Polarization of Coherent Curvature Radition in Pulsars

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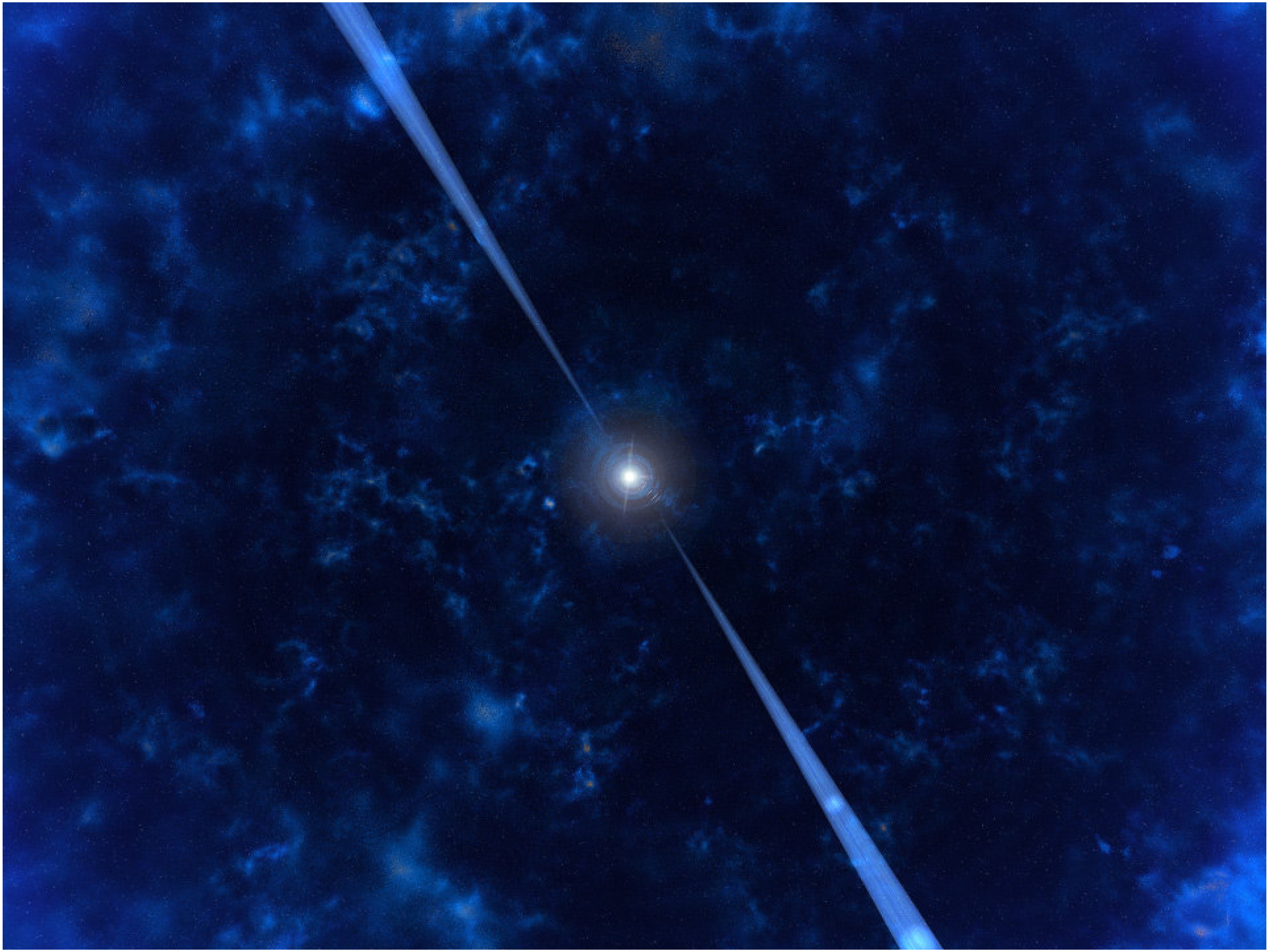
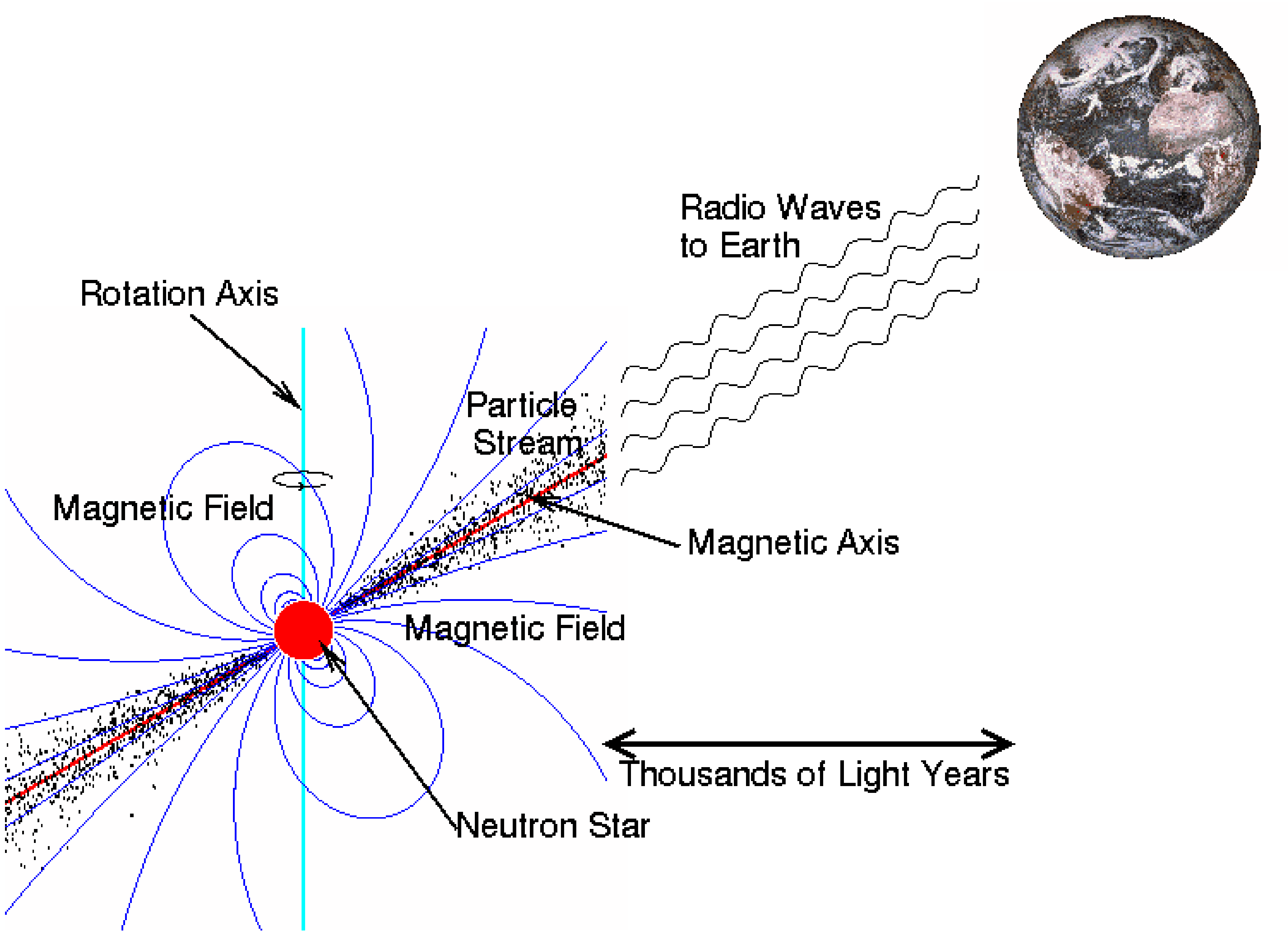
