Wavefront Sensing using Polarization Shearing Interferometer

A report on the work done for my Ph.D

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CONTENTS

Major contributions

1. Babinet Compensator based PSI Wavefront sensor
2. Theoretical Simulations for PSI
3. Wavefront Reconstruction from PSI Interferometric Data
4. Laboratory Experiment & Results

Wavefront Sensing

Direct Sensing of the wavefront is not possible.

Wavefront can be derived either from Geometric or Interferometric methods from Intensity measurements at the focal plane or pupil plane.

Existing Methods:

Shack Hartmann Wavefront Sensor

Curvature Sensor

Pyramid Wavefront Sensing

Lateral Shearing Interferometer

What does a Wavefront Sensor do ?

A plane wavefront from a distant star gets distorted due to atmospheric turbulence and system errors.

A wavefront sensor measures these errors.

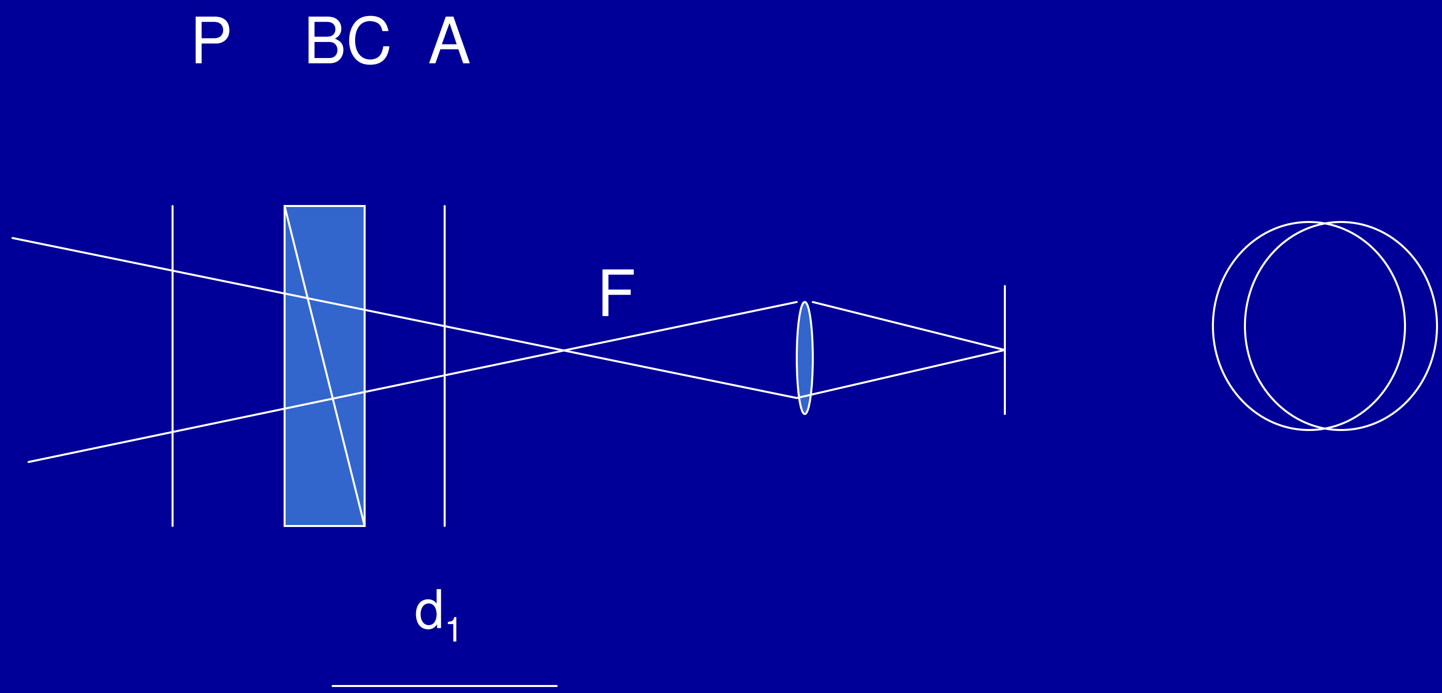
Basic Requirements of a wavefront sensor:

To sense the Wavefront with enough spatial resolution and enough speed for real time compensation.

