Wavefront Sensing using Polarization Shearing Interferometer

A report on the work done for my Ph.D

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Major contributions

1. Babinet Compensator based PSI Wavefront sensor
2. Theoretical Simulations for PSI
3. Wavefront Reconstruction from PSI Interferometric Data
4. Laboratory Experiment & Results
Wavefront Sensing

Direct Sensing of the wavefront is not possible.

Wavefront can be derived either from Geometric or Interferometric methods from Intensity measurements at the focal plane or pupil plane.

Existing Methods:

Shack Hartmann Wavefront Sensor
Curvature Sensor
Pyramid Wavefront Sensing
Lateral Shearing Interferometer
What does a Wavefront Sensor do?

A plane wavefront from a distant star gets distorted due to atmospheric turbulence and system errors. A wavefront sensor measures these errors.

Basic Requirements of a wavefront sensor:

To sense the Wavefront with enough spatial resolution and enough speed for real time compensation.
Polarization Shearing Interferometer Using Two Crossed Babinet Compensators

It works on the principle of Lateral Shearing Interferometry Technique.

Now consider the case of a single BC
Shear = 2(n_e - n_o) \tan \alpha f
Two Babinet Compensators BC1 & BC2 are crossed and kept either side of the focus.

Shear in the X – direction  \( S \)

Shear in the Y – direction  \( T \)

Resultant Shear  \( = \sqrt{S^2 + T^2} \)
The intensity distribution at the detector plane can be written as

\[ I(x, y) = A + B \cos(\phi(x, y)) \]

Where \( B(x, y) \) - Intensity modulations due to imperfections in the polarizer and analyzer and the birefringent material transmission or reflection.

The local fringe phase \( \phi(x,y) = \frac{2\pi}{\lambda} \Delta w(x,y) \)

\( \Delta w (x, y) = w (x + S/2, y + T/2) - w (x - S/2, y - T/2) \),

For small shear approximation \( \Delta w \) can be written as

\[ \Delta W = \frac{\partial w}{\partial x} s + \frac{\partial w}{\partial y} t \]
Theoretical Simulations

A wavefront over a circular region of unit radius can be expressed using Zernike Polynomials

\[ Z_{i\text{ even}} = \sqrt{n+1}R_n^m(\rho)\sqrt{2}\cos(m\theta), \text{ for } m \neq 0 \]
\[ Z_{i\text{ odd}} = \sqrt{n+1}R_n^m(\rho)\sqrt{2}\sin(m\theta), \text{ for } m \neq 0 \]
\[ Z_i = \sqrt{n+1}R_n^0(\rho), \text{ for } m=0 \]

\[ R_n^m(\rho) = \sum_{s=0}^{n-m} \frac{(-1)^s (n-s)!}{s!(\frac{n+m}{2} - s)!\left(\frac{n-m}{2} - s\right)!} \rho^{n-2s} \]

The values of n and m satisfy m < n and n-m is even.
Any wavefront distortion over a circular aperture of unit radius can be expanded as a sum of Zernike modes

\[ W(\rho, \theta) = \sum_{m=1}^{\infty} d_m Z_m(\rho, \theta) \]

Where \( d_m \) is the coefficient of \( m^{th} \) polynomial \( Z_m \)

\[
\begin{align*}
Z_1 &= 1 \\
Z_2 &= 2\rho \cos \theta \\
Z_3 &= 2\rho \sin \theta \\
Z_4 &= \sqrt{3}(2\rho^2 - 1)
\end{align*}
\]
Expression of the first 15 Zernike modes

\[ Z_1 = 1 \]
\[ Z_2 = 2\rho \cos \theta \]
\[ Z_3 = 2\rho \sin \theta \]
\[ Z_4 = \sqrt{3}(2\rho^2 - 1) \]
\[ Z_5 = \sqrt{6}\rho^2 \sin 2\theta \]
\[ Z_6 = \sqrt{6}\rho^2 \cos 2\theta \]
\[ Z_7 = \sqrt{8}(3\rho^2 - 2\rho) \sin \theta \]
\[ Z_8 = \sqrt{8}(3\rho^2 - 2\rho) \cos \theta \]
\[ Z_9 = \sqrt{8} \sin 3\theta \]
\[ Z_{10} = \sqrt{8} \cos 3\theta \]
\[ Z_{11} = \sqrt{5}(6\rho^4 - 6\rho^2 + 1) \]
\[ Z_{12} = \sqrt{10}(10\rho^4 - 3\rho^2) \cos 2\theta \]
\[ Z_{13} = \sqrt{10}(10\rho^4 - 3\rho^2) \sin 2\theta \]
\[ Z_{14} = \sqrt{10} \cos 4\theta \]
\[ Z_{15} = \sqrt{10} \sin 4\theta \]

Piston
Tip & tilt
Defocus
Astigmatism 3 rd order
Coma
Trefoil
Spherical Aberration
Astigmatism 5 th order
Astigmatism 7 th order
Graphical representation of few Zernike Modes

