Solar Seismic Model and the Neutrino Fluxes

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Plan of the talk

- **Introduction**
- **Helioseismology : Theory & Observations**
- **Standard Solar Model**
- **Seismic Model**
- **Results of the Seismic Model**
- **Summary**

Introduction

- **Sun is a nearest star and a great astrophysical laboratory from which one can test great physical ideas**
- **It is a main sequence star which is in hydrostatic equilibrium**
- **Physical parameters : Mass = $2x10^{33}$ g, Luminosity=$3.9x10^{33}$ erg/s, Radius= $6.96x10^{10}$ cm , Surface temperature=5780 K**
- **If we understand the sun, we can also understand the stars, galaxies and ultimately the whole universe**

Helioseismology-Theory

- **Helio –Sun, Helioseismology-Seismology of the sun**
- **It is a tool to probe the internal structure, dynamics & magnetic field of the sun**
- **In the sun there are mainly 3 restoring forces : pressure (p), gravity (g) and magnetic field (MHD); pressure and gravity are the strongest restoring forces in the sun; periods : ~ 5 min; 40-160 min**
- **Global oscillations of the star can be represented by spherical harmonic functions, $V_{lmn}(r)Y_l^m$, `l' is degree, `m ' is azimuthal order and `n' is the radial order**
- **Wavelength of oscillations is $[R_0/l(l+1)]$ and p-modes which have very high `l' values trapped close to the surface and very low `l' values trap close to the center**
- **Gravity (g) modes are trapped mostly in the radiative interior and are evanescent in the convective envelope**

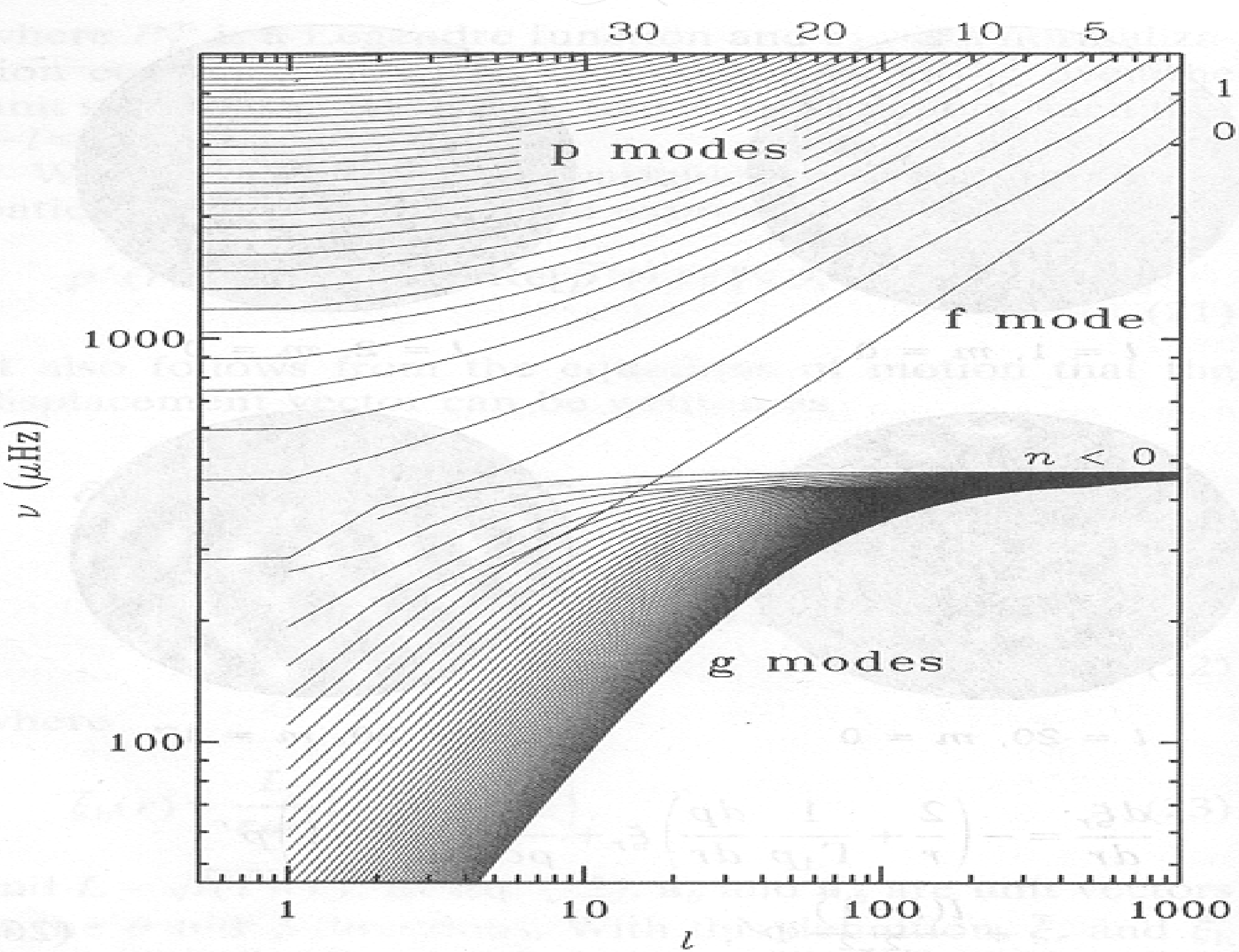
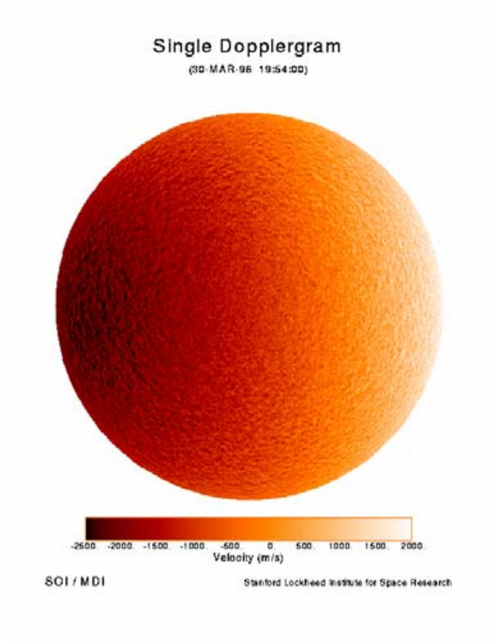
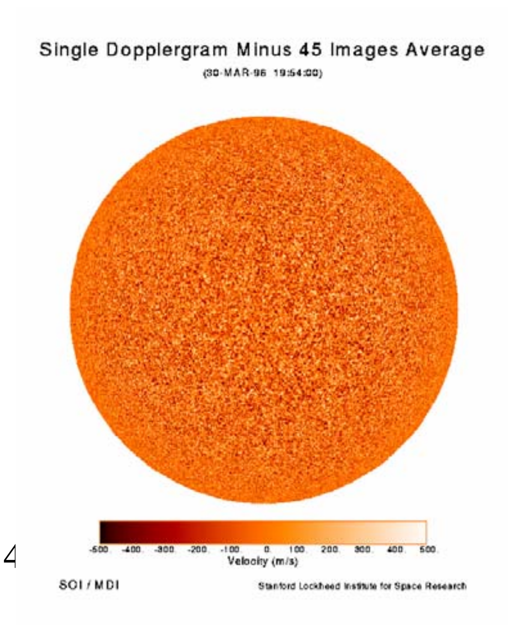


FIG. 3. Cyclic frequencies $\nu = \omega/2\pi$, as functions of degree l, computed for a normal solar model. Selected values of the radial order n have been indicated.

Helioseismology : observations

- **On the surface of the sun, oscillations are detected by the measurements of Doppler shift in the spectral line**
- **Observed frequencies : 2-5.7 mHz; Amplitudes : ~ 20 cm/sec**
- **Accurate frequencies, line widths and amplitudes are obtained from the observed `p' modes**


Single Dopplergram
(30-MAR-96 19:54:00)
-2500. -2000. -1500. -1000. -500. 0 500. 1000. 1500. 2000.
Velocity (m/s)
SOI / MDI
Stanford Lockheed Institute for Space Research


