

Exercises

1. Let X_1, X_2, \dots, X_n be iid with density $f_\theta(x)$ given by

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(x-\theta)^2}{2}\right\}, \quad -\infty < x < \infty.$$

Find the maximum likelihood estimator of θ .

2. Let X_1, X_2, \dots, X_n be iid with common continuous distribution having density $f_\theta(x)$ given by

$$f_\theta(x) = \frac{1}{\theta} \exp\left\{-\frac{x}{\theta}\right\}, \quad x \geq 0;$$

$f_\theta(x) = 0$ otherwise. ($\theta > 0$.)

(i) Find $E(X_i)$ and $\text{Var}(X_i)$.

(ii) Find the maximum likelihood estimator of θ .

3. Let X_1, X_2, \dots, X_n be iid with density $f_\lambda(x)$ given by

$$f_\lambda(x) = \lambda \exp\{-x\lambda\}, \quad x \geq 0;$$

$f_\lambda(x) = 0$ otherwise. ($\lambda > 0$.) Find $E(X_i)$ and $\text{Var}(X_i)$. Find the maximum likelihood estimator of λ .

4. Let X_1, X_2, \dots, X_n be iid with Normal distribution with mean θ and variance 64. Consider the problem of testing Null Hypothesis $H_0 : \theta = 2$ against the alternate hypothesis $H_1 : \theta = 1$.

Suggest a critical region for the test. Specify the exact critical region with level of significance 0.05. Further suppose that we have $n = 25$ observations x_1, x_2, \dots, x_{25} and we have

$$\sum_{i=1}^{25} x_i = 24.8.$$

Compute the p -value and write down the conclusion.

5. Let X_1, X_2, \dots, X_n be iid with Normal distribution with mean θ and variance 64. Consider the problem of testing Null Hypothesis $H_0 : \theta = 1$ against the alternate hypothesis $H_1 : \theta = 2$.

Suggest a critical region for the test. Specify the exact critical region with level of significance 0.05. Further suppose that we have $n = 25$ observations x_1, x_2, \dots, x_{25} and we have

$$\sum_{i=1}^{25} x_i = 24.8.$$

Compute the p -value and write down the conclusion.

6. Let X_1, X_2, \dots, X_{64} be iid $P(X_i = 0) = 1 - p$ and $P(X_i = 1) = p$. Consider the problem of testing Null Hypothesis $H_0 : p = 0.5$ against the alternate hypothesis $H_1 : p = 0.45$.

Suggest a critical region for the test. Specify the exact critical region with level of significance 0.05. Further suppose that we have

$$\sum_{i=1}^{64} X_i = 28.$$

Write down the conclusion.

7. Let X_1, X_2, \dots, X_{36} be iid with Normal distribution with mean μ and variance 25. Write down the critical region for testing $H_0 : \mu = 0$ against the alternative $H_1 : \mu = 1$ for level of significance 5%. Find the power of the test when (i) $n=16$, (ii) $n=25$, (iii) $n=36$, (iv) $n=64$ and (v) $n=100$.

(use software or normal tables).