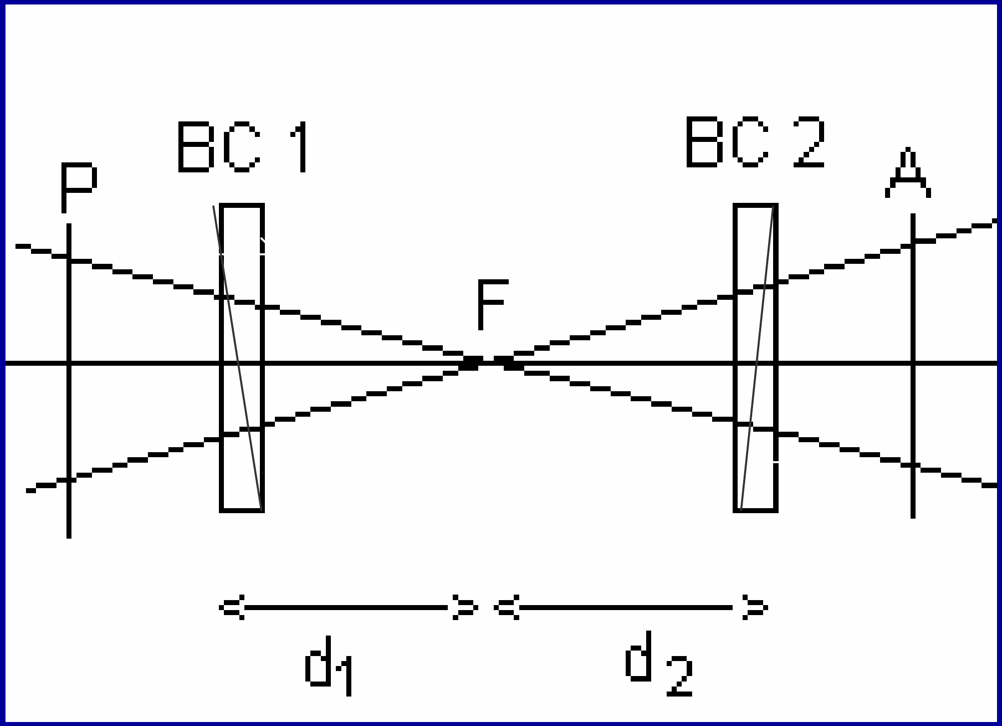
Polarization Shearing Interferometer
Using Two Crossed Babinet Compensators

It works on the principle of Lateral Shearing Interferometry Technique.

Now consider the case of a single BC



$$\text{Shear} = 2(n_e - n_o)\tan\alpha\, f$$



Two Babinet Compensators BC1 & BC2 are crossed and kept either side of the focus

Shear in the X – direction S

Shear in the Y – direction T

Resultant Shear $= \sqrt{S^2 + T^2}$

The intensity distribution at the detector plane can be written as

$$I(x, y) = A + B\cos(\phi(x, y))$$

Where B(x, y) - Intensity modulations due to imperfections in the polarizer and analyzer and the birefringent material transmission or reflection.

The local fringe phase $\phi(x,y) = 2\pi/\lambda \quad \Delta w(x,y)$

$\Delta w(x, y) = w(x + S/2, y + T/2) - w(x - S/2, y - T/2)$,

For small shear approximation Δw can be written as $$\Delta W = \frac{\partial w}{\partial x}s + \frac{\partial w}{\partial y}t$$

Theoretical Simulations

A wavefront over a circular region of unit radius can be expressed using Zernike Polynomials

$$Z_{i\,even} = \sqrt{n+1}R_n^m(\rho)\sqrt{2}\cos(m\theta),\ for\ \ \mathrm{m} \neq 0$$

$$Z_{i\,odd} = \sqrt{n+1}R_n^m(\rho)\sqrt{2}\sin(m\theta),\ for\ \ \mathrm{m} \neq 0$$

$$Z_i = \sqrt{n+1}R_n^0(\rho),\qquad for\ \ \mathrm{m{=}0}$$

$$R_n^m(\rho) = \sum_{s=0}^{\frac{n-m}{2}} \frac{(-1)^s (n-s)!}{s!\left(\frac{n+m}{2}-s\right)!\left(\frac{n-m}{2}-s\right)!}\rho^{n-2s}$$

The values of n and m satisfy m < n and n-m is even.

Any wavefront distortion over a circular aperture of unit radius can be expanded as a sum of Zernike modes