Single Dopplergram Minus 45 Images Average
(30-MAR-96 19:54:00)
-500. -400. -300. -200. -100. 0. 100. 200. 300. 400. 500.
Velocity (m/s)
SOI / MDI
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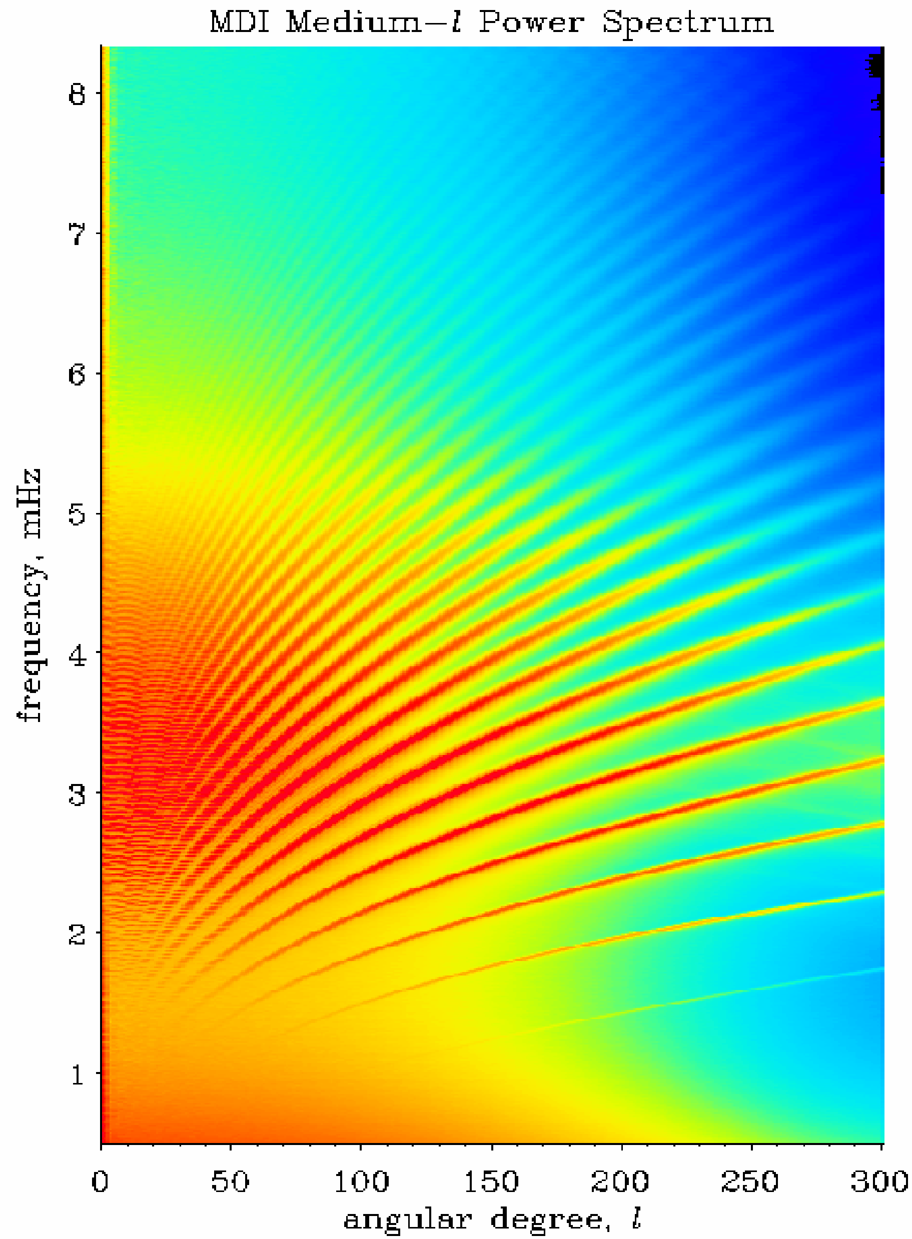
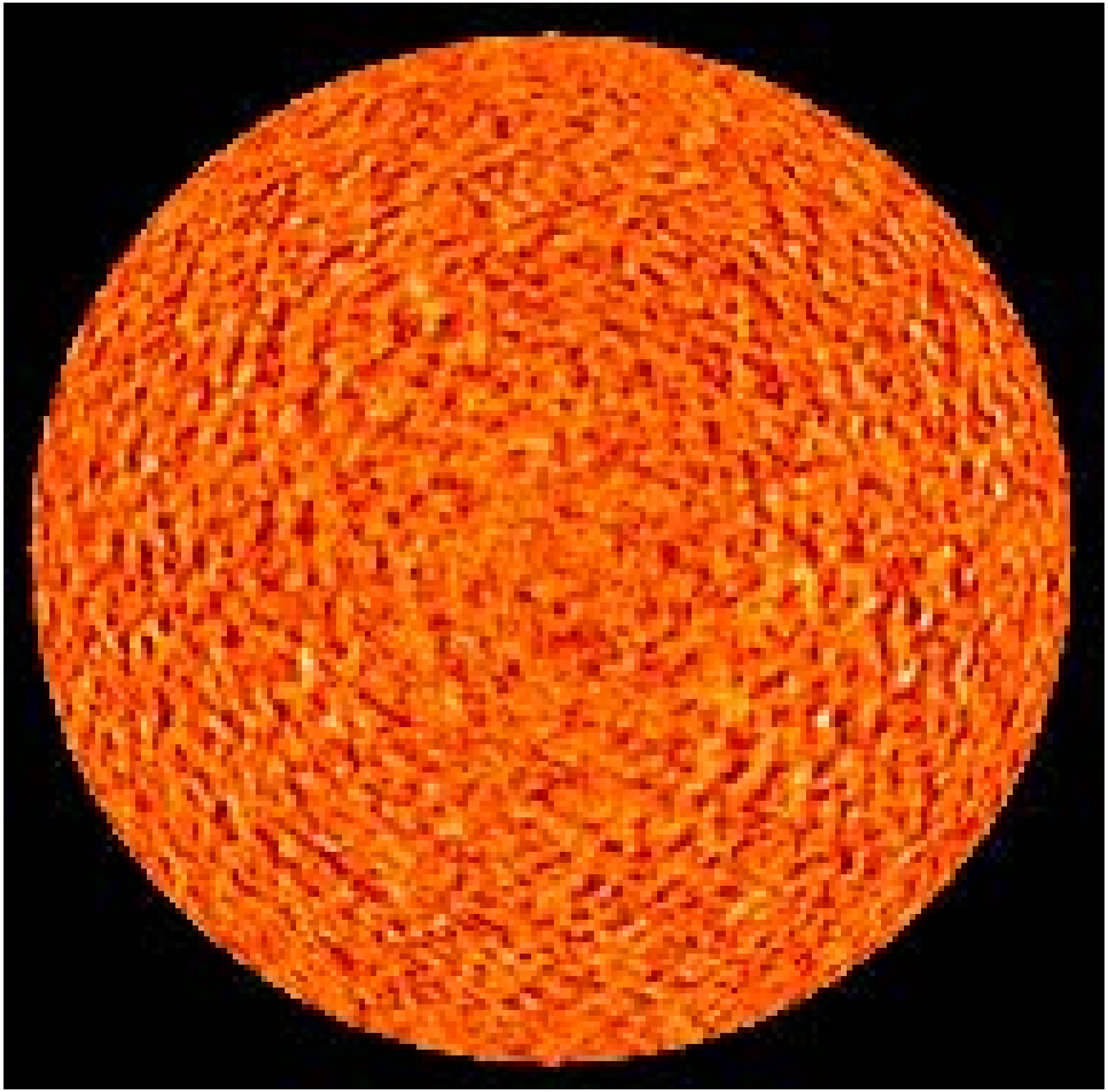

MDI Medium−l Power Spectrum
frequency, mHz
1
2
3
4
5
6
7
8
0 50 100 150 200 250 300
angular degree, l



Standard Solar Model

- Standard solar Model : Using the surface properties like mass, radius and luminosity, solar model is constructed
- Approximations : Spherically symmetric, Hydrostatic & radiative equilibrium, energy generation by nuclear energy generation & chemical abundance changes are solely due to the nuclear reactions
- Keeping the mass constant and with the initial uniform chemical composition, stellar structure equations are evolved to the present age (4.6 billion yrs) which satisfy the luminosity and the radius.
- Such a model yields physical parameters at the center : T=15 million K, P=$2.3\text{E}\times10^{17}$ dynes/cm^2, density=148 g/cm^3, H2 abundance = 0.34, Helium abundance = 0.64

Solar Structure Equations

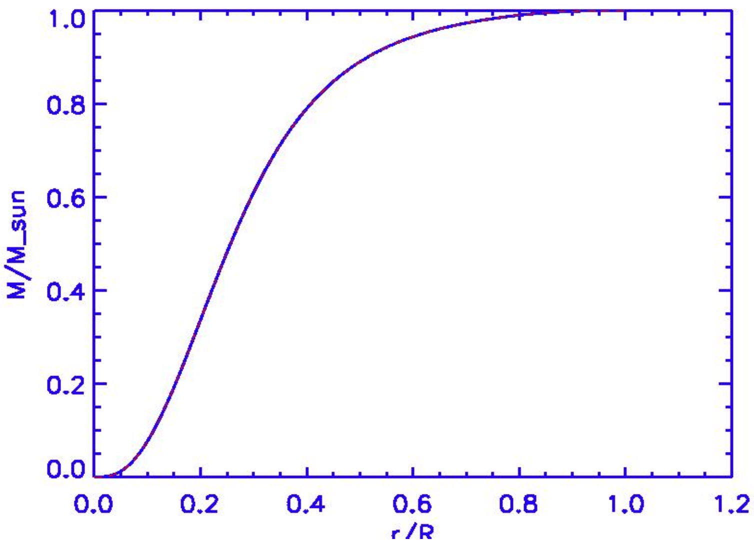
$$\frac{dM_r}{dr} = 4\pi r^2 \qquad (1)$$