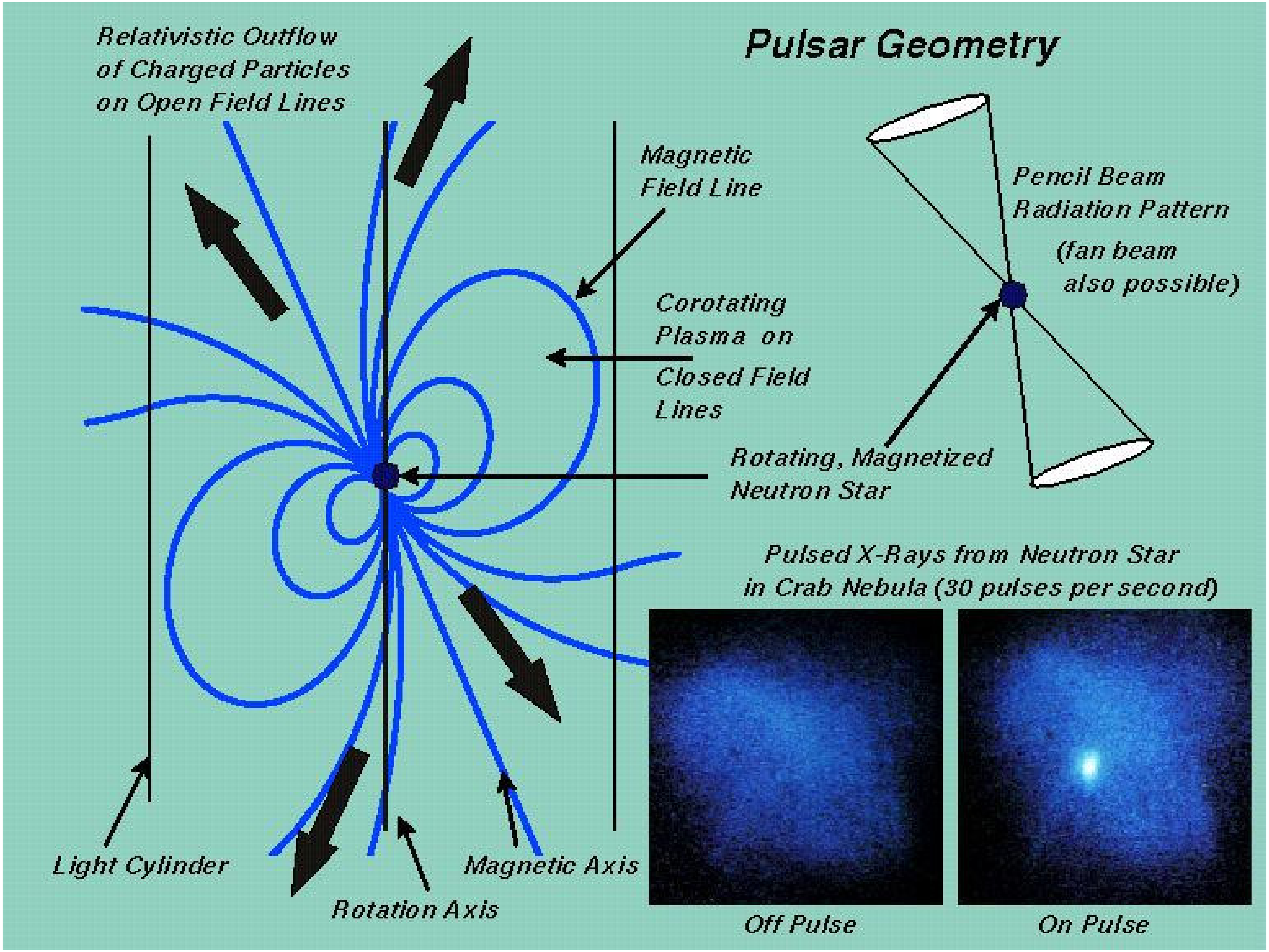
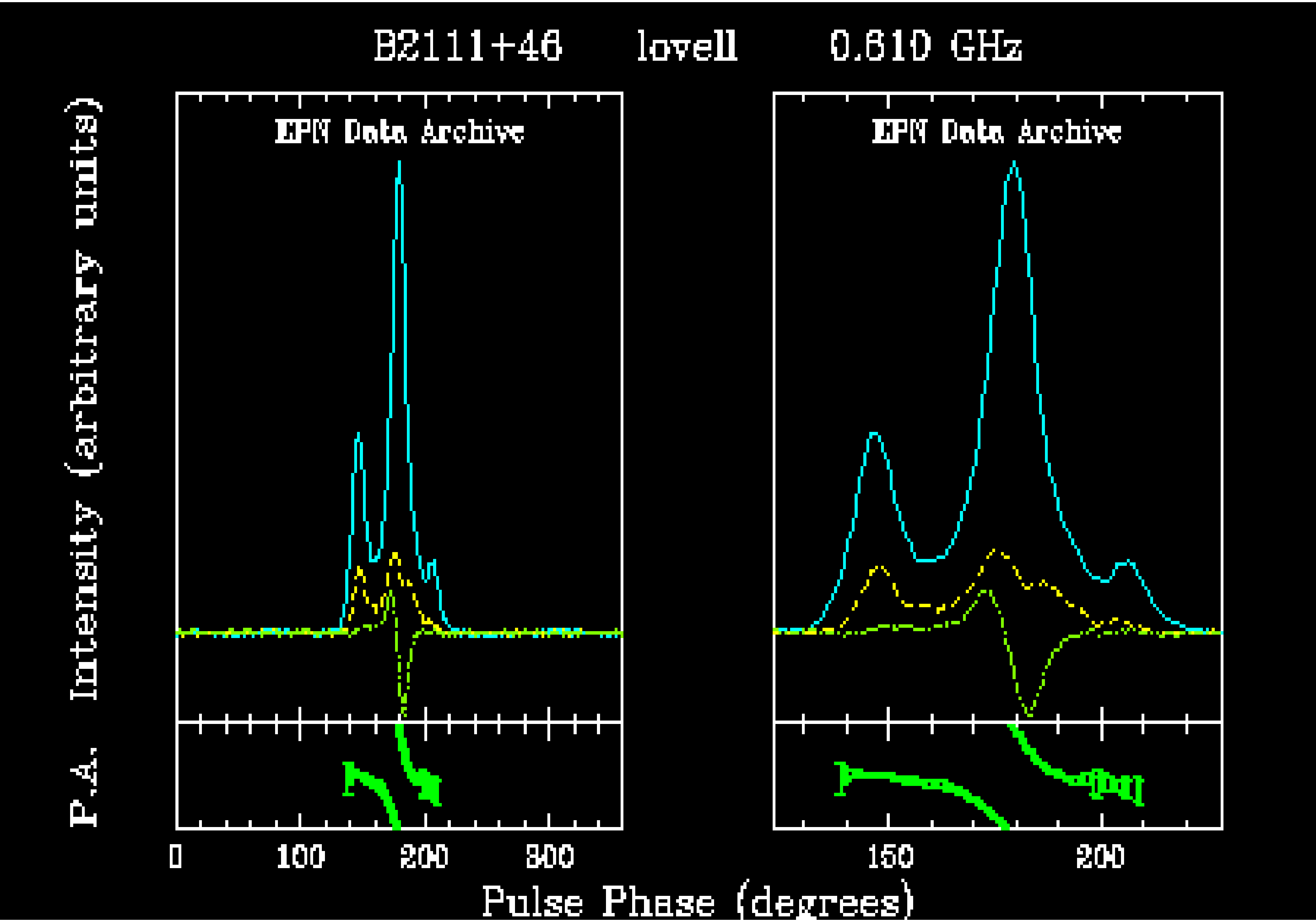
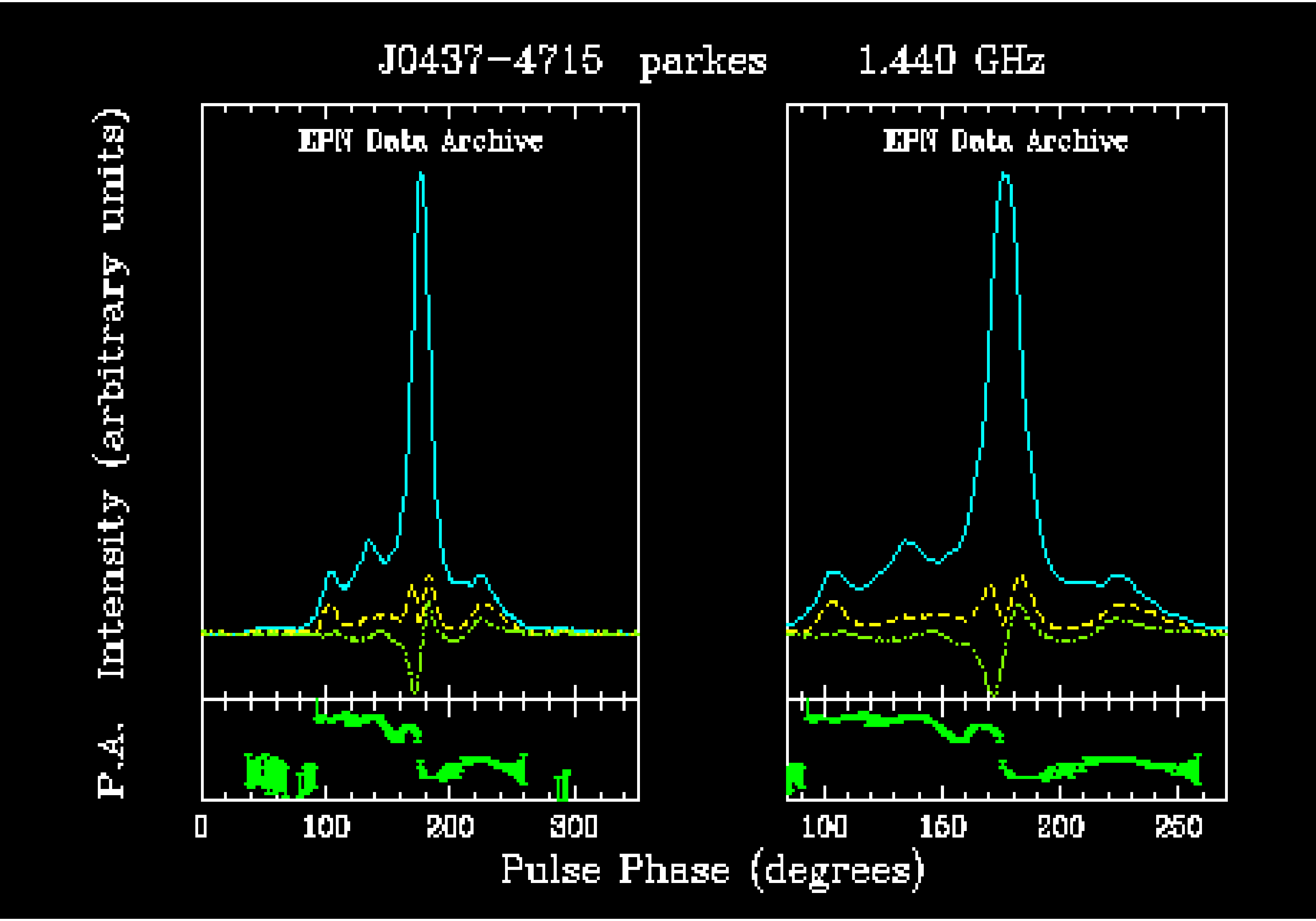