$$W(\rho,\theta)=\sum_{m=1}^{\infty} d_m Z_m(\rho,\theta)$$

Where d_m is the coefficient of m^{th} polynomial Z_m

$$Z_1 = 1$$
Piston

$$Z_2 = 2\rho\cos\theta$$
$$Z_3 = 2\rho\sin\theta$$
tip & tilt

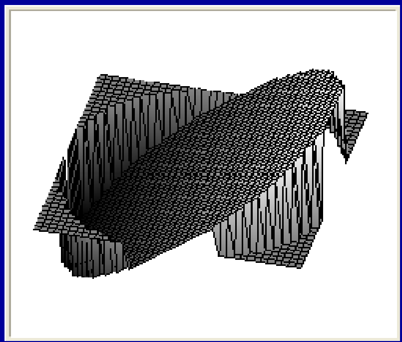
$$Z_4 = \sqrt{3}(2\rho^2 - 1)$$
Defocus

Expression of the first 15 Zernike modes

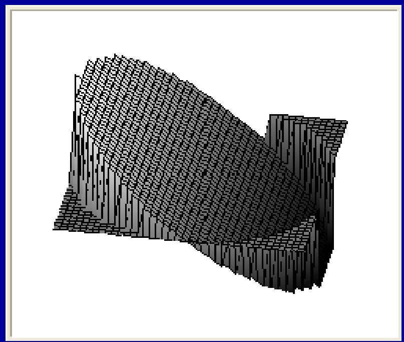
$Z_1 = 1$	Piston
$Z_2 = 2\rho\cos\theta$	Tip & tilt
$Z_3 = 2\rho\sin\theta$	
$Z_4 = \sqrt{3}(2\rho^2 - 1)$	Defocus
$Z_5 = \sqrt{6}r^2\sin 2\theta$	Astigmatism 3 rd order
$Z_6 = \sqrt{6}r^2\cos 2\theta$	
$Z_7 = \sqrt{8}(3r^2 - 2r)\sin\theta$	Coma
$Z_8 = \sqrt{8}(3r^2 - 2r)\cos\theta$	
$Z_9 = \sqrt{8}\sin 3\theta$	Trefoil
$Z_{10} = \sqrt{8}\cos 3\theta$	
$Z_{11} = \sqrt{5}(6r^4 - 6r^2 + 1)$	Spherical Aberration
$Z_{12} = \sqrt{10}(10r^4 - 3r^2)\cos 2\theta$	Astigmatism 5^{th} order
$Z_{13} = \sqrt{10}(10r^4 - 3r^2)\sin 2\theta$	
$Z_{14} = \sqrt{10}\cos 4\theta$	Astigmatism 7^{th} order
$Z_{15} = \sqrt{10}\sin 4\theta$	

Graphical representation of few Zernike Modes

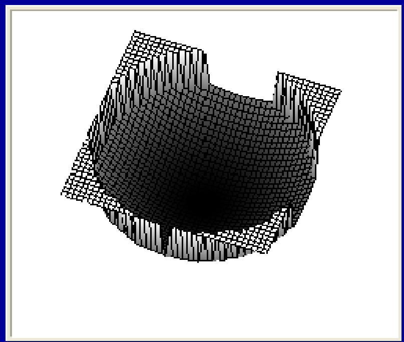
X-tilt



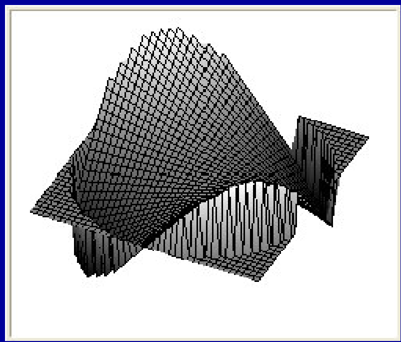
Y-tilt



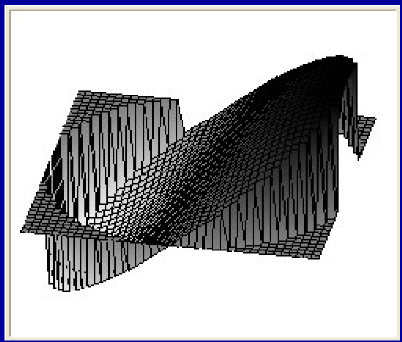
Defocus



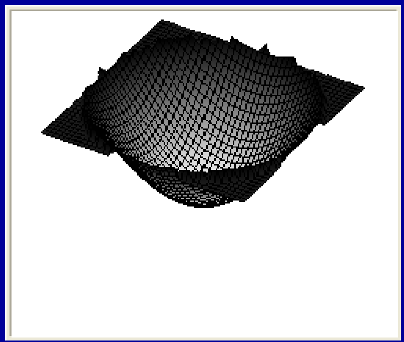
Astigmatism



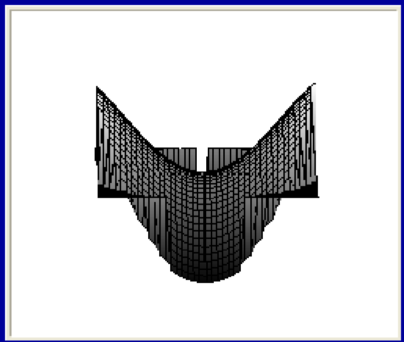
Coma 3^{rd} order



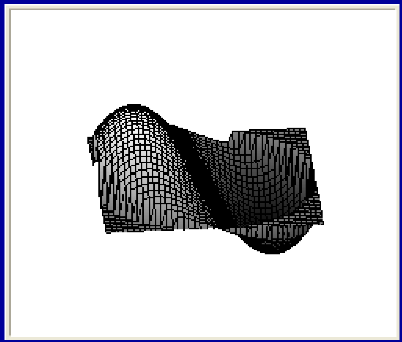
Spherical



Astigmatism 5^{th}



Coma 5^{th} order



$$\nabla W = \sum_{j=1}^{\infty} d_j \nabla Z_j$$

Since the derivative of the wavefront contains the derivative of the Zernike polynomial