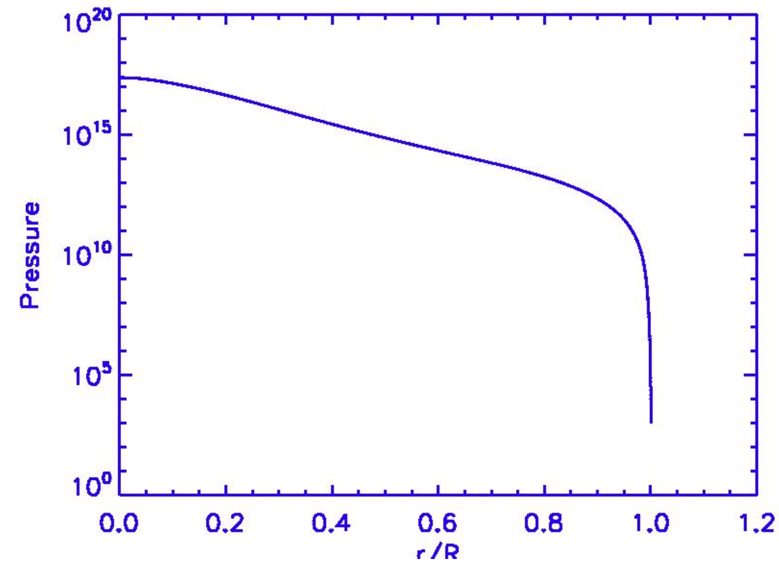
$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \qquad (2)$$

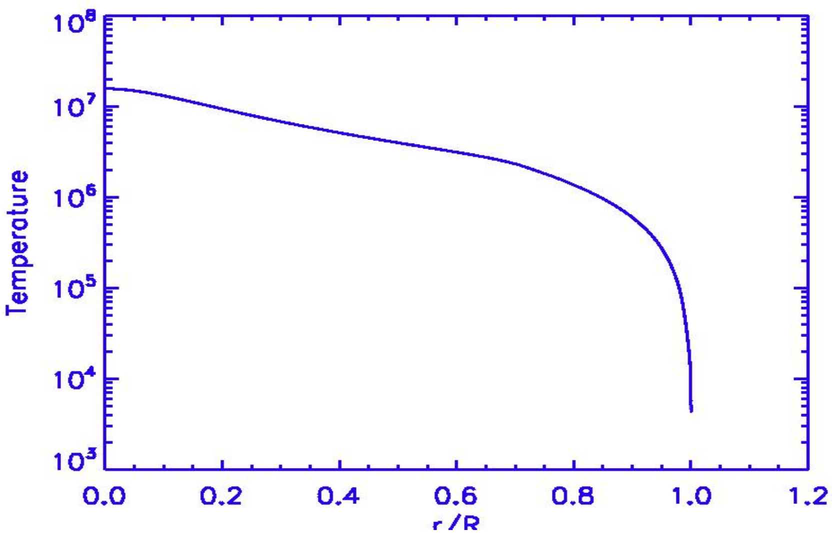
$$\frac{\mathrm{d}L_r}{\mathrm{dr}} = 4\pi r^2 \rho\varepsilon \qquad (3)$$

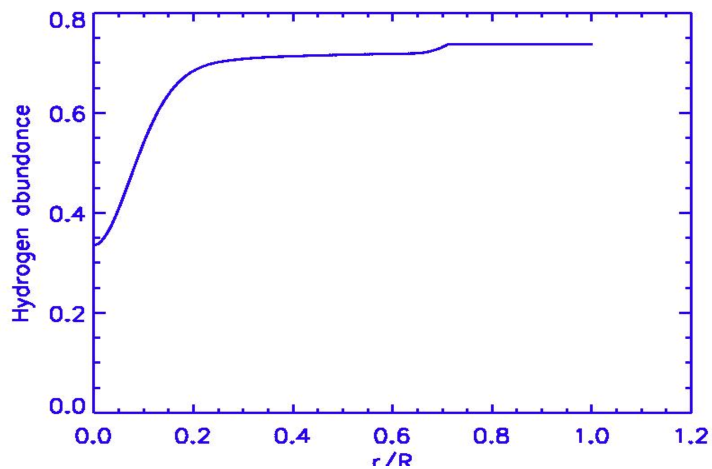
$$\frac{\mathrm{dT}}{\mathrm{dr}} = \begin{cases} \frac{-3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2} & \text{if raddiative} \\ \frac{\mathrm{dT}}{\mathrm{dr}} & \text{if convective} \end{cases} \qquad (4)$$

Results from the Standard Solar Mode (In CGS Units)





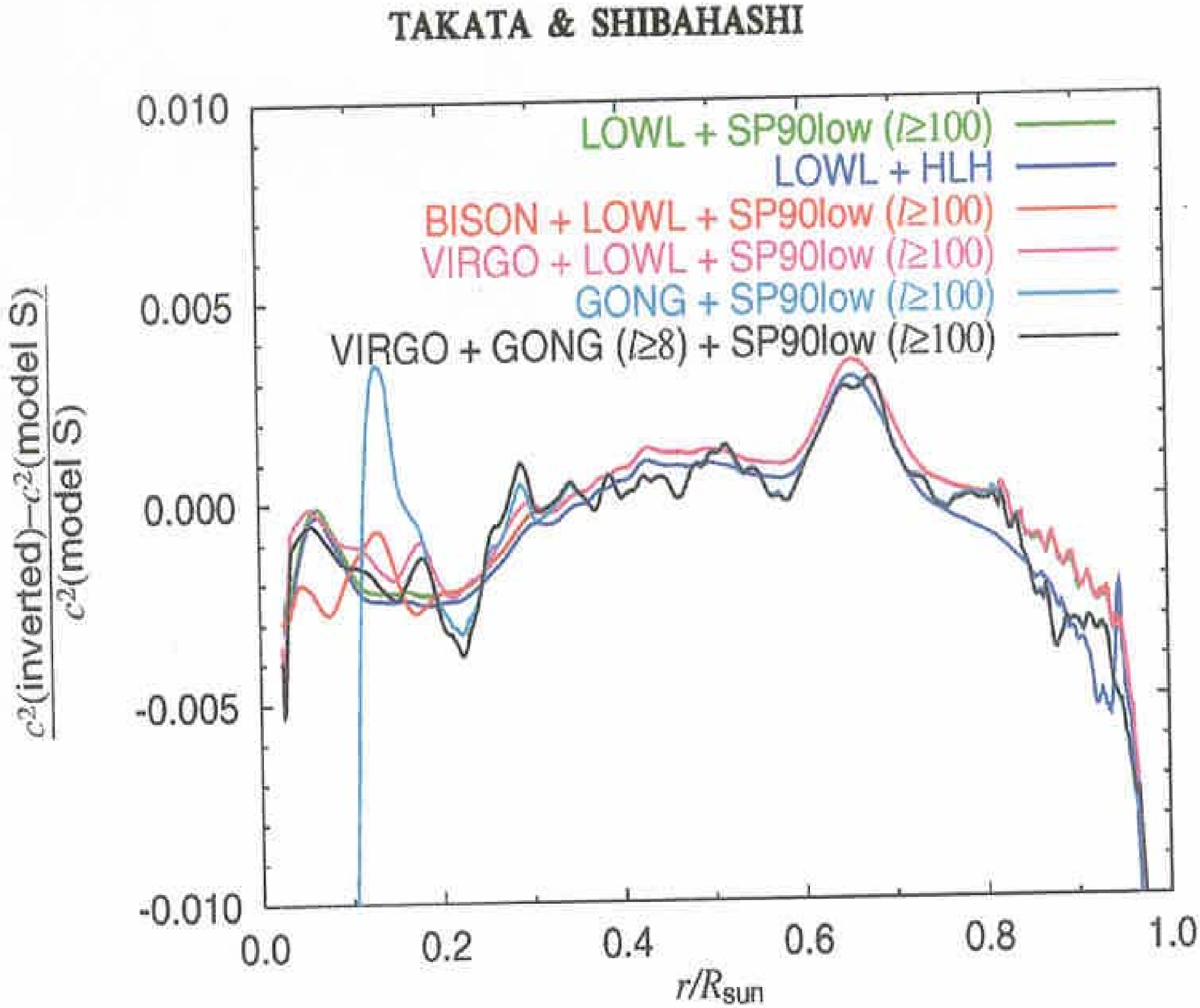




Inversion of the sound speed

- Asymptotic Relation : For high radial or high degree modes, we have the following relation
- Observed Frequency (nu) = F(c^2)
- From the observed frequencies one can infer the sound speed
- Non-Asymptotic Method **(Antia, Chitre, Basu 1998)**

$$\frac{\delta\omega_i}{\omega_i} = \int K^i_{c^2,\rho}\frac{\delta c^2}{c^2}dr + \int K^i_{\rho,c^2}\frac{\delta\rho}{\rho}dr + \frac{F_s(\omega_i)}{E_i}$$