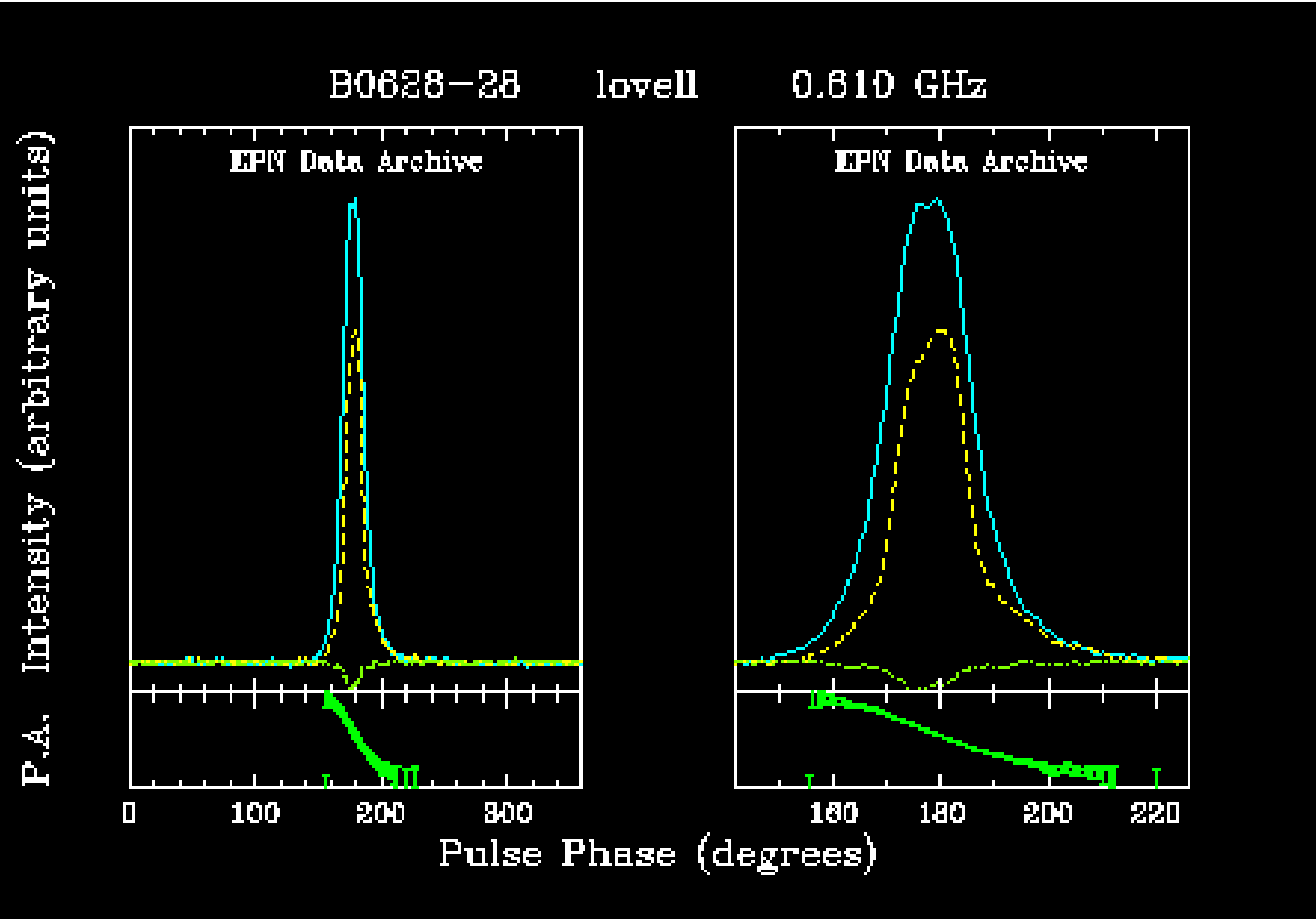


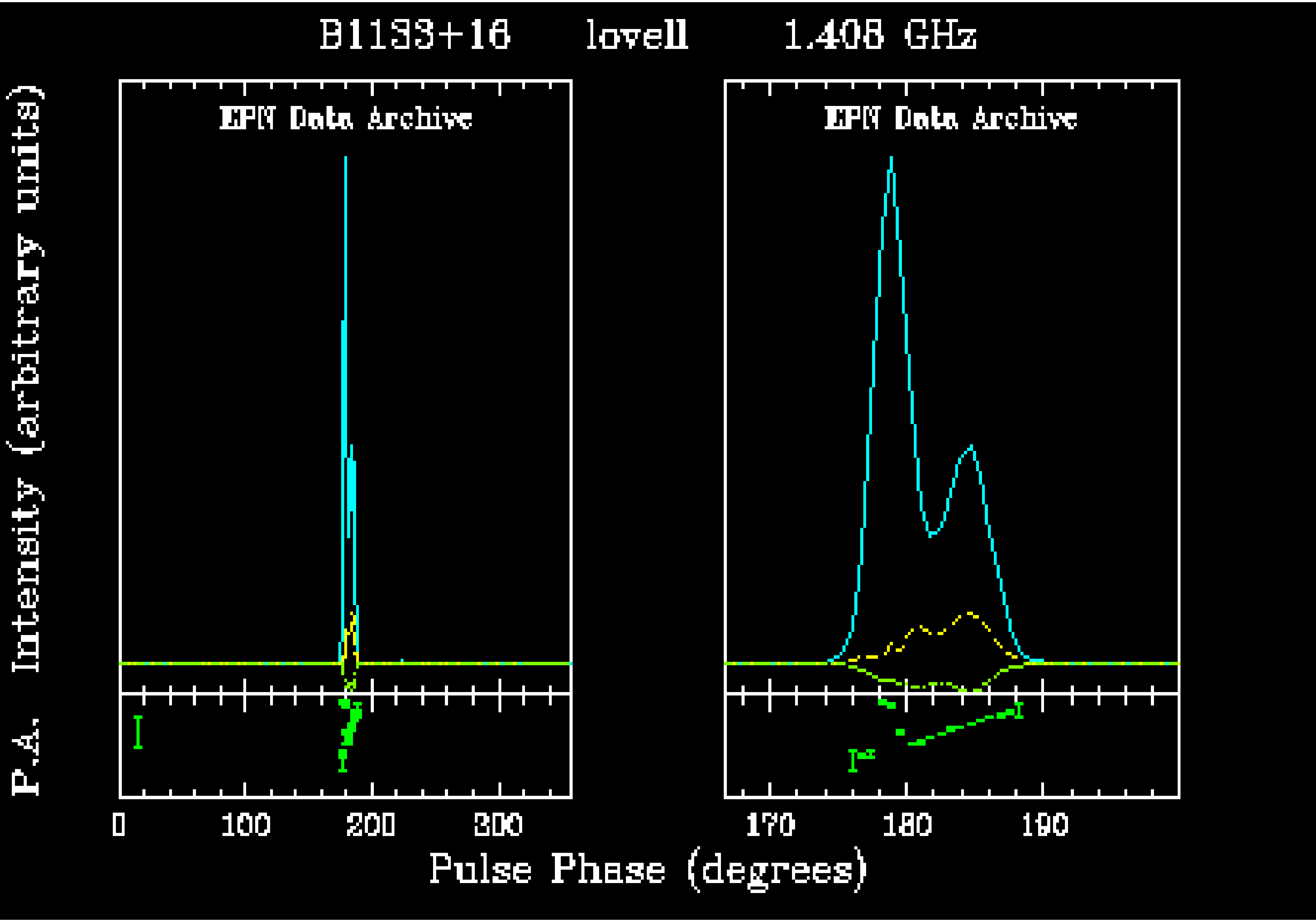
Radio Waves
to Earth
Rotation Axis
Particle
Stream
Magnetic Field
Magnetic Axis
Magnetic Field
Thousands of Light Years
Neutron Star


Pulsar Geometry
Relativistic Outflow
of Charged Particles
on Open Field Lines
Magnetic
Field Line
Corotating
Plasma on
Closed Field
Lines
Rotating, Magnetized
Neutron Star
Pencil Beam
Radiation Pattern
(fan beam
also possible)
Pulsed X-Rays from Neutron Star
in Crab Nebula (30 pulses per second)
Light Cylinder
Rotation Axis
Magnetic Axis
Off Pulse
On Pulse


B2111+46 lovell 0.610 GHz
EPN Data Archive
EPN Data Archive
Intensity (arbitrary units)
P.A.
0
100
200
300
150
200
Pulse Phase (degrees)


J0437-4715 parkes 1.440 GHz
EPN Data Archive
EPN Data Archive
Intensity (arbitrary units)
P.A.
0
100
200
300
100
150
200
250
Pulse Phase (degrees)


B0628-28 lovell 0.610 GHz
EPN Data Archive
EPN Data Archive
Intensity (arbitrary units)
P.A.
0
100
200
300
160
180
200
220
Pulse Phase (degrees)


B1133+16 lovell 1.408 GHz
EPN Data Archive
EPN Data Archive
Intensity (arbitrary units)
P.A.
0
100
200
300
170
180
190
Pulse Phase (degrees)

- The observations such as polarization angle swing, sense reversing circular polarization and high brightness temperature favor the coherent curvature radiation as the emission mechanism of pulsars.
- We consider the detailed geometry of the emission region in non-rotating or very slowly rotating dipole magnetic field, and study the polarization features of coherent curvature radiation.
- The emitted radiation is highly beamed in the direction of velocity of source within an angular width of $\approx 2/\gamma$, where γ is the Lorentz factor.

