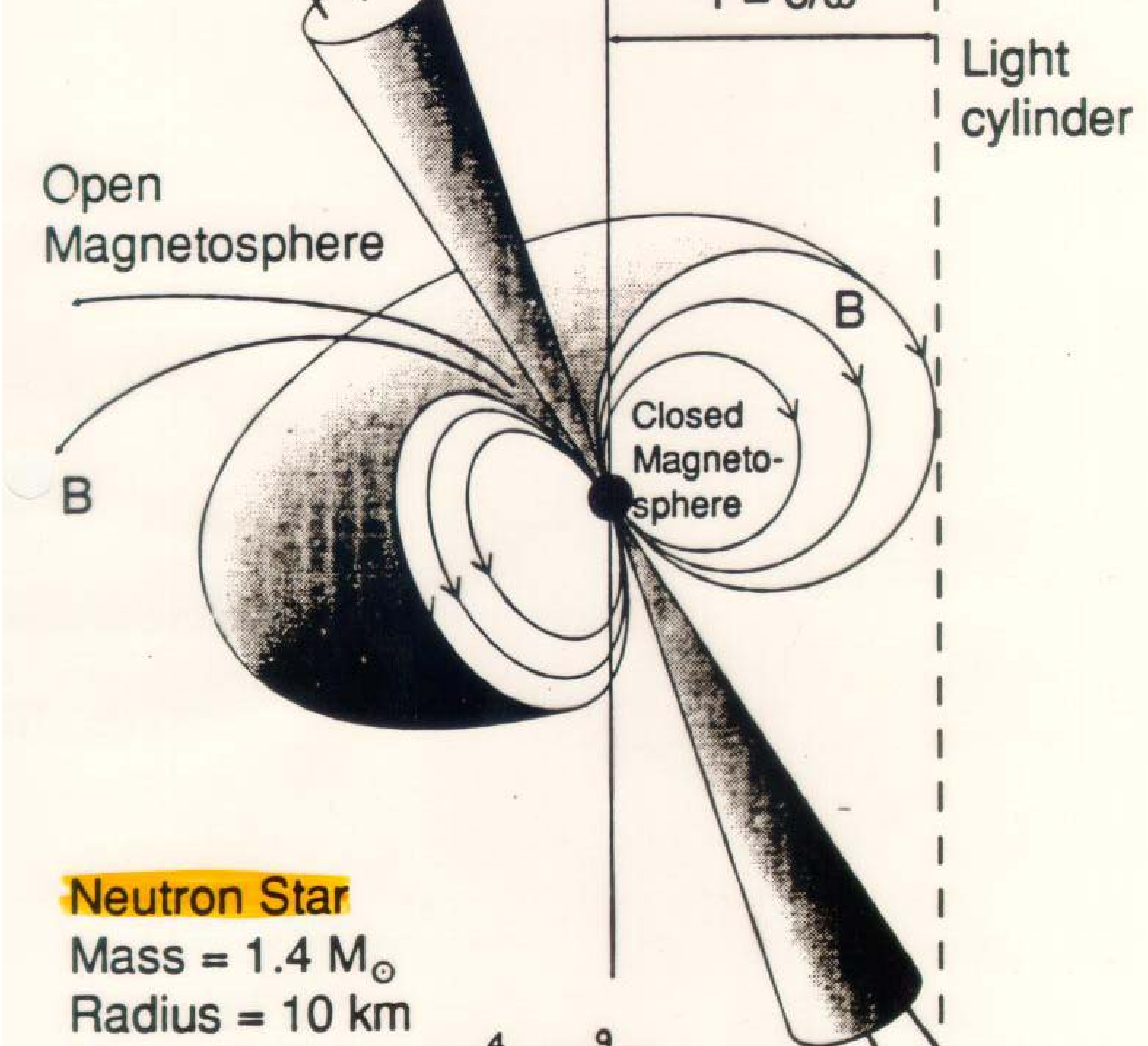
DYNAMICS OF CHARGED PARTICLES IN PULSAR MAGNETOSPHERE

by

Reji Mathew C. Thomas


Light
cylinder
Open
Magnetosphere
B
B
Closed
Magneto-
sphere
Neutron Star
Mass = 1.4 $M_\odot$
Radius = 10 km

2D MODEL OF PARTICLE MOTION

- We assume pulsar magnetic field is mainly dipolar. The field lines are projected on to the equitorial plane (2D plane). The field lines are approximated to be straight lines.
- The equation of motion is given by Gangadhara (1996). Consider it's radial component:

$$\frac{d}{dt}\left(m\frac{dr}{dt}\right) = m\Omega^{*2}r \tag{1}$$

Relativistic mass of particle: $\mathrm{m} = \mathrm{m}_0\gamma$

Lorentz factor

$$\gamma = \left(1 - \frac{\dot{\mathrm{r}}^2}{\mathrm{c}^2} - \frac{\mathrm{r}^2\Omega^{*2}}{\mathrm{c}^2}\right)^{-1/2}$$

$\dot{r} = dr/dt$

Particle angular velocity: $\Omega^* = \Omega\sqrt{1-(b^2/r^2)}$, $\mathrm{b} = \mathrm{d}_0\cos\theta_0$

we rewrite Eq. (1) as

$$s\frac{d^2s}{dt^2} + \frac{[2s^2 - D^2/(1+D^2)]}{1-s^2}\left(\frac{ds}{dt}\right)^2 - s^2\Omega^2 + \Omega^2\frac{D^2}{1+D^2} = 0 \tag{2}$$

$$s = \frac{\Omega}{c}\frac{r}{\sqrt{1+D^2}}$$

$$D = \Omega d_0 \cos\theta_0/c.$$

The solution of the 2D equation:

$$r(t) = \frac{c\sqrt{1+D^2}}{\Omega}\mathrm{cn}(\lambda - \Omega \mathrm{t})$$

- The particle parameters such as β, γ, ρ, and the radiation parameters such as Stokes parameters: I, Q, U and V are computed.