TAKATA & SHIBAHASHI
LOWL + SP90low (l≥100)
LOWL + HLH
BISON + LOWL + SP90low (l≥100)
VIRGO + LOWL + SP90low (l≥100)
GONG + SP90low (l≥100)
VIRGO + GONG (l≥8) + SP90low (l≥100)
0.010
0.005
0.000
-0.005
-0.010
0.0
0.2
0.4
0.6
0.8
1.0
r/R_{sun}
$\frac{c^2(\text{inverted})-c^2(\text{model S})}{c^2(\text{model S})}$

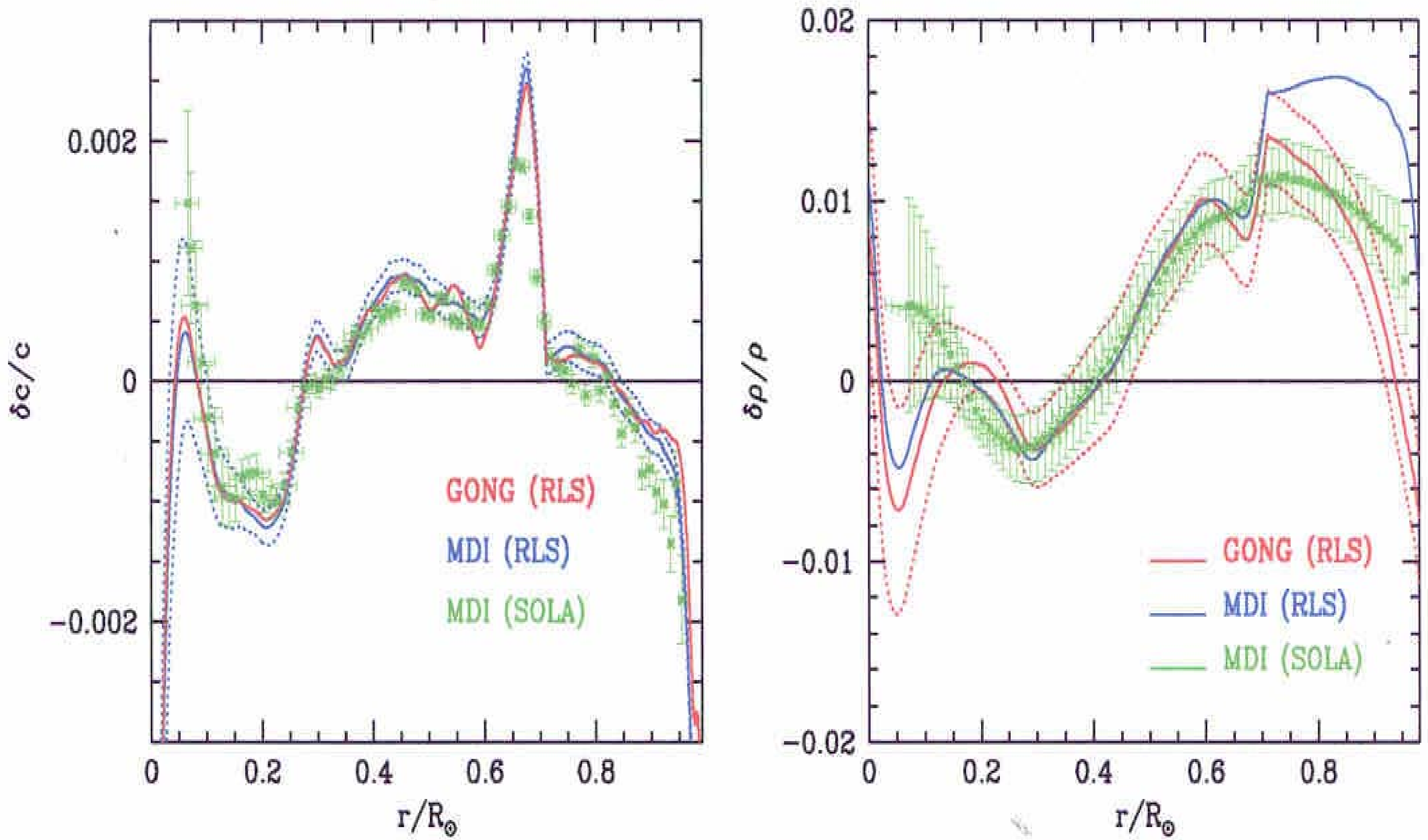


Fig. 1: Relative difference in sound speed and density between the Sun and Model S (Christensen-Dalsgaard et al. 1996) as inferred using different data sets and inversion techniques as marked in the figures. The SOLA inversion results are from Basu (1998).

Solar Seismic model

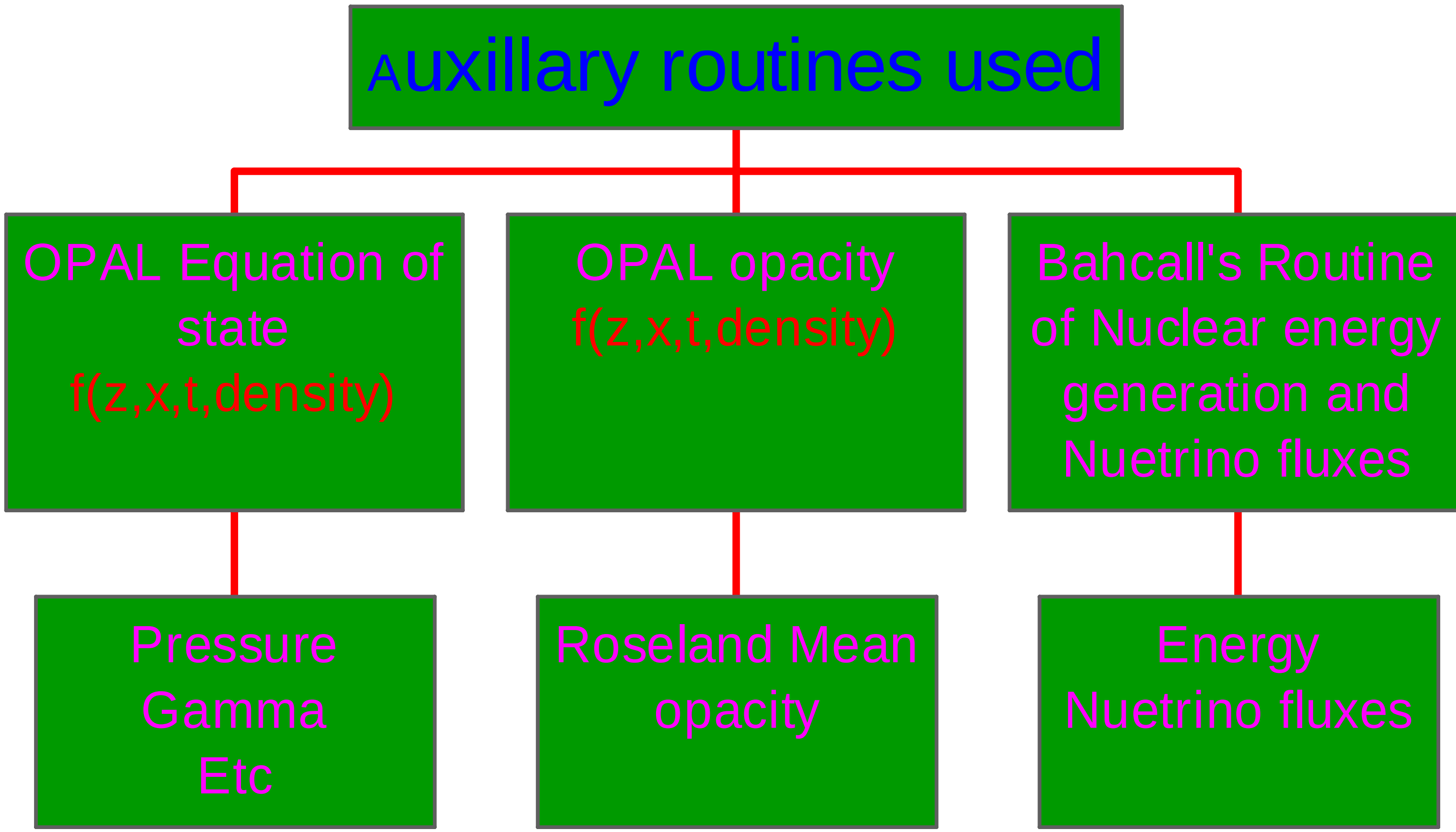
- **Seismic model can be constructed by solving the stellar structure equations with the imposition of the sound speed inferred from the helioseismology and the equation of state**
- **In order to close the system of equations (1-4), we need auxiliary equations : the equation of state, equation of opacity, the equation for nuclear reaction rate**
- **C(P,T,X) & X(P,T,C) yield P,T & X can be determined uniquely**
- **By satisfying observed luminosity and mass , structure equations are solved**
- **This method gives a snap-shot model of the present day sun**
- **Advantages over the standard solar model : (I) no need of history of the sun and (ii) no need to compute the convective flux**
- **Dalsgaard's model is used for the comparison of the seismic model**
- **Helium abundance and depth of the convection zone are obtained as part of the solution**