Radiation Electric Fields

- $E' = E'_{||}\hat{\epsilon}'_{||} + E'_{\perp}\hat{\epsilon}'_{\perp}$
-

$$\begin{aligned} E'_{||} &= -\mu\, C_{||}\; L_1(\mu) \\ E'_{\perp} &= C_{\perp}\; L_2(\mu)\,, \end{aligned} \tag{1}$$

-

$$\begin{aligned} C_{\perp} &= \frac{\omega e^{i\omega R/c}}{2\sqrt{\pi}\,R\,c^2}(J_0\, s_0\, \zeta_0\, \eta_0)\frac{\sin[(k-k_{\rm p})s_0/2]}{(k-k_{\rm p})s_0/2}\,\frac{\sin(k\eta_0\mu/2)}{k\eta_0\mu/2}\left(\frac{36\rho}{\omega^2\, c}\right)^{1/3} \\ C_{||} &= C_{\perp}\sqrt{\frac{c_1}{3}} \\ c_1 &= \frac{1}{2}\left(\frac{6\,\omega^2\rho^2}{c^2}\right)^{1/3} = 6^{1/3}\left(\frac{\rho\,\omega}{c}\right)^{2/3}\,, \end{aligned} \tag{2}$$

The angle μ

- $$\mu \simeq C_\theta \, \theta'_{\rm b} + C_\phi \, \phi'_{\rm b} + C_{\phi 0} \; , \qquad (3)$$

- $$\begin{aligned} C_\theta &= -\frac{\sin\Gamma \sin\alpha \sin\phi'}{(\cos\zeta\sin\alpha - \cos\alpha\sin\zeta\cos\phi')^2 + \sin^2\zeta\sin^2\phi'} - \\ &\quad \sigma \sin\alpha \, \cos\theta'_p \cos\phi' \\ C_\phi &= \frac{\sin\Gamma \sin\zeta(\cos\alpha\sin\zeta - \cos\zeta\sin\alpha\cos\phi')}{(\cos\zeta\sin\alpha - \cos\alpha\sin\zeta\cos\phi')^2 + \sin^2\zeta\sin^2\phi'} - \\ &\quad \sigma \, \sin\alpha \sin\theta'_p \sin\phi' \\ C_{\phi 0} &= \sigma(\cos\alpha\cos\zeta + \sin\alpha\sin\theta'_{\rm p}\cos\phi') \; , \end{aligned} \qquad (4)$$

- $$\sigma = \frac{\rho_{\rm i}}{r_{\rm L}} \sin\alpha \, \cos\phi_{\rm n} \cos\Gamma \; .$$

The Polarization of Radiation

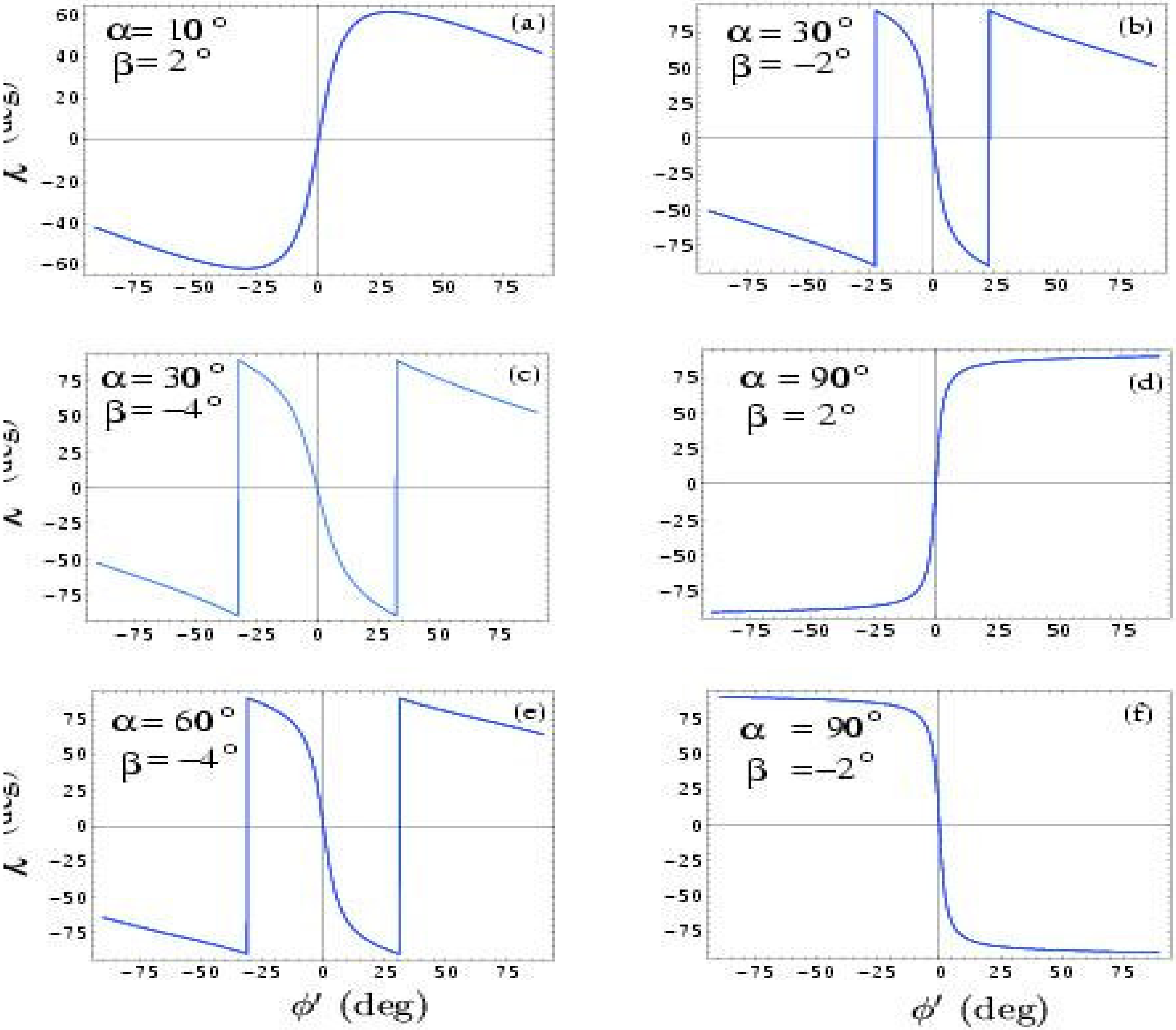
- Using the definition of $\mathbf{E}'$, and

$$
\begin{aligned}
ES &= ST \cdot [(M \bigotimes M) \cdot (\mathbf{E}' \bigotimes \mathbf{E}'^{*})] \\
\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} &= \begin{bmatrix} I' \\ -U' \sin(2\chi) + Q' \cos(2\chi) \\ U' \cos(2\chi) + Q' \sin(2\chi) \\ V' \end{bmatrix} \\
I' &= \left\langle E'_{||} E'^{*}_{||} \right\rangle + \left\langle E'_{\perp} E'^{*}_{\perp} \right\rangle \\
Q' &= \left\langle E'_{||} E'^{*}_{||} \right\rangle - \left\langle E'_{\perp} E'^{*}_{\perp} \right\rangle \\
U' &= \left\langle E'_{\perp} E'^{*}_{||} \right\rangle + \left\langle E'^{*}_{\perp} E_{||} \right\rangle \\
V' &= -i \left(\left\langle E'_{\perp} E'^{*}_{||} \right\rangle - \left\langle E'^{*}_{\perp} E_{||} \right\rangle \right) .
\end{aligned} \tag{5}
$$

