

Questions

(1) Why there are a very small number of planets around pulsars?

(2) What happened to the companion of isolated millisecond pulsars?

Effect of Gravitational Tidal Force

Potential energy of a gaseous sphere of radius R_P and mass M_P

$$E_{P} = -G \int_{0}^{R_{P}} \frac{m(r)}{r} \frac{dm(r)}{dr} dr$$

For uniform density:
$$E_P = -\frac{3}{5} \frac{GM_P^2}{R_P}$$

For a polytropic gas configuration with index n=1 ($P \propto \rho^2$)

$$E_P = -\frac{3}{4} \frac{GM_P^2}{R_P}$$

Effect of Gravitational Tidal Force

Under the effect of strong gravity of the pulsar, the iso-potentials around the planet at high altitude are no longer spherical and are open to infinity when the potential reaches the value of the Roche lobe. Therefore the energy needed to escape the planetary surface can be defined by the energy needed to reach this Roche lobe.

The modified potential energy of the planet can be written as $:E_P'=E_P+\Delta_{tidal}E_P$

where

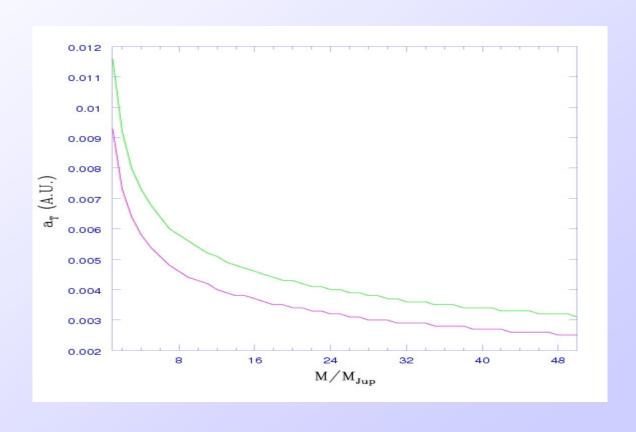
$$\Delta_{tidal} E_P = \int_0^{R_P} \frac{3^{4/3}}{2} \frac{GM^{1/3} m(r)^{2/3}}{a} \frac{dm(r)}{dr} dr$$

$$= \frac{3^{7/3}}{10} \frac{GM^{1/3} M_P^{5/3}}{a}$$

$$E_P' = -\frac{3}{4} \frac{GM_P^{-2}}{R_P} + \frac{3^{7/3}}{10} \frac{GM_P^{-1/3}M_P^{-5/3}}{a}$$

Radius of Tidal Destruction Zone

$$a_T = \frac{2}{5} 3^{4/3} R_P \left(\frac{M}{M_P}\right)^{1/3}$$



Effect of X-Ray Irradiation

The rate of energy absorbed by a planet from the pulsar with accretion

luminosity L_{Acc} is

$$\frac{dE_{Acc}}{dt} = 0.5\pi (1 - \omega_x) L_{Acc} \left(\frac{R_P}{a}\right)^2$$

Let
$$-dE'_{P} = dE_{Acc}$$

Then the typical life time of the planet or the time taken to fill the whole potential well T can be derived as

$$T = \frac{3Ga^{2} \left(\frac{M_{P}}{R_{P}}\right)^{2}}{\pi (1 - \omega_{X}) L_{Acc}} \left[\frac{1}{2R_{P}} - \frac{3^{4/3}}{5a} \left(\frac{M}{M_{P}}\right)^{1/3} \right]$$

where
$$a > a_T$$

Answers

(1) Why there are a very small number of planets around pulsars?

They are eaten by their host pulsar !!

| Pulsar planet | M _p sini (M _J) | a (A.U.) | Conclusion |
|----------------------|---------------------------------------|----------|-----------------------------------|
| PSR 1257+12b | 7X10 ⁻⁵ | 0.19 | Rocky planet |
| PSR 1257+12c | 0.013 | 0.36 | Either rocky or |
| PSR 1257+12d | 0.012 | 0.46 | L_{Acc} <10 ²⁸ erg/s |
| PSR B1620-26b | 2.5 | 23.0 | survives |

(2) What happened to the companion of isolated millisecond pulsars?

The companion, a Jupiter like planet, filled up the Roche lobe and donated its angular momentum to the normal pulsar making it a millisecond pulsar through the usual way and completely evaporated within a few Gyr.