Solution of the structure equations with the constraint of sound speed

- **Since the sound speed is a thermodynamically determined quantity, it is a function of two other variables such as P and T, along with the mass fraction of each chemical composition X_{I}**
- **If we consider hydrogen and helium separately as X and Y respectively and treat all other elements collectively as Z, then only two of them are treated as an independent variable.**

- In order to solve the structure equations, the auxiliary equations such as equation of state, opacity and equation of nuclear energy generation are needed

Solution of the structure equations



Computation of Hydrogen abundance and density from the constraint of the sound speed in the radiative zone

Correct Values

$$c_0^2 = c_0^2(\rho, X) \qquad (1)$$

$$P_0 = P(\rho, X) \qquad (2)$$

Values with different initial conditions

$$c^2 = c^2(\rho + \delta\rho, X + \delta X)$$

$$c^2 = c_0^2 + \frac{\partial c^2}{\partial \rho}\delta\rho + \frac{\partial c^2}{\partial X}\delta X \qquad (3)$$

$$P = P(\rho + \delta\rho, X + \delta X)$$

$$P = p_0 + \frac{\partial P}{\partial \rho}\delta\rho + \frac{\partial P}{\partial X}\delta X \qquad (4)$$

Seismic Model

Radiative zone

All the four structure
Equations are used

Convective envelope

Only first two equations
of the structure equations
are used

Convection imposes uniform abundance

C=C(density,temp)
P=P(density,temp)

Initial Conditions

Seismic Model

Radiative Zone

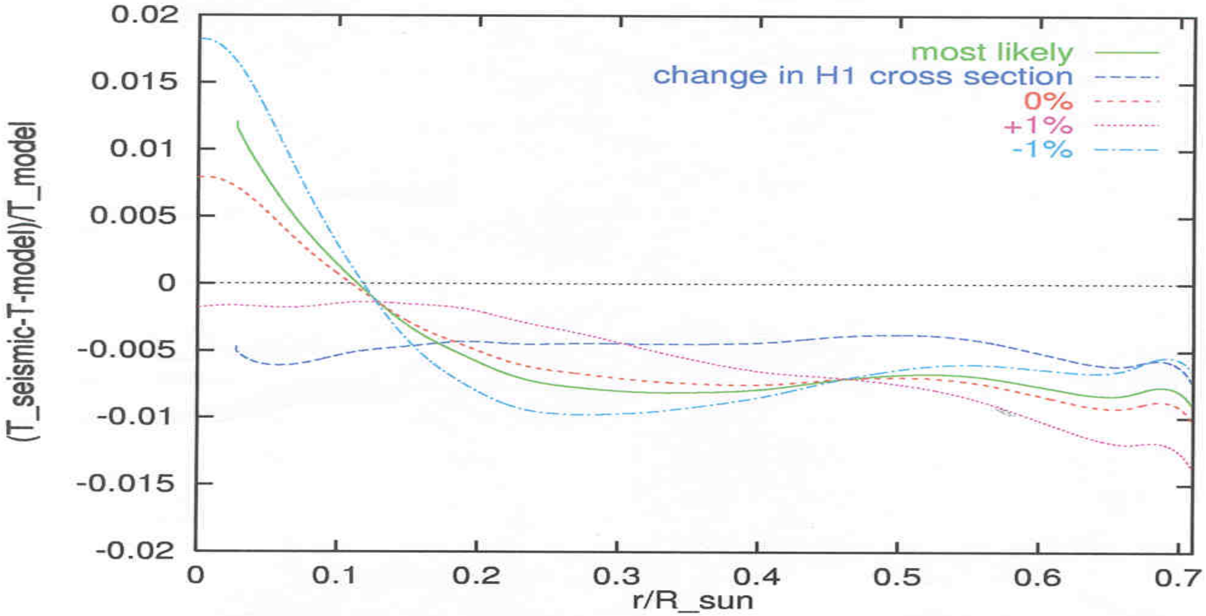
Convective Envelope
Mass and Pressure at the base
of the convection zone
from the radiative solution

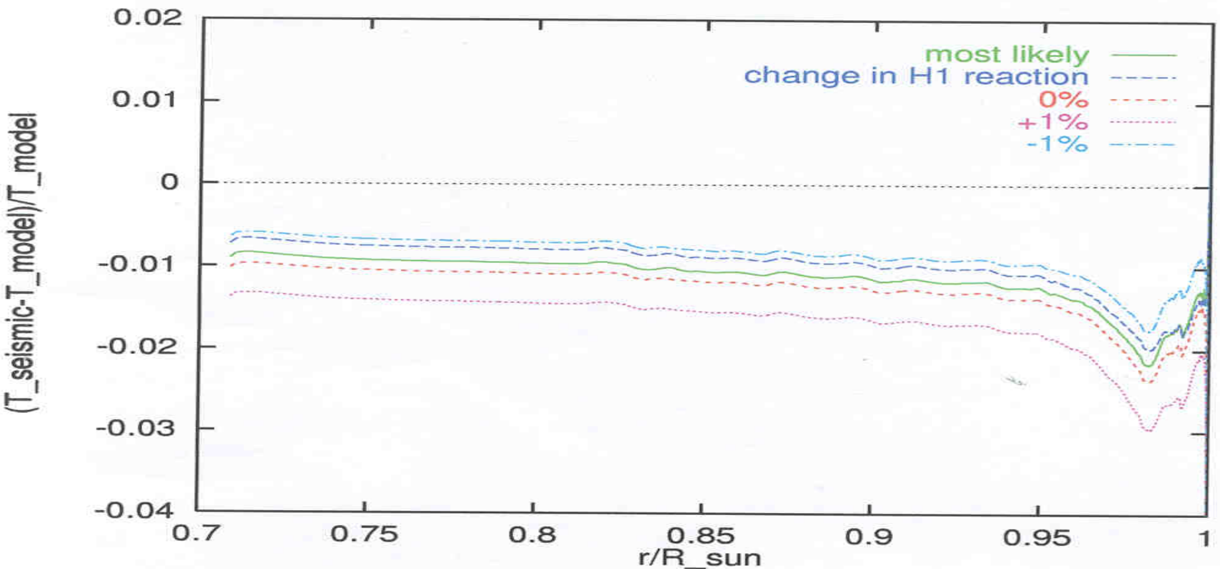
Near Center
M=0.0,L=0.0,P=P_c,T=T_c

Near Base of the convective Envelope
(Del)_ad=(Del)_rad,L=L_sun
P=P_b,T=T_b

The whole Solution should satisfy
the constraint of observed
Mass=M_sun
chemical abundance Z/X=0.0245