Polarization angle

- $$\Psi = \chi = \tan^{-1}\left(\frac{\sin\chi}{\cos\chi}\right), \tag{6}$$

$$\begin{aligned} \cos\chi &= \hat{e}_{\phi t}\cdot\hat{\epsilon}_{||}\,, \\ \sin\chi &= (\hat{e}_{\phi t}\times\hat{\epsilon}_{||})\cdot\hat{n}\,. \end{aligned} \tag{7}$$


(a)
α= 10°
β= 2°
(b)
α = 30°
β = −2°
(c)
α= 30°
β= −4°
(d)
α = 90°
β = 2°
(e)
α= 60°
β= −4°
(f)
α = 90°
β =−2°
φ′ (deg)
φ′ (deg)

Addition of Stokes parameters

- The net emission, which observer receives along $\hat{n}$, will have incoherently added contributions from the neighboring field lines
- Thus the radiation in the direction of $\hat{n}$ should be integrated over a small solid angle $d\Omega = \sin\theta \, d\theta \, d\mu$.
- We assume (i) the width of bunch η_0 is much smaller than the wavelength λ of the radio waves, so that the radiation emitted by a bunch is coherent, and (ii) the bunches, within the beaming region, are closely spaced, so that the net emission becomes smooth and continuous.

$$I_{\rm sm} = \frac{\gamma}{2}\int_{-\infty}^{\infty} S_{\rm M} I \, d\mu \tag{8}$$

$$Q_{\rm sm} = \frac{\gamma}{2}\int_{-\infty}^{\infty} S_{\rm M} Q \, d\mu \tag{9}$$

$$U_{\rm sm} = \frac{\gamma}{2}\int_{-\infty}^{\infty} S_{\rm M} U \, d\mu \tag{10}$$

$$V_{\rm sm} = \frac{\gamma}{2}\int_{-\infty}^{\infty} S_{\rm M} V \, d\mu \, . \tag{11}$$

$$S_{\rm M} = \exp[-(\phi'_{\rm b} + \phi'_{\rm c})^2/\sigma^2_{\phi'} - (\theta'_{\rm b} + \theta'_{\rm c})^2/\sigma^2_{\theta'}] \, , \tag{12}$$

- The resultant expressions after μ integrations

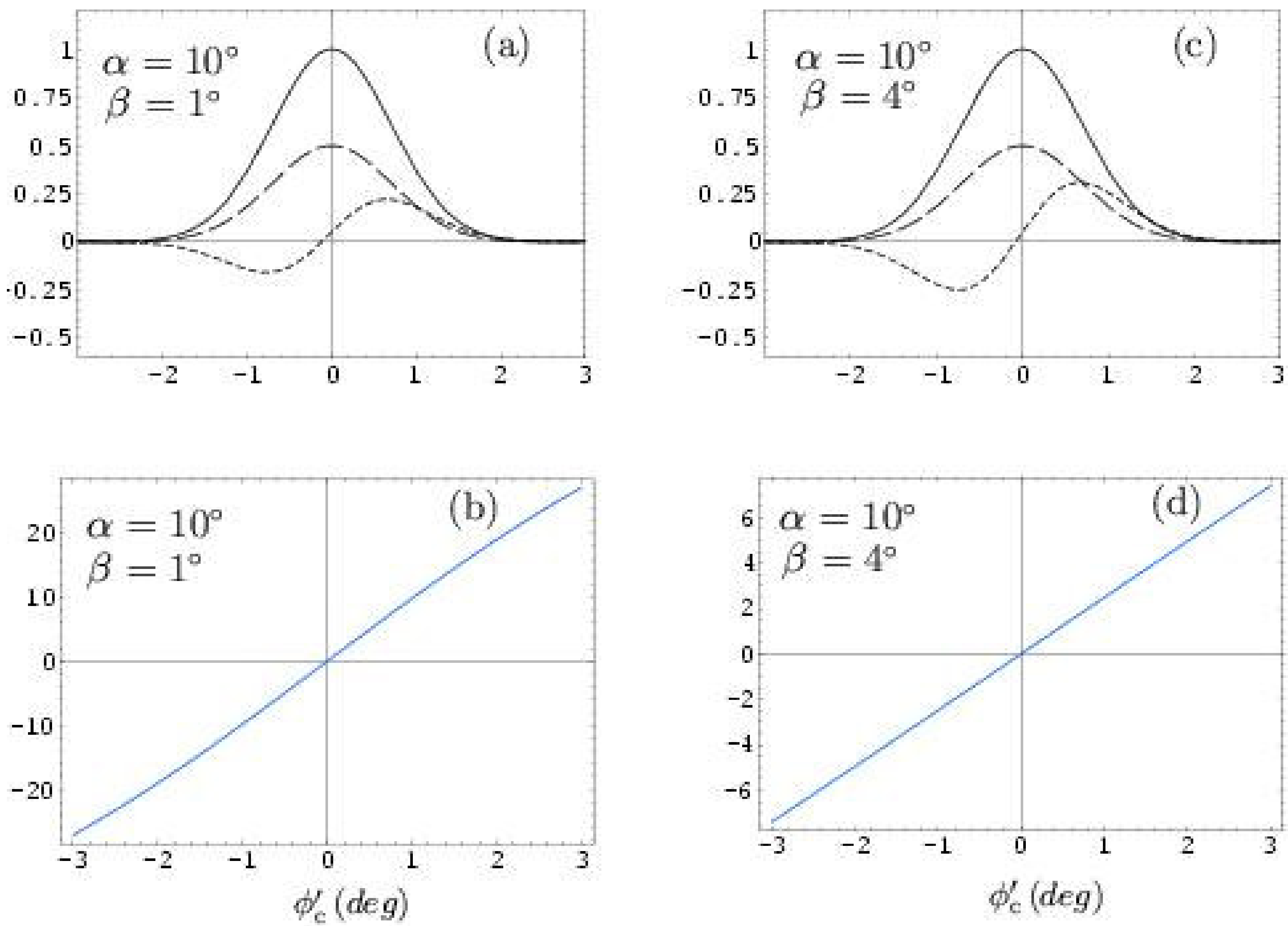
$$I_{\rm sm} = C\ G_{\rm M} \tag{13}$$

$$L_{\rm sm} = \sqrt{Q_{\rm sm}^2 + U_{\rm sm}^2} = \frac{C}{2}\, G_{\rm M} \tag{14}$$