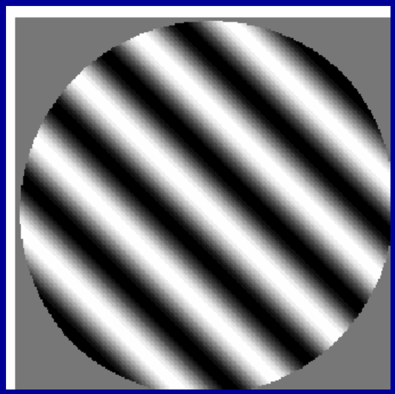
The Derivatives of the Zernike polynomials can be expressed as linear combination of Zernike Polynomial (Noll 1976)

$$\nabla Z_j = \sum_{j'} \gamma_{jj'} Z_{j'}$$

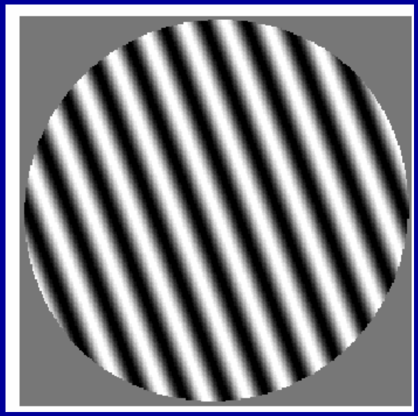
Where d_j are the Zernike coefficients

$$\Delta w(x,y) = \frac{\partial w}{\partial x} S + \frac{\partial w}{\partial y} T = \sum_{j=1}^{\infty} d_j \left(s \sum_{j'} \gamma_{x\,jj'} Z_{j'} + t \sum_{j'} \gamma_{y\,jj'} Z_{j'} \right)$$

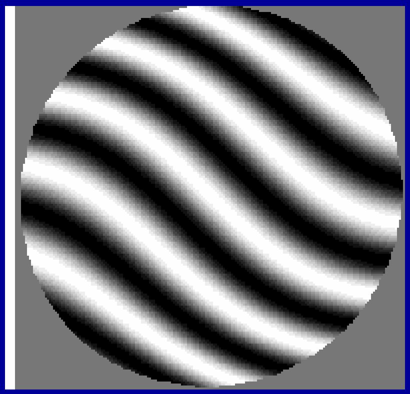
For different values for Zernike coefficients corresponding to different aberrations interferograms were simulated