Results using the sound speed inferred from Tokyo group





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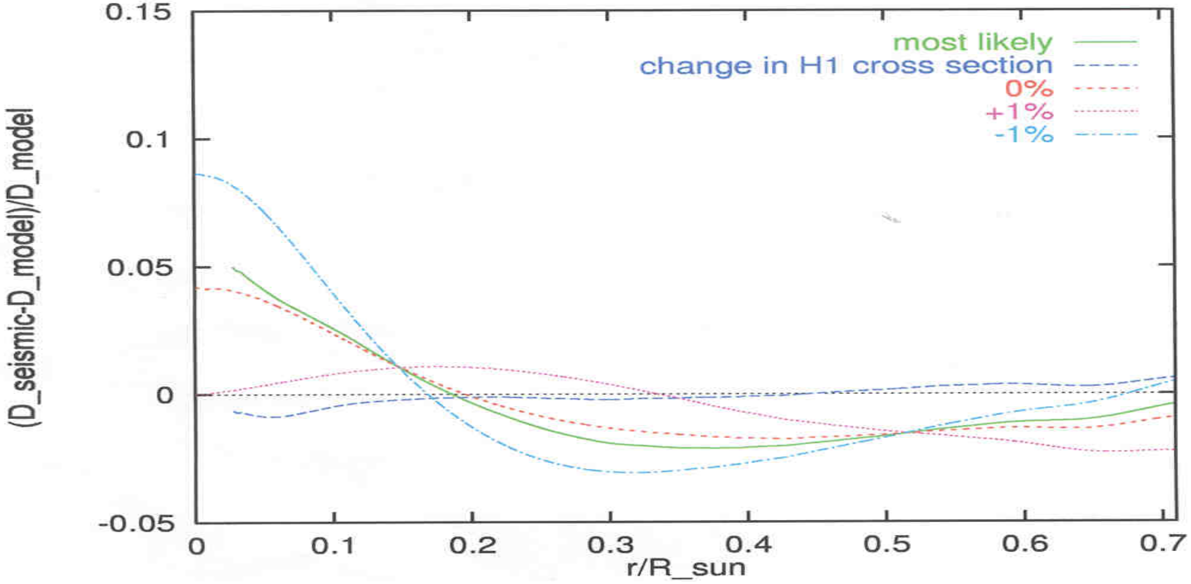
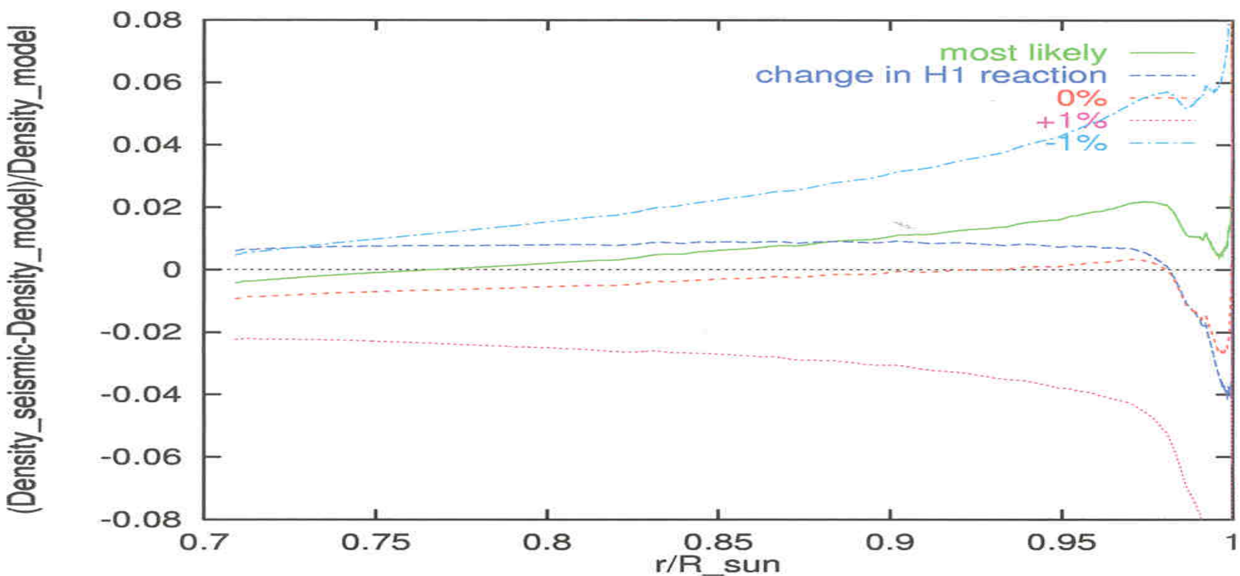
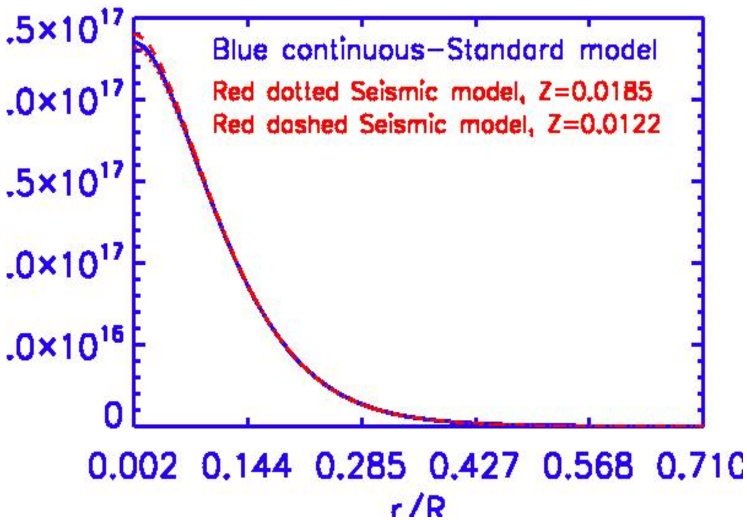
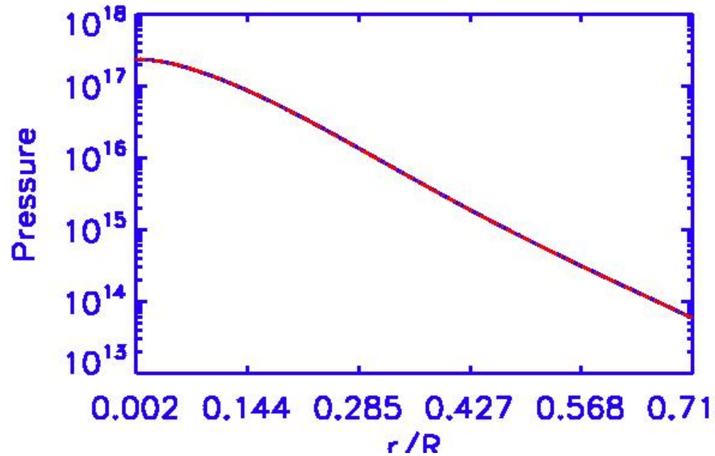
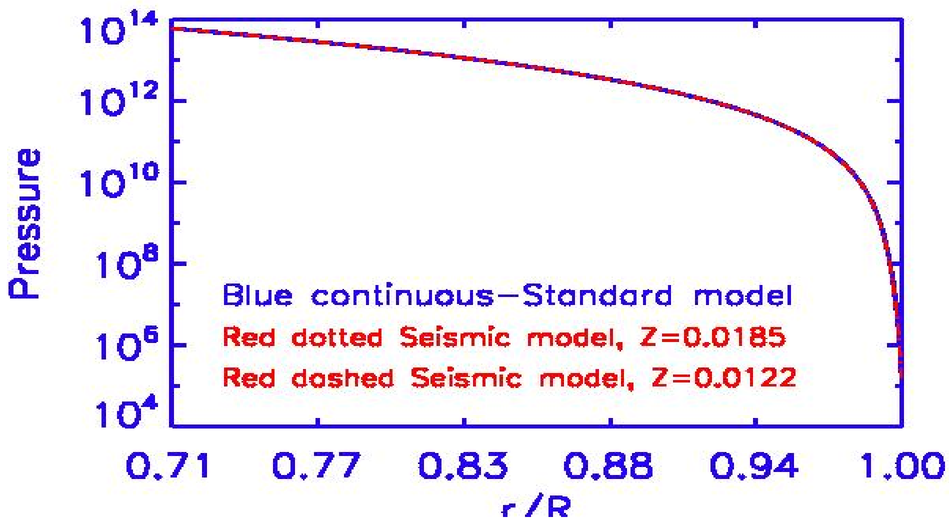




Recent results using the sound speed inferred by Antia and Basu







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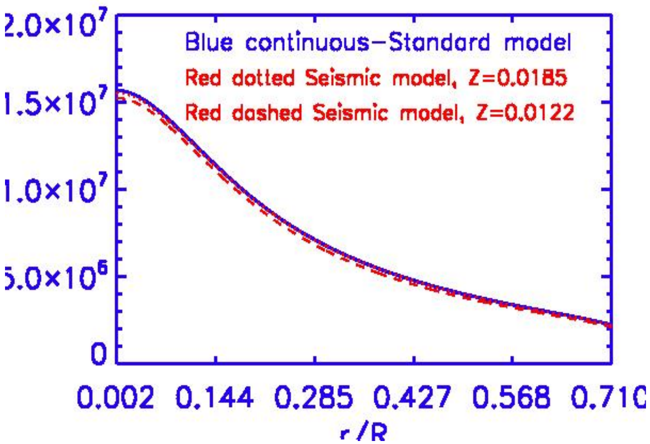
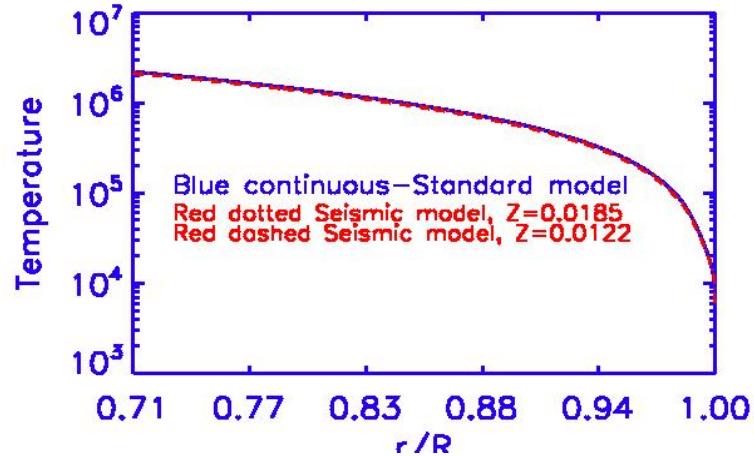
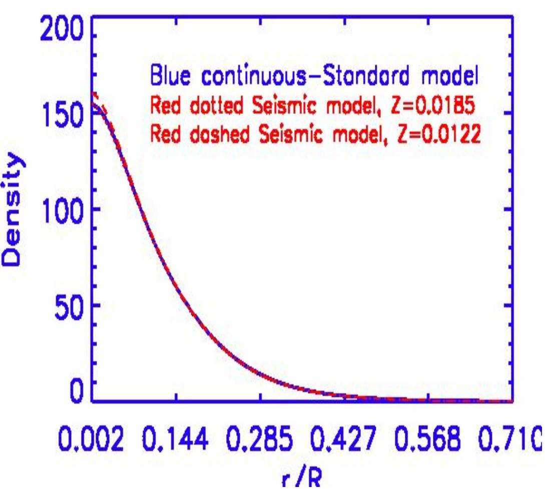
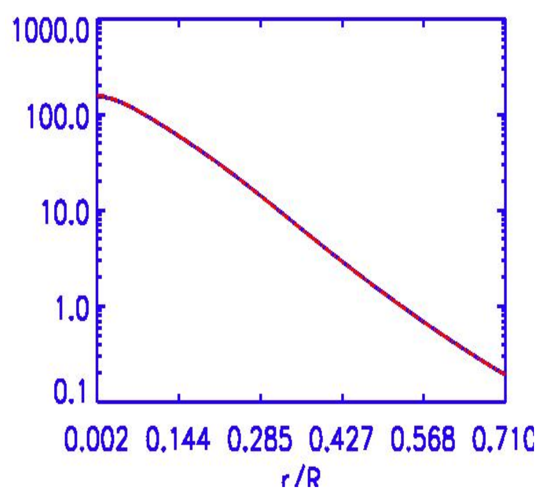
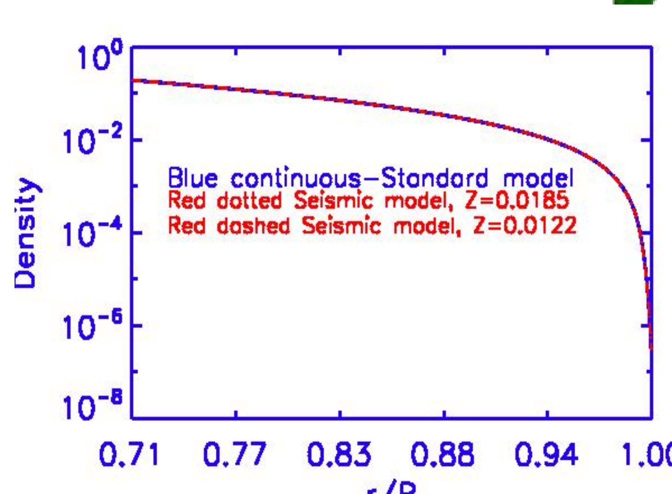


