Effect of Gravity on Alfven Waves with Flows

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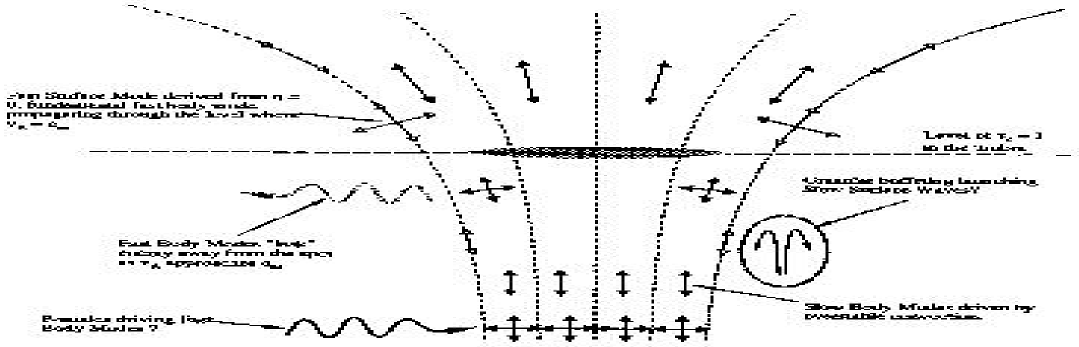
Sketch of a Sunspot with different types of waves taken from Roberts (1990)



The basic geometry

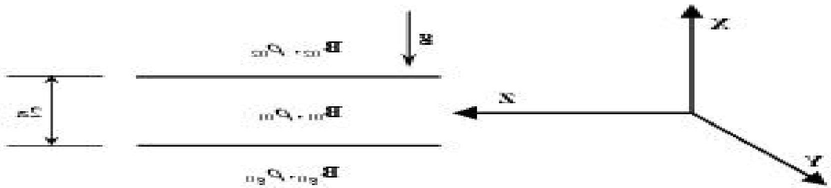


Figure 1. The Geometry.

Different Types of Modes

Three important MHD Waves in the Sun are:

1. The Alfven Wave
2. The Fast Magnetosonic Wave
3. The Slow Magnetosonic Wave

$$V_f > V_A > V_s$$

In addition to the above, we have the Surface Waves too. Other types of waves come into play depending on external forcing such as density stratification, rotation etc.

The Basic Equations of motion are the continuity, momentum, induction equation , the energy equation

Basic Equations

$$\frac{\partial \rho}{\partial t} + U_0 \frac{\partial \rho}{\partial x} = -\left[\rho_0(z)(\nabla \cdot \mathbf{v}) + v_z \frac{\partial \rho_0(z)}{\partial z}\right]$$

$$\rho_0(z)\left[\frac{\partial \mathbf{v}}{\partial t} + U_0 \frac{\partial \mathbf{v}}{\partial z}\right] = -\nabla p + \rho \mathbf{g} + \frac{1}{\mu}(\nabla \times \mathbf{b}) \times \mathbf{B}_0$$

$$\frac{\partial p}{\partial t} + U_0 \frac{\partial p}{\partial x} = -C^2 \rho_0(z)(\nabla \cdot \mathbf{v}) + g \rho_0(z) v_z$$

$$\frac{\partial \mathbf{b}}{\partial t} + U_0 \frac{\partial \mathbf{b}}{\partial x} = -\mathbf{B}_0(\nabla \cdot \mathbf{v}) + B_0 \frac{\partial \mathbf{v}}{\partial x}$$

$$\nabla \cdot \mathbf{b} = 0$$

$\rho_0(z)$ is the density, $C = \sqrt{\gamma P_0(z)/\rho_0(z)}$ is the sound speed. $P_0(z)$ and γ are the pressure and ratio of specific heats and g is the acceleration due to gravity.

The case of flow (non zero) is rather complicated to begin with. The solution of the Wave equation involves Hyper geometric Functions. In this talk I shall restrict myself with the simple case of no flow. The case of uniform flow is being worked out and the dispersion relation will be solved numerically for different sets of parameters pertaining to the model.

Let us start with the case $U_0 = 0$

The boundary conditions are :

$$p_1 + \frac{\mathbf{B}_{01} b_{z1}}{\mu} - \rho g = p_g + \frac{\mathbf{B}_{g0} b_{gz}}{\mu}$$

total pressure continuous. Similarly, the normal velocity component must be continuous. i.e.,

$$v_{z1} = v_{gz}$$

Applying the boundary conditions at the interface wherein there is a discontinuity in the density and the magnetic field, the dispersion relation (for details look into the book by Chandrasekhar (Hydrodynamic and Hydromagnetic Stability)

The dispersion relation can be simplified to yield

$$\left[\epsilon_1\epsilon_2+\epsilon_1\epsilon_g+\epsilon_1 g\left\{\frac{\rho_{g0}-\rho_{02}}{\omega}\right\}\right]+\left[\epsilon_1^2+\epsilon_2\epsilon_g+\epsilon_2 g\left\{\frac{\rho_{g0}-\rho_{02}}{\omega}\right\}+\epsilon_g g\left\{\frac{\rho_{01}-\rho_{02}}{\omega}\right\}\right] tanh(2ka)=0$$

In this slide we deal with some special cases before solving the original dispersion relation. In the case of small and large ka (long and short wavelengths) the dispersion relation is relatively easy to solve.

Introduce normalised $G = g/kV_A^2$.

Limiting cases : $ka \to 0$

$$\epsilon_2 + \epsilon_g + g\{\frac{\rho_{g0} - \rho_{02}}{\omega}\} = 0$$

$ka \to \infty$

$$\epsilon_1(\epsilon_2 + \epsilon_1) = 0$$

$$\epsilon_1\left[\epsilon_g + g\{\frac{\rho_{g0} - \rho_{02}}{\omega}\}\right] + \epsilon_2\left[\epsilon_g + g\{\frac{\rho_{g0} - \rho_{01}}{\omega}\} + \epsilon_g g\{\frac{\rho_{g0} - \rho_{02}}{\omega}\}\right] = 0$$

The expression for the different epsilons given in the dispersion relation.

$$\epsilon_1(k,\omega) = \frac{\mathbf{B}_{01}^2 k}{\mu\omega} - \frac{\rho_{01}\omega}{k}$$

$$\epsilon_2(k,\omega) = \frac{\mathbf{B}_{02}^2 k}{\mu\omega} - \frac{\rho_{02}\omega}{k}$$

$$\epsilon_g(k,\omega) = \frac{\mathbf{B}_{0g}^2 k}{\mu\omega} - \frac{\rho_{0g}\omega}{k\tau}$$

where $\tau = \rho_{g0}/\rho_{02}$

In summary, the gravitational stratification effect on magnetohydrodynamic waves in a single interface has been studied in the penumbral region of the sunspot. The effect of flows is being considered. However, in the present talk , the case of no flow is dealt with . The dispersion relation with flow will be derived and the different mode structure of the waves will be studied for the relevant set of parametric values in due course.

Thank You