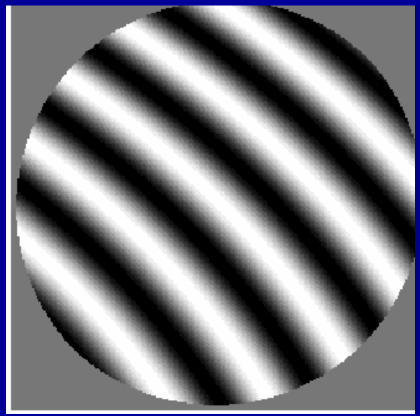
Defocus d1 = d2



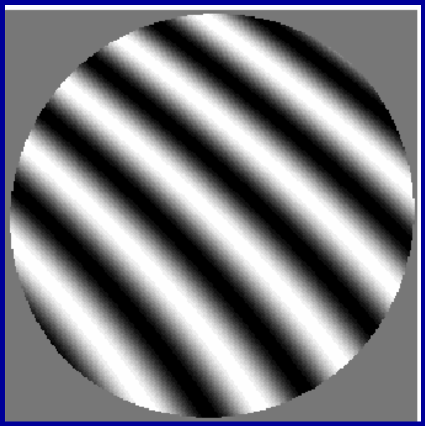
Defocus d1≠ d2



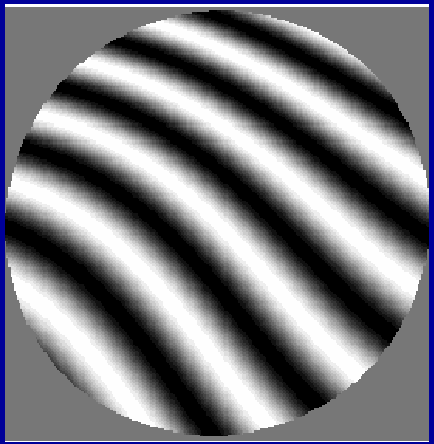
Spherical aberration=1λ



Astigmatism = 1λ

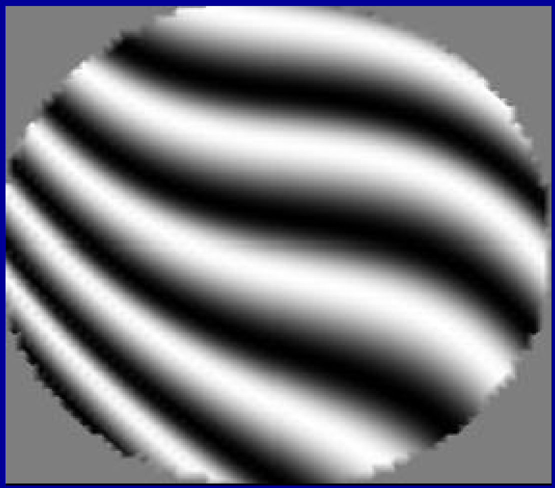


Coma = 1λ

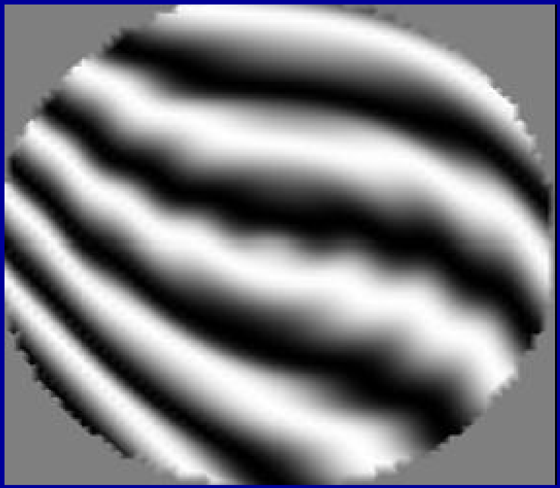


All aberrations

Introducing noise, ripples due to polishing marks and noise due to atmospheric turbulence corresponding to changing r_o



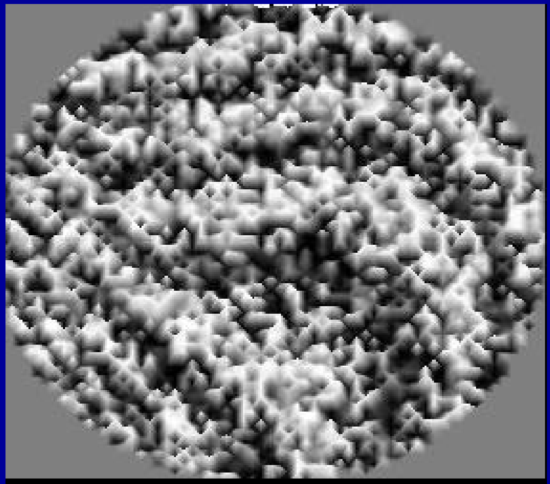
With System errors



With ripple errors

Errors due to turbulence

Including noise



More noise &
Small r_o – worst case

Reconstruction of Wavefront

The aberrated wavefront has to be deduced from the interferogram.

$$I(x, y) = A + B\cos(\phi(x, y))$$

Where

$$\phi(x, y) = \frac{2\pi}{\lambda}\Delta w(x, y)$$

In this case, the defocus term corresponds to the spatial carrier frequency in the interferogram.

The high frequency noise is removed in the Fourier domain

Inverse FT results in Intensity with 2π ambiguity

Suitable Phase unwrapping Technique has to be employed to remove the ambiguity

The phase variations corresponds to $\Rightarrow \frac{2\pi}{\lambda}\Delta w(x, y)$

Recalling the Equation

$$\Delta w(x, y) = \sum_{j=1}^{n} d_j \left(s\sum_{j'} \gamma_{x\, jj'} Z_{j'} + t\sum_{j'} \gamma_{y\, jj'} Z_{j'} \right)$$

The number of measurements is generally more than the number of unknowns, so a Least Squares solution is adopted.

The linear relationship can be written as $A = XB$

The Zernike coefficients are determined by $X = (B^T B)^{-1} B^T A$

By knowing the Zernike coefficients the wavefront is reconstructed using

$$W(\rho,\theta) = \sum_{m=1}^{\infty} d_m Z_m(\rho,\theta)$$

The common measure of the wavefront quality is the Strehl Ratio

Strehl Ratio – Ratio of intensity at the Gausian image point in the presence of aberrations to the intensity with no aberrations

Strehl Ratio = 1 Diffraction limited case

The performance of the system is considered to be good if the the Strehl ratio is > 0.8