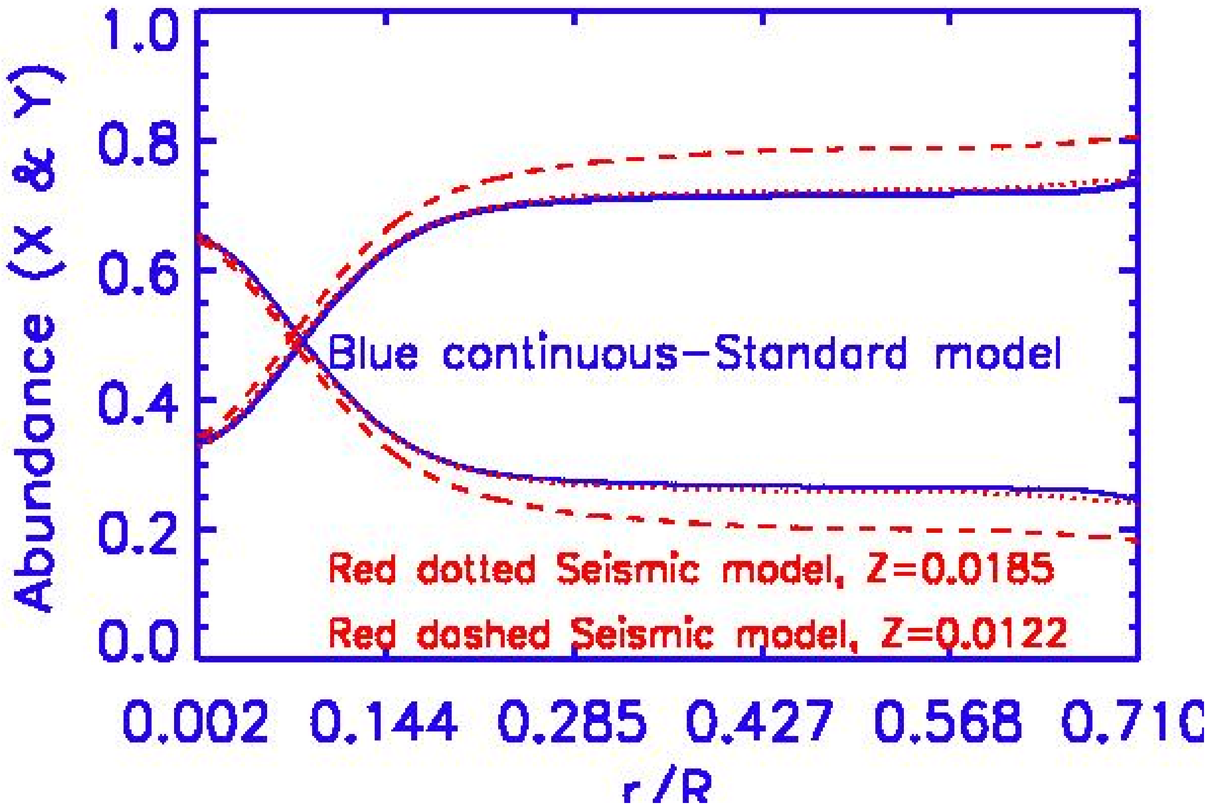




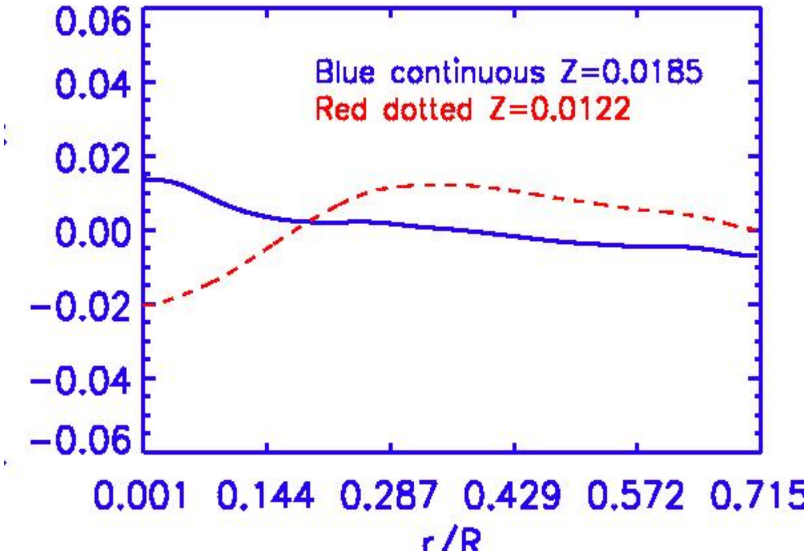


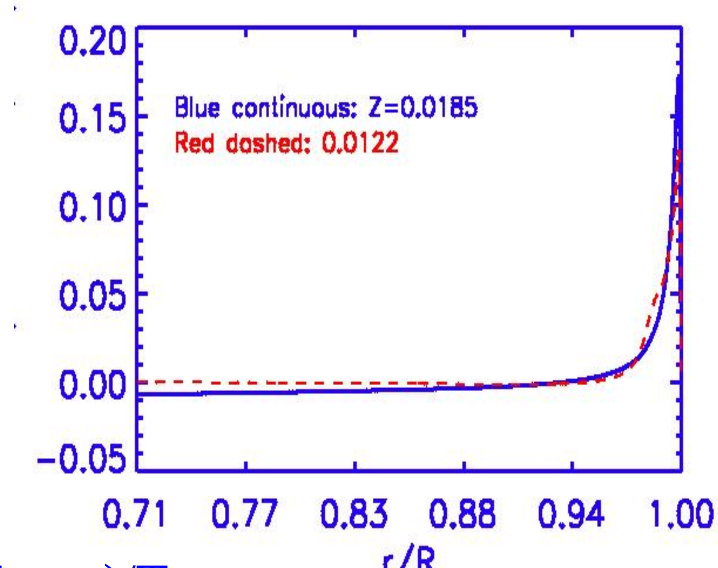


Results using the sound speed inferred by Antia and Basu

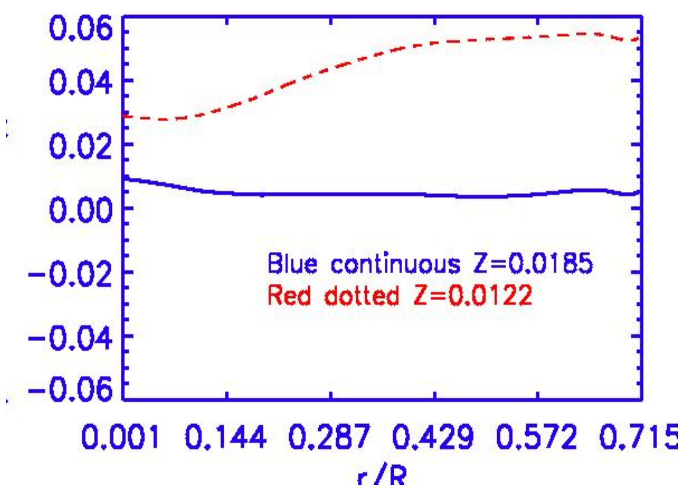


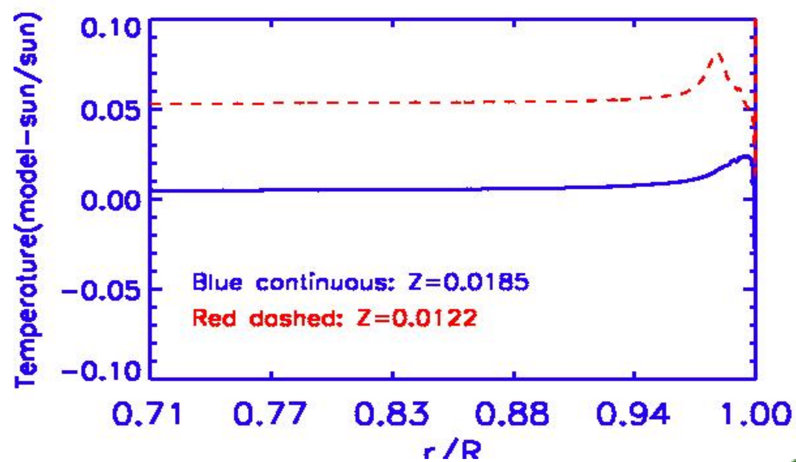
(P_model-P_sun)/P_sun



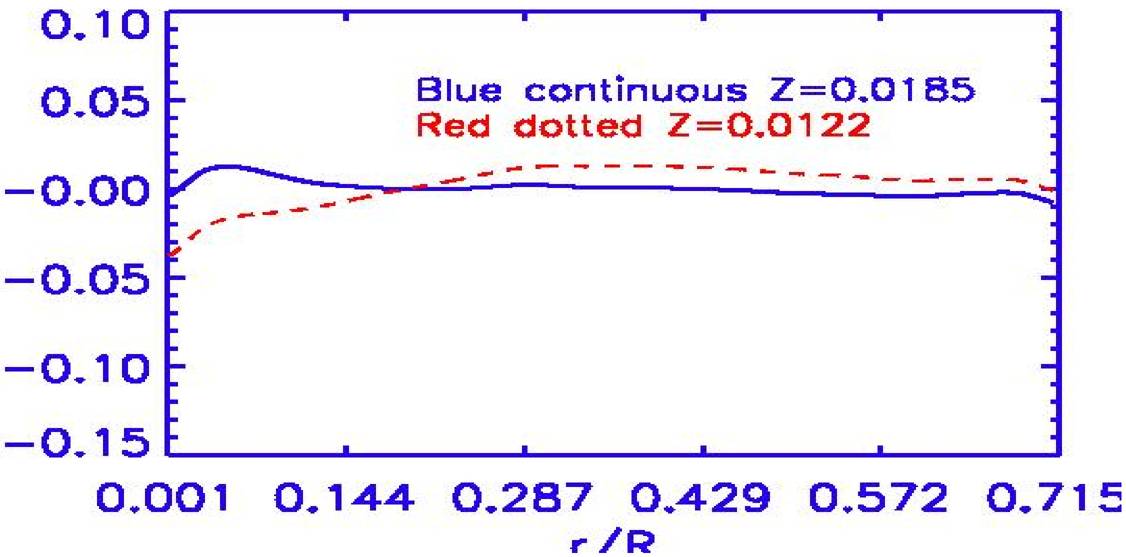


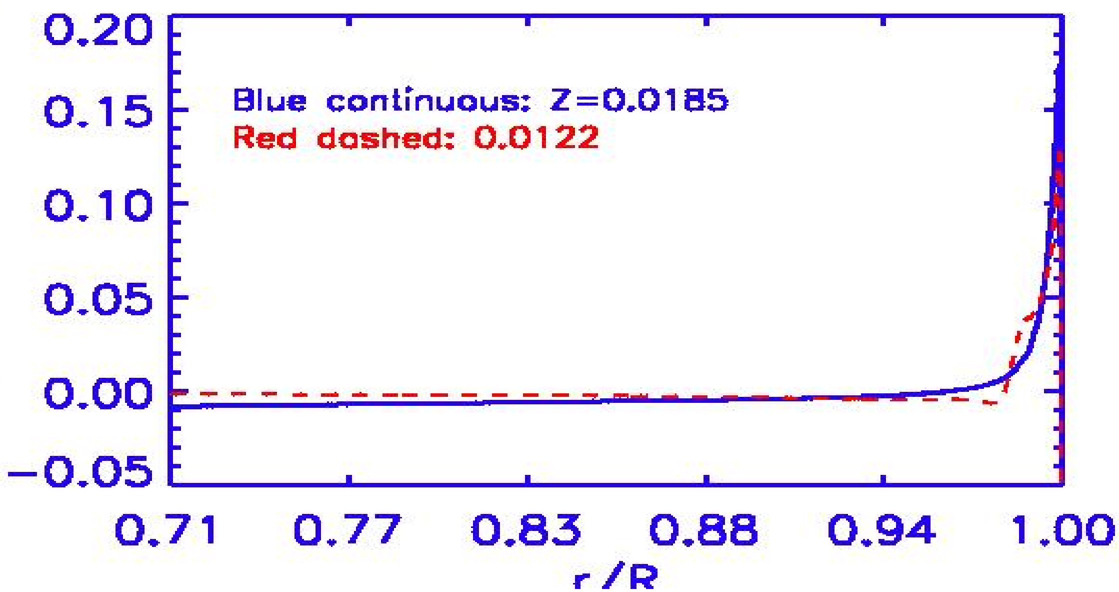
(T_model-T_sun)/T_sun





(Density_model-Density_sun)/Density_sun





Neutrino Fluxes (10^10 cm^{-2}sec^{-1})

Source	Fluxes SSM	Fluxes Seism 1	Fluxes Seism 2	Fluxes Seism3	Fluxes Seism 4	Fluxes Seism 5	Fluxes Seism 6		
	Z=0.0185 Z/X=0.0247 Bcz=0.709	Z=0.0185 , Z/x=0.0247 Bcz=0.709	Z=0.0165 Z/X=0.0215 Bcz=0.709	Z=0.015 Z/X=0.0188 Bcz=0.709	Z=0.015 z/X=0.0194 Bcz=0.71	Z=0.0122 z/X=0.0151 Bcz=0.71	Z=0.0122 z/X=0.0158 Bcz=0.72		
pp	6.0	6.01	6.05	6.07	6.07	6.1	6.1		
pep	0.014	0.015	0.014	0.015	0.015	0.015	0.015		
hep	8e-07	0.03e-08	0.03e-08	1.3e-07	1.28e-07	1.32e-07	1.32e-07		
7Be	0.47	0.44	0.42	0.41	0.408	0.379	0.379		
8B	5.8e-04 (2.8e-04) (2.35e-04)	4.49e-04	4.03e-04	3.76e-04	3.76e-04	3.18e-04	3.18e-04		
13N	0.06	0.057	0.047	0.04	0.04	0.029	0.0287		
15O	0.05	0.052	0.042	0.036	0.036	0.025	0.025		
17F	5.2e-04	4.02e-4	3.2e-04	2.7e-04	2.8e-04	1.92e-04	1.92e-04		

Summary

- **With the constraint of the helioseismologically inferred sound speed, we develop a seismic model that consists of two parts; radiative core and convective envelope.**
- **In the region of the radiative core, we use four basic solar structure equations.**
- **By considering the mass and the pressure obtained from the solution of the radiative core, as the initial conditions, we solve the equations of conservation of mass and the hydrostatic equilibrium in the convective envelope and satisfy the outer boundary of one solar mass.**