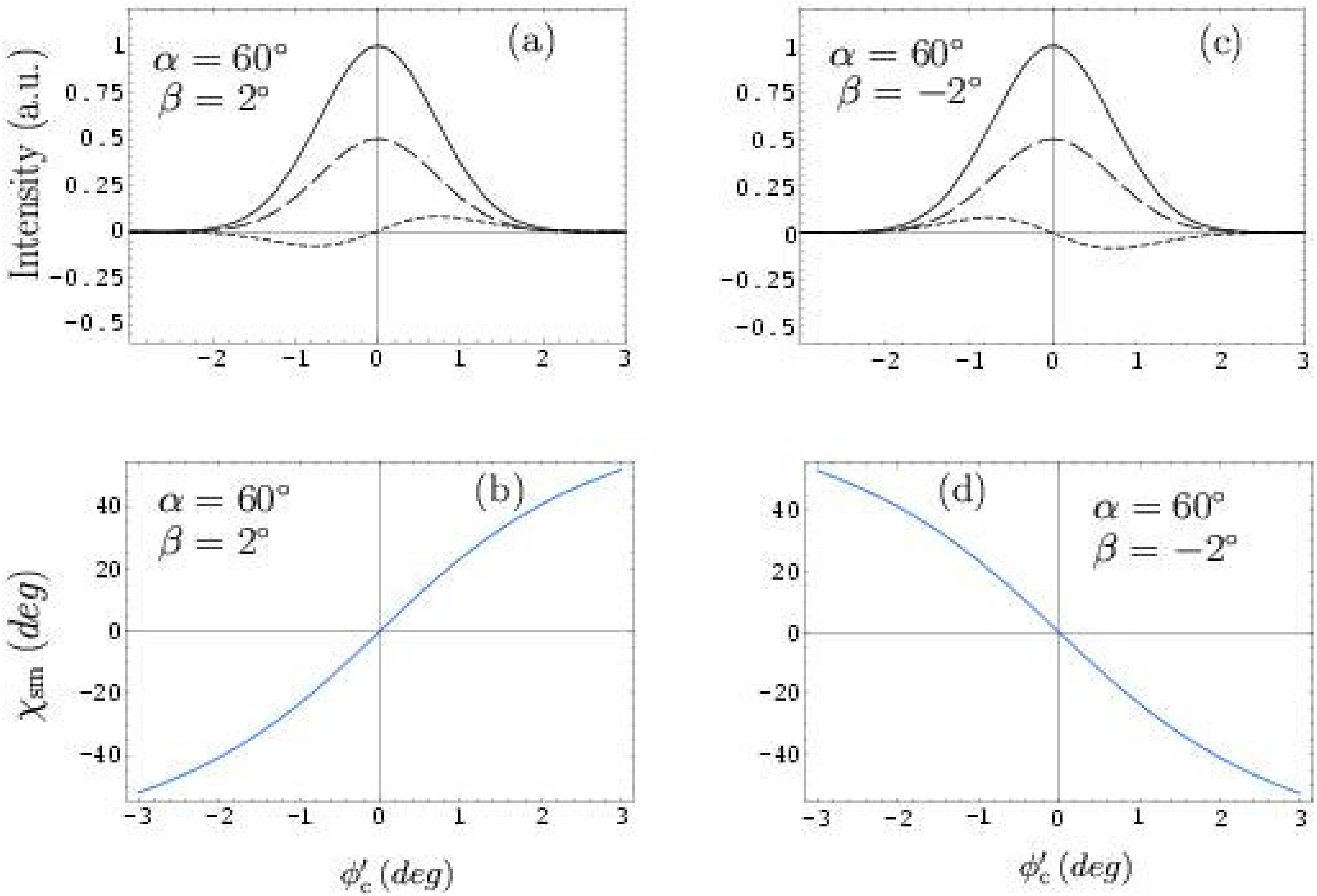
$$\chi_{\rm sm} = \frac{1}{2}\arctan\left(\frac{U_{\rm sm}}{Q_{\rm sm}}\right) \tag{15}$$

$$V_{\rm sm} = 3\,\Gamma\left(\frac{1}{3}\right)\ 2^{-4/3}\ c_1^{-1/2}\ C\ G_{\rm M} \left[\frac{\phi'_{\rm c}}{\sigma^2_{\phi'}|C_\phi|} + \frac{\theta'_{\rm c}}{\sigma^2_{\theta'}|C_\theta|}\right], \tag{16}$$

$$
\begin{aligned}
C &= \frac{\gamma}{2}|C_{||}|^2 \frac{\pi}{\sqrt{c_1}} \left[\frac{2^{1/3}}{3}\right] \Gamma\left(\frac{2}{3}\right) \\
G_{\mathrm{M}} &= \exp\left[-\left(\frac{\theta'_{\mathrm{c}}}{\sigma_{\theta'}}\right)^2 - \left(\frac{\phi'_{\mathrm{c}}}{\sigma_{\phi'}}\right)^2\right] . \quad (17)
\end{aligned}
$$



(a)
$\alpha = 10°$
$\beta = 1°$
(b)
$\alpha = 10°$
$\beta = 1°$
(c)
$\alpha = 10°$
$\beta = 4°$
(d)
$\alpha = 10°$
$\beta = 4°$
$\phi'_c\,(deg)$
$\phi'_c\,(deg)$



(a)
$\alpha = 60°$
$\beta = 2°$
Intensity (a.u.)
1
0.75
0.5
0.25
0
-0.25
-0.5
-2
-1
0
1
2
3
(c)
$\alpha = 60°$
$\beta = -2°$
(b)
$\alpha = 60°$
$\beta = 2°$
$\chi_{\rm sm}$ (deg)
40
20
0
-20
-40
-3
$\phi'_{\rm c}\,(deg)$
(d)
$\alpha = 60°$
$\beta = -2°$
$\phi'_{\rm c}\,(deg)$

