amplitude due to an oscillatory velocity perturbation along the bottom

$$v_z(t) = v_0 \sin(2\pi (t - t_0)\nu)$$

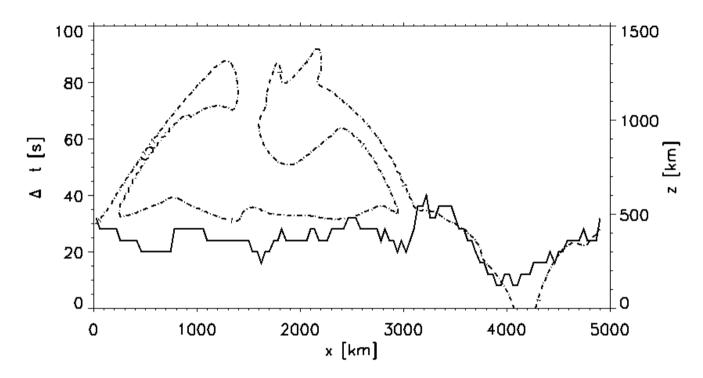
with an amplitude of  $v_0 = 50$  m/s and a frequency of  $\nu_0 = 20$  mHz from t = 1200 s to t = 1450 s. Note the fast magnetic wave that gets refracted.



From Steiner, Vigeesh, Krieger et al.: 2007, Astron. Nachr. / AN 328, 323

Wave propagation in a magnetically structured atmosphere (cont.)

Wave travel time vs. canopy height.

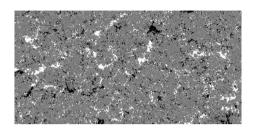


Wave travel time across the layer from z = 200 km to z = 420 km as a function of horizontal distance (thick solid curve). Superposed is the contour of  $\beta = 1$  (magnetic and thermal equipartition), for which the height is indicated in the right hand side ordinate (dash-dotted curve). Note that the travel time markedly decreases where the low  $\beta$  region intrudes this layer.

– toc – ref

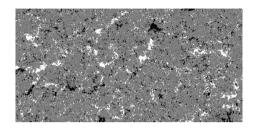


## 6. Conclusion

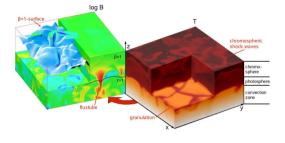


High resolution polarimetry with Hinode

# 6. Conclusion

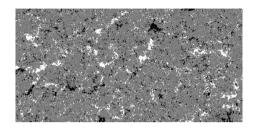


High resolution polarimetry with Hinode

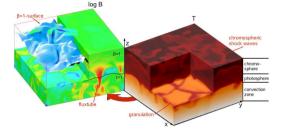


State of the art 3-D MHD-code for the numerical simulation of near surface magnetoconvection

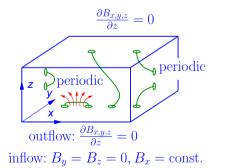
# 6. Conclusion



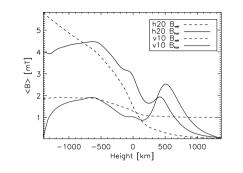
### High resolution polarimetry with Hinode



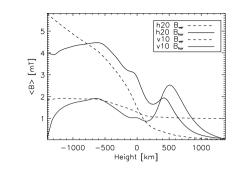
State of the art 3-D MHD-code for the numerical simulation of near surface magnetoconvection



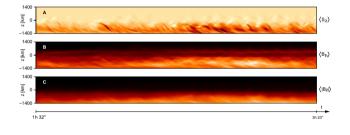
Two runs with different initial and boundary conditions



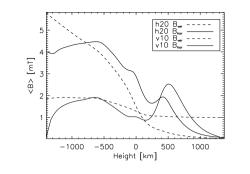
Strong horizontal fields in numerical simulations of the internetwork



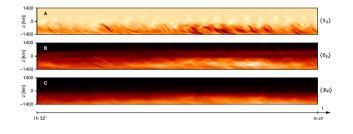
Strong horizontal fields in numerical simulations of the internetwork



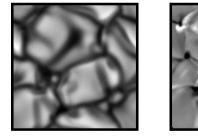
 $\tau=1\mbox{-surface constitutes}$  a separatrix for the Poynting flux



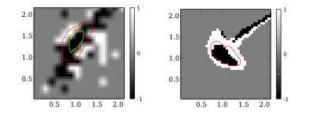
Strong horizontal fields in numerical simulations of the internetwork



 $\tau=1\mbox{-surface constitutes}$  a separatrix for the Poynting flux



Synthesized Stokes parameters yield  $\langle B_{\rm hor} \rangle / \langle B_{\rm ver} \rangle = 1.5 \dots 2.8$ 



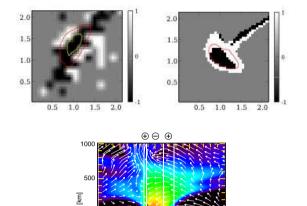
Peculiar pattern of positive area asymmetry of Stokes-V profiles in the periphery of internetwork magnetic elements .....

-1000

500

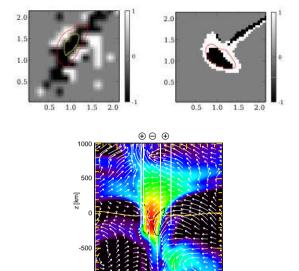
toc — ref

1000 1500 x [km] 2000



Peculiar pattern of positive area asymmetry of Stokes-V profiles in the periphery of internetwork magnetic elements .....

..... due to their funnel shaped magnetic boundary layer



-500 -1000 

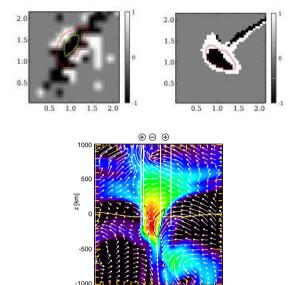
1000 1500 x [km] Peculiar pattern of positive area asymmetry of Stokes-V profiles in the periphery of internetwork magnetic elements .....

..... due to their funnel shaped magnetic boundary layer

The magnetic field of the internetwork chromosphere

— toc — ref

-1000



1000 1500 x [km]

500

2000

 $\frac{1000}{50}$ 

2000

toc — ref

3000

x [km]

4000

5000

Peculiar pattern of positive area asymmetry of Stokes-V profiles in the periphery of internetwork magnetic elements .....

..... due to their funnel shaped magnetic boundary layer

The magnetic field of the internetwork chromosphere

Feasibility of atmospheric magneto-seismology

## **Table of content**

- 1. Observations with Hinode
- 2. Numerical simulation of near surface magnetoconvection
  - 2.1. MHD-simulation from the convection zone to the chromosphere
- 3. Structure and development of the horizontal magnetic field
  - 3.1. Comparison with polarimetry from Hinode
- 4. The structure of internetwork magnetic elements
- 5. The magnetic field of the internetwork chromosphere
  - 5.1. Wave propagation in a magnetically structured atmosphere
- 6. Conclusions