- X-tilt
- Y-tilt
- Defocus
- Astigmatism
- Coma 3\textsuperscript{rd} order
- Spherical
- Astigmatism 5\textsuperscript{th}
- Coma 5\textsuperscript{th} order
The Derivatives of the Zernike polynomials can be expressed as linear combination of Zernike Polynomial (Noll 1976)

\[ \nabla W = \sum_{j=1}^{\infty} d_j \nabla Z_j \]

Since the derivative of the wavefront contains the derivative of the Zernike polynomial

The Derivatives of the Zernike polynomials can be expressed as linear combination of Zernike Polynomial (Noll 1976)

\[ \nabla Z_j = \sum_{j} \gamma_{jj} Z_j \]

Where \( d_j \) are the Zernike coefficients

\[ \Delta w(x, y) = \frac{\partial w}{\partial x} S + \frac{\partial w}{\partial y} T = \sum_{j=1}^{\infty} d_j \left( s \sum_{j} \gamma_{xjj} Z_j + t \sum_{j} \gamma_{yjj} Z_j \right) \]
For different values for Zernike coefficients corresponding to different aberrations interferograms were simulated.

- Defocus $d_1 = d_2$
- Defocus $d_1 \neq d_2$
- Spherical aberration $= 1\lambda$
- Astigmatism $= 1\lambda$
- Coma $= 1\lambda$
- All aberrations
Introducing noise, ripples due to polishing marks and noise due to atmospheric turbulence corresponding to changing $r_o$. 

- With System errors
- With ripple errors
- Errors due to turbulence

Including noise

More noise & Small $r_o$ – worst case
Reconstruction of Wavefront

The aberrated wavefront has to be deduced from the interferogram.

\[ I(x, y) = A + B \cos(\phi(x, y)) \]

Where

\[ \phi(x, y) = \frac{2\pi}{\lambda} \Delta w(x, y) \]

In this case, the defocus term corresponds to the spatial carrier frequency in the interferogram.

The high frequency noise is removed in the Fourier domain.
Inverse FT results in Intensity with $2\pi$ ambiguity

Suitable Phase unwrapping Technique has to be employed to remove the ambiguity

The phase variations corresponds to

\[ \Rightarrow \frac{2\pi}{\lambda} \Delta w(x, y) \]

Recalling the Equation

\[ \Delta w(x, y) = \sum_{j=1}^{n} d_j \left( s \sum_{j} \gamma_{x, jj} Z_j + t \sum_{j} \gamma_{y, jj} Z_j \right) \]

The number of measurements is generally more than the number of unknowns, so a Least Squares solution is adopted.

The linear relationship can be written as

\[ A = XB \]

The Zernike coefficients are determined by

\[ X = (B^T B)^{-1} B^T A \]
By knowing the Zernike coefficients the wavefront is reconstructed using

\[ W(\rho, \theta) = \sum_{m=1}^{\infty} d_m Z_m(\rho, \theta) \]

The common measure of the wavefront quality is the Strehl Ratio

Strehl Ratio – Ratio of intensity at the Gaussian image point in the presence of aberrations to the intensity with no aberrations

Strehl Ratio = 1 Diffraction limited case

The performance of the system is considered to be good if the Strehl ratio is > 0.8
Root Mean Square deviation of the wavefront is calculated as

\[
\text{rms} = \sqrt{\int_0^{2\pi} \int_0^1 \left( W(\rho, \theta) - \overline{W(\rho, \theta)} \right)^2 \rho d\rho d\theta}
\]

\[
= \sqrt{\sum_{i=0}^{n} C_i^2}
\]

Where \( C_i \) are the Zernike coefficients
Laboratory Experiment and results

Adaptive optics experimental setup and optical layout
Interferogram Recorded in the Lab.
Interferogram Profile

Interferogram Profile Noise removed
Interferogram profile – Defocus term removed

Local phase variations with \( \pi \) ambiguity removed

Local phase variations with \( \pi \) ambiguity removed

Wavefront Derivative map
Wavefront Derived from PSI

Wavefront Derived from SH
Zernike Coefficients derived from PSI Interferogram

<table>
<thead>
<tr>
<th>d_n</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>d_2</td>
<td>0.17590</td>
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<tr>
<td>d_3</td>
<td>0.35190</td>
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<tr>
<td>d_4</td>
<td>-0.2967</td>
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<td>d_5</td>
<td>0.31182</td>
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<tr>
<td>d_6</td>
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<td>d_7</td>
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<td>d_20</td>
<td>-0.19966</td>
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<td>d_21</td>
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Comparative Results:

<table>
<thead>
<tr>
<th>Method</th>
<th>RMS</th>
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</thead>
<tbody>
<tr>
<td>PSI Method</td>
<td>0.42 ( \lambda )</td>
</tr>
<tr>
<td>OPD method</td>
<td>0.43 ( \lambda )</td>
</tr>
<tr>
<td>Shack Hartmann</td>
<td>0.38 ( \lambda )</td>
</tr>
</tbody>
</table>
Conclusion:

Polarization Shearing Proves to be better Wavefront Sensing Technique

- High Spatial Resolution
- Simple set up and easy alignment
- X- shear and Y-shear combined in a Single record
- No reference optics required
- Excellent linearity
- Measurement accuracy better than 0.1 arc sec.
Thank You