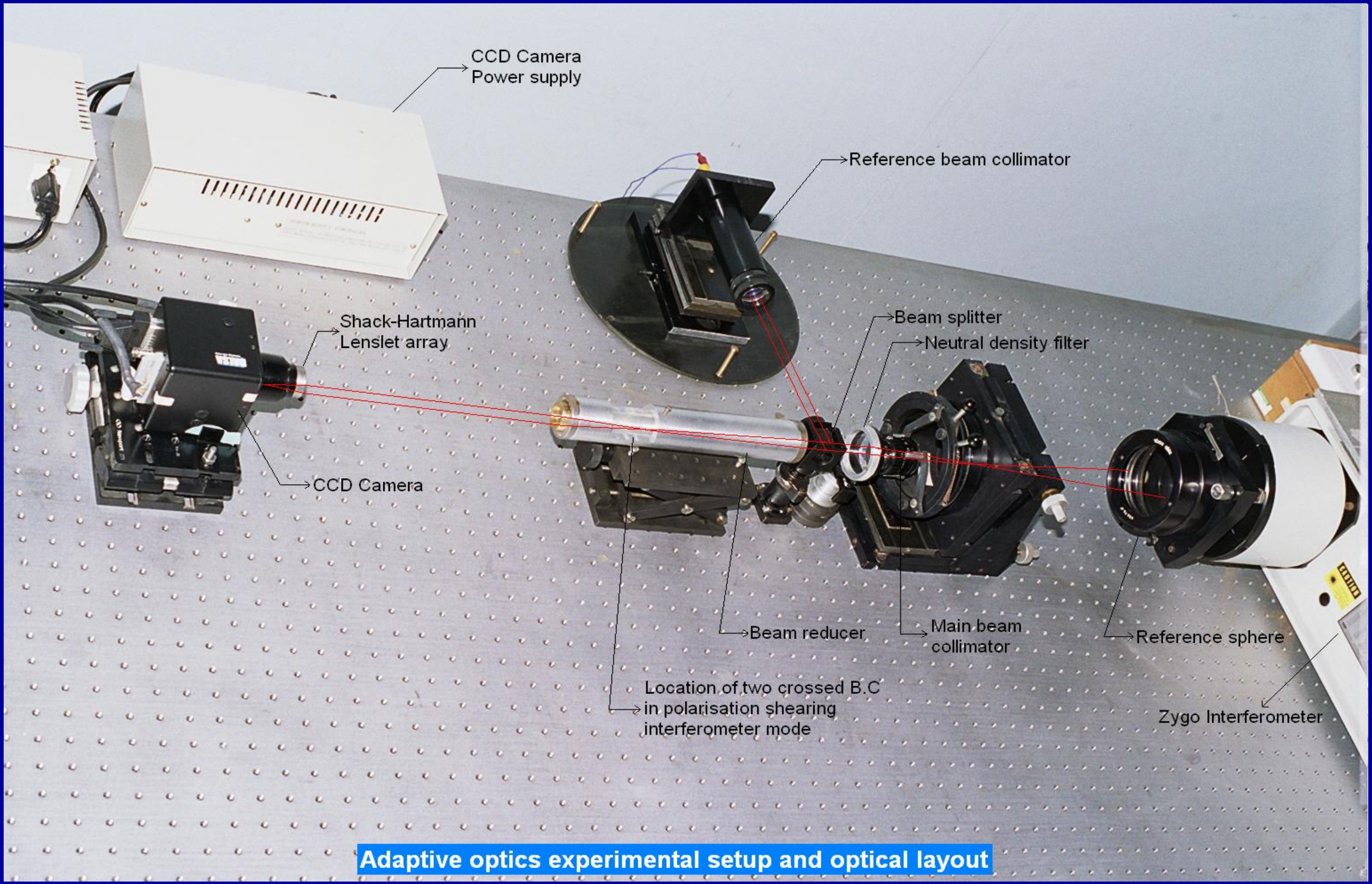
Root Mean Square deviation of the wavefront is calculated as

$$rms = \sqrt{\int_0^{2\pi}\int_0^{1}\left(W(\rho,\theta) - \overline{W(\rho,\theta)}\right)^2 \rho d\rho d\theta}$$