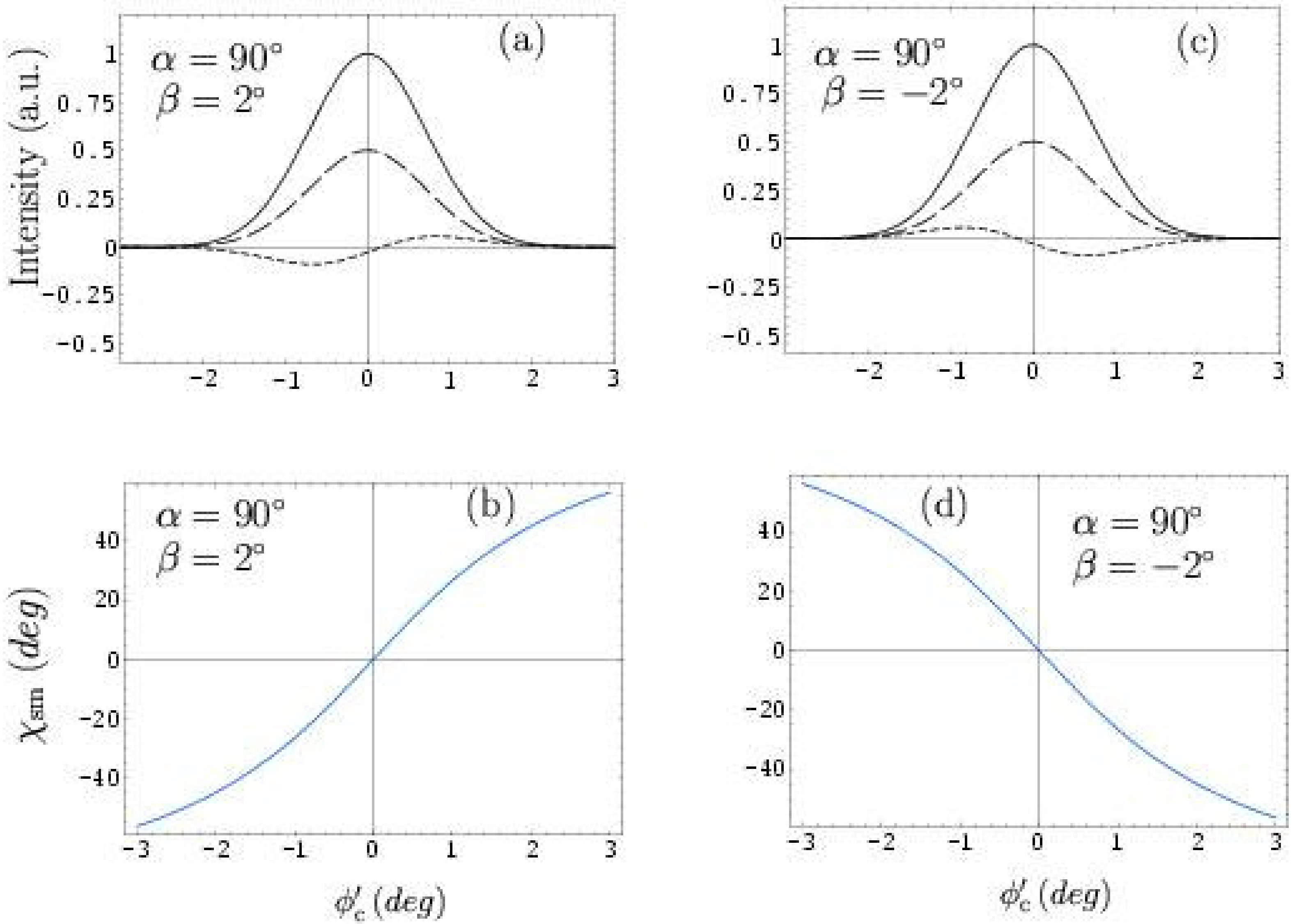
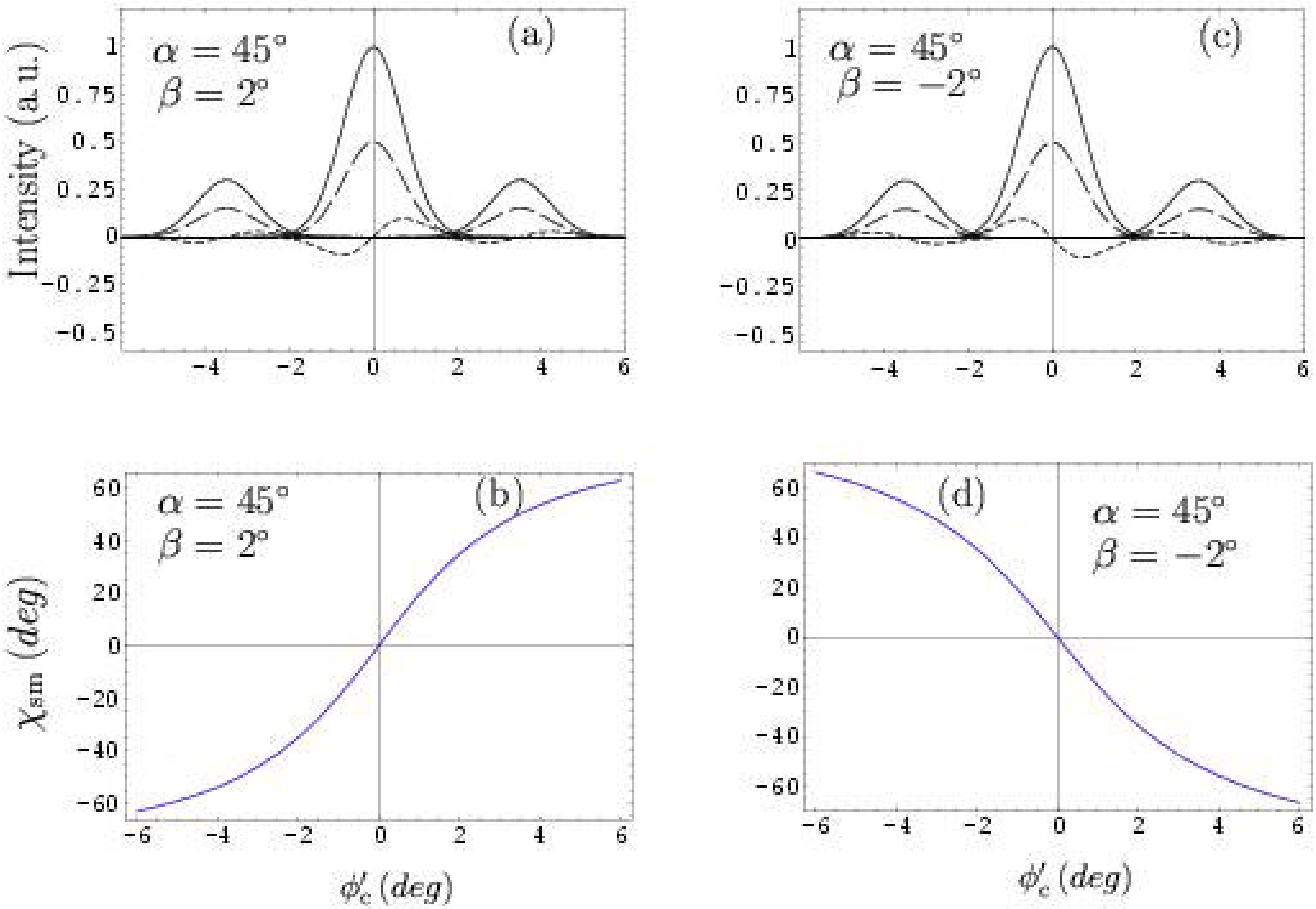


Figure 14: The polarization parameters I_{sm} (thick line), L_{sm} (large dashed line) and V_{sm} (small



(a)
$\alpha = 45°$
$\beta = 2°$
Intensity (a.u.)
1
0.75
0.5
0.25
0
-0.25
-0.5
-4 -2 0 2 4 6
(c)
$\alpha = 45°$
$\beta = -2°$
(b)
χ_{sm} (deg)
60
40
20
0
-20
-40
-60
-6 -4 -2 0 2 4 6
ϕ'_c (deg)
(d)

Results and Discussions

- We have generalized the coherent curvature radiation model to include the detailed geometry (Gangadhara 2004) of the emission region in pulsar magnetosphere.
- Our model takes into account of (i) the detailed geometry of emission region, (ii) the incoherent addition within the beaming region having angular width $2/\gamma$, and (iii) possible modulations (gaussian) in the emission region.
- We transferred the coherent radiation fields from magnetic coordinates to laboratory frame.

- Since the arc-length corresponding to $2/\gamma$ is much larger than the wavelength, we have incoherently added the radiation fields from different bunches.
- Individual components of a profile might have a basic unit of gaussian structure. The component shapes suggest that the emissions corresponding to each component is gaussian modulated.
- Depending up on the values of $\sigma_{\phi'}$ and $\sigma_{\theta'}$ the two terms inside the square bracket (Eq. ??) describe the various behaviors of the circular polarization with respect to the pulse phase.

- If there is no modulation the $\sigma_{\phi'} \to \infty$ and $\sigma_{\theta'} \to \infty$ and the circular polarization $V_{\rm sm} \to 0$.
- If $\sigma_{\phi'} < \sigma_{\theta'}$ then the azimuthal term dominates over the polar term, and the circular polarization behaves like a sense reversing (antisymmetric) type with respect to pulse phase $\phi'_{\rm c}$.
- Relatively narrow pulse component should exhibit a sense reversing circular polarization, since $\sigma_{\phi'}$ is small (example: PSR B0329+54 which has a narrow core component with clearly sense reversing circular polarization)

Thanks