- **We obtain the temperature in the convective envelope without solving the differential equation.**
- **It is found that seismic model satisfying one solar mass at the surface varies strongly on the nature of the sound speed profile near the center and a weak function of either depth of the convection zone or heavy elemental abundance Z/X.**
- **It is found that one solar mass at the surface is satisfied for the sound speed profile which deviates at the center by ~ 0.22% from the sound speed profile of model S (Cristensen Dalsgaard, 1996), if we adopt the value of nuclear cross section factor S_11 value of 4.07e-22 keV barns.**
- **The extent of the convection zone is determined so that obtained temperature gradient matches the adiabatic temperature gradient at the base of the convective envelope. This consistency enables us to estimate the depth of the convection zone to be 0.718R_o if we accept the same nuclear cross section factor**
- **In order that thus obtained temperature gradient should not be sub adiabatic, we decreased the value of nuclear cross section factor S_11 to 4.02e-22 keV barns and depth of the convective envelope to 0.709R_0**
- **This indicates that Solar convective envelope is deeper than the depth of the solar convective envelope obtained from the standard solar model**
- **The resulting chemical abundances at the base of the convective envelope are obtained to be X=0.755, Y=0.226 and Z=0.0185**

Summary

Recent results

- **Using the sound speed of Antia and Basu and for the low Z, Neutrino fluxes substantially reduce (compared to the neutrino fluxes obtained from the standard solar model) and are very close to the observed flux.**
- **However, the helium abundance at the base of the convective envelope is very low ~ 0.18 inconsistent with the other cosmic abundances (~0.23)**