$$= \sqrt{\sum_0^n C_i^2}$$

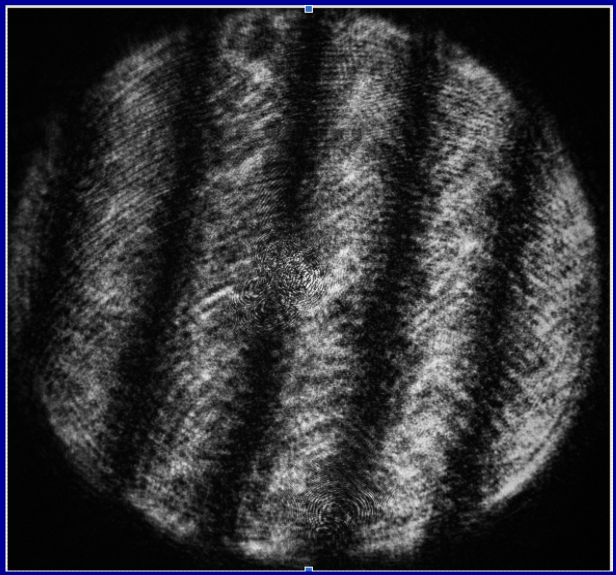
Where C_i are the Zernike coefficients

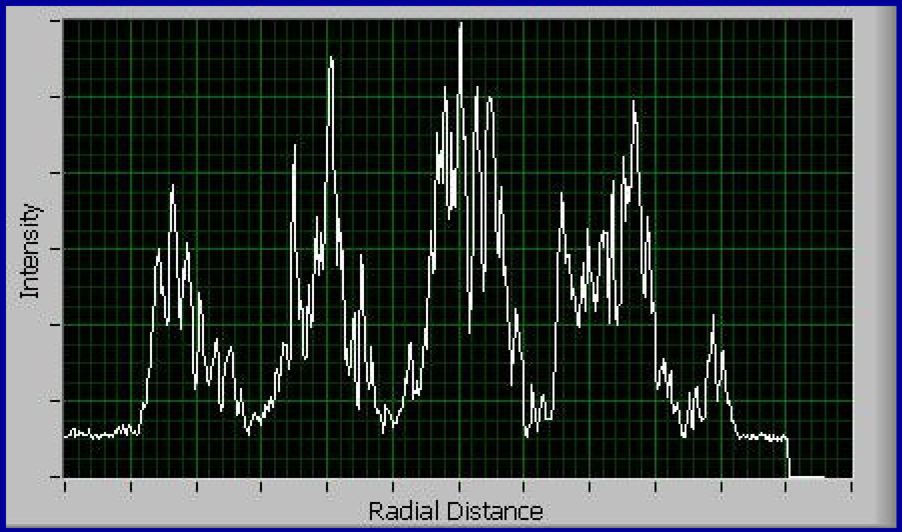
Laboratory Experiment and results



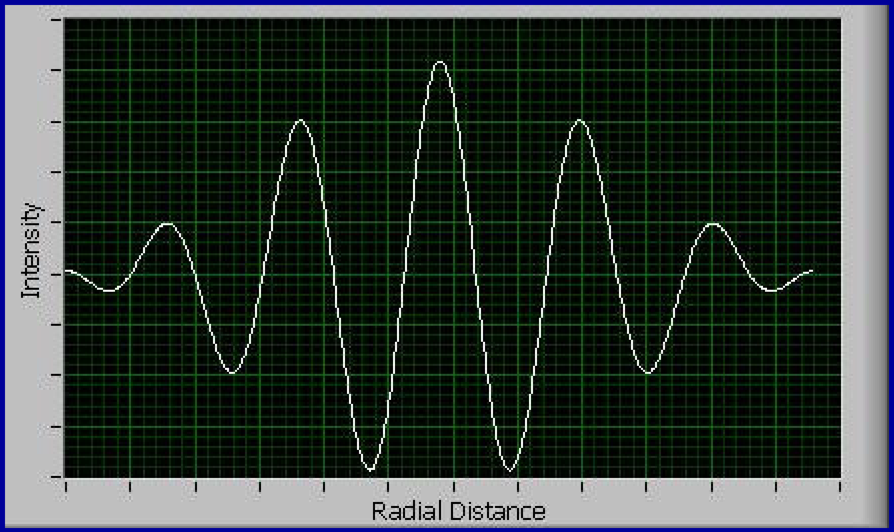
Adaptive optics experimental setup and optical layout

Interferogram Recorded in the Lab.

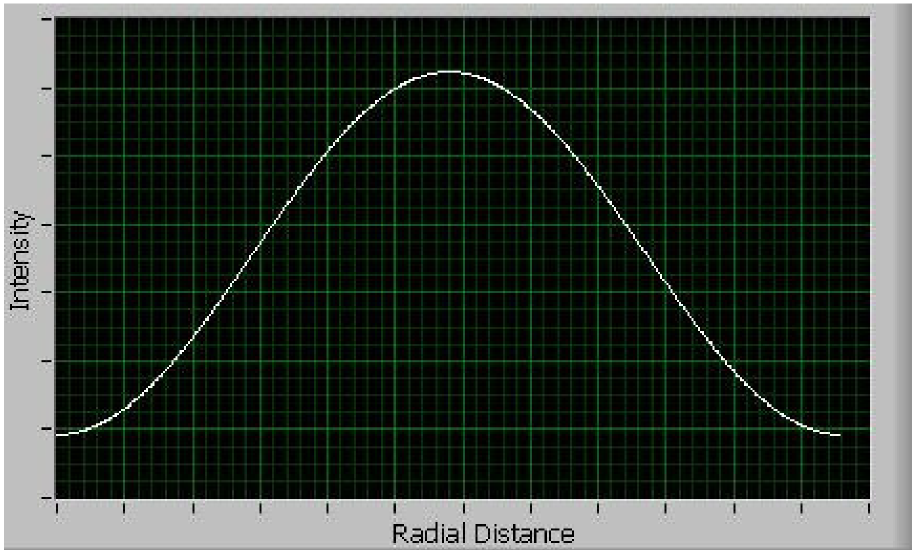




Interferogram Profile



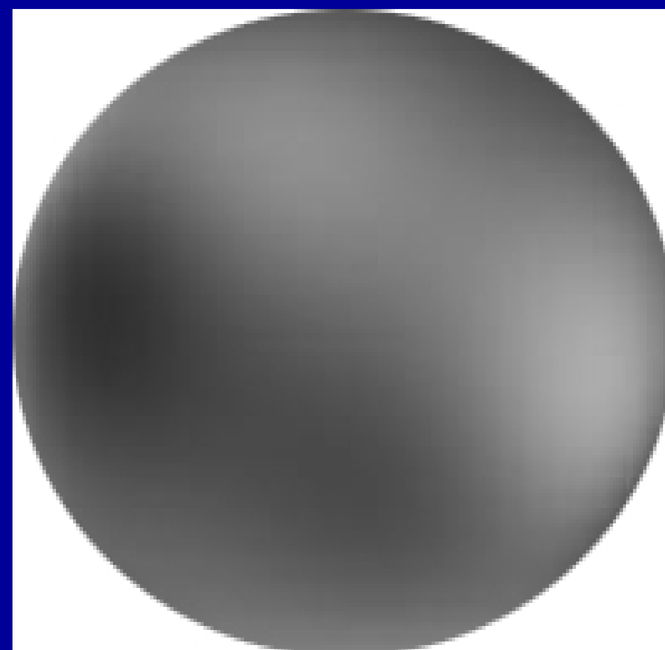
Interferogram Profile Noise removed



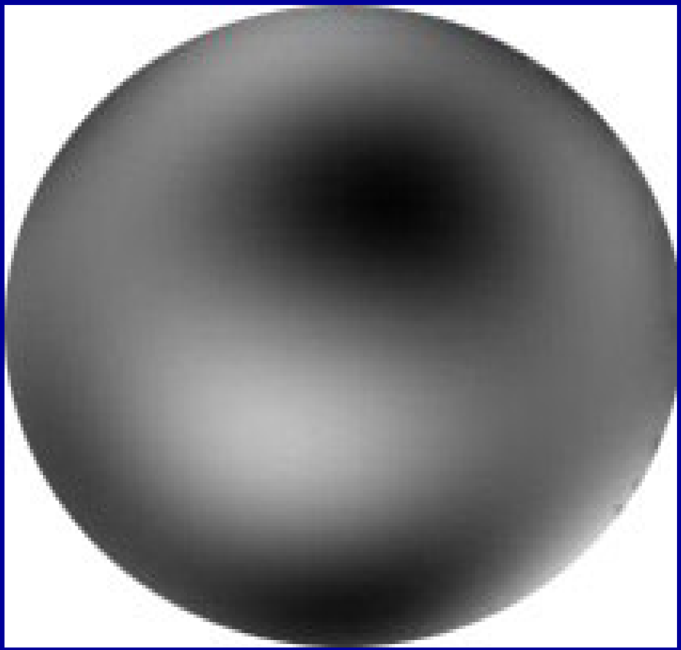
Interferogram profile – Defocus term removed

Local phase variations with π ambiguity

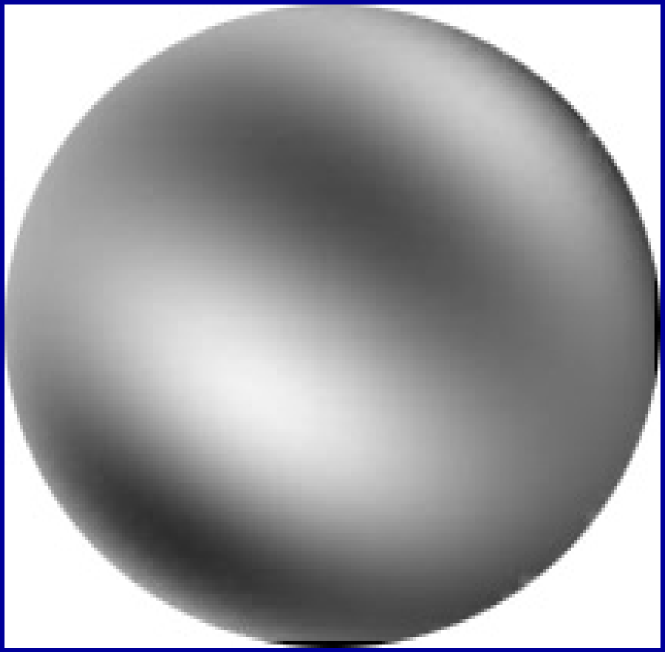
Local Phase variations with π ambiguity removed



Wavefront Derivative map



Wavefront Derived from PSI



Wavefront Derived from SH

Zernike Coefficients derived from PSI Interferogram

d_2	0.17590
d_3	0.35190
d_4	-0.2967
d_5	0.31182
d_6	0.7654
d7	0.1703
d8	0.00125
d9	-0.1715
d_{10}	-0.5239
d_{11}	0.03686
d_{12}	0.26354
d_{13}	0.021735
d_{14}	0.351213
d_{15}	-0.7507
d_{16}	-0.21565
d_{17}	0.07800
d_{18}	0.23942
d_{19}	0.18915
d_{20}	-0.19966
d_{21}	0.30373

Comparative Results:

Method	RMS
PSI Method	0.42 λ
OPD method	0.43 λ
Shack Hartmann	0.38 λ

Conclusion :

Polarization Shearing Proves to be better Wavefront Sensing Technique

- High Spatial Resolution
- Simple set up and easy alignment
- X- shear and Y-shear combined in a Single record
- No reference optics required
- Excellent linearity
- Measurement accuracy better than 0.1 arc sec.

Thank You