(Thomas, R. M. C., Gangadhara, R. T. 2005, A& A, 437, 537)

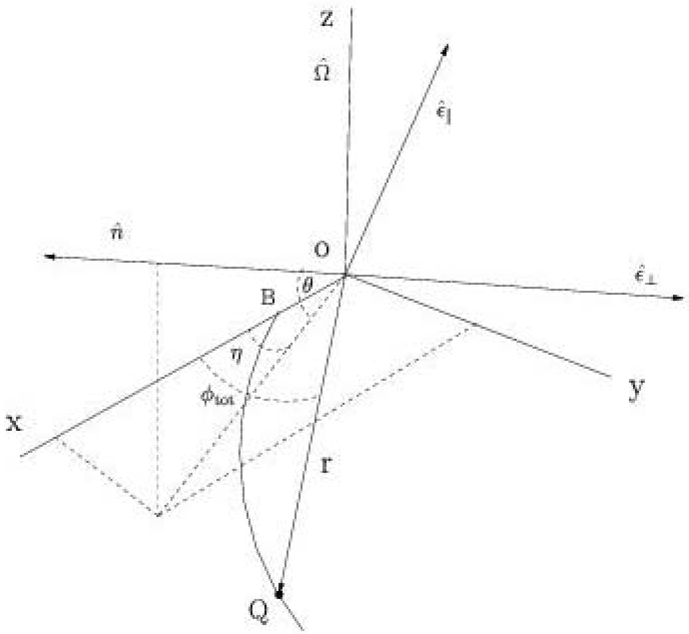


Figure 1: The coordinate system in which the particle motion is considered. The curve BQ represents the particle trajectory in the x-y plane.

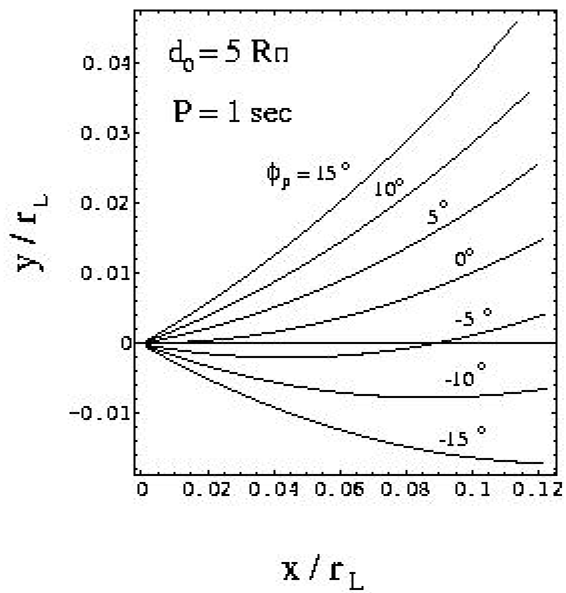


Figure 2: Particle trajectories during the time interval $0 \leq t \leq 0.02$ sec in laboratory frame. The corresponding field lines lie with in the range $-15^o \leq \phi_p \leq 15^o$, at an interval of 5^o. Assumed neutron star radius $R_n = 10$ Km.

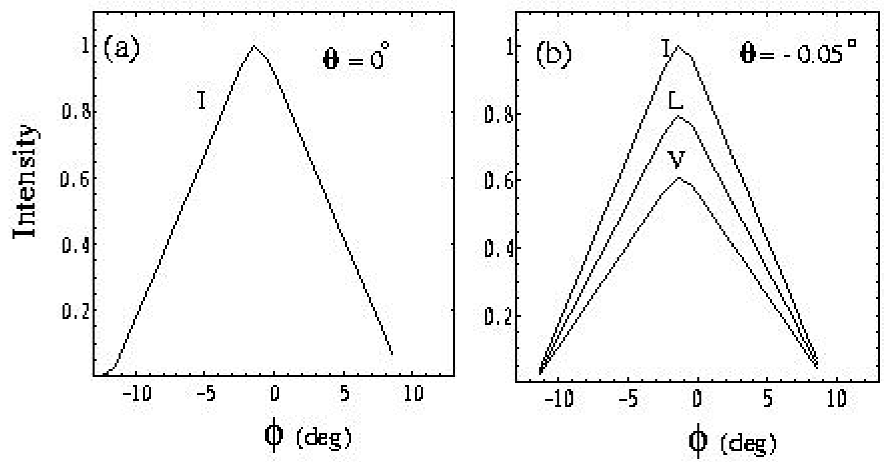


Figure 3: The simulated profiles: panel (a) for $\theta = 0°$ and panel (b) for $\theta = -0.05°$. The parameter ϕ is the rotation phase. Used $\gamma_0 = 100$ and $d_0 = 10$ Km.

3D MODEL OF PARTICLE MOTION

- The radial equation of motion considering the dynamics of particle in r, θ and ϕ coordinates are derived from the following classical equation.

$$\frac{d\,p_{\rm lab}}{dt} = q\,\mathbf{E} + \mathbf{F}_{\rm B} \tag{3}$$

- $$\gamma = \left[1 - \frac{\dot{r}^2}{c^2} - \left(\frac{r}{c}\frac{d\theta'}{dt}\right)^2 - \left(\frac{r\,\sin\theta'}{c}\frac{d\phi_p}{dt}\right)^2\right]^{-1/2} , \tag{4}$$

- $p_{\rm lab} = m v_{\rm lab}$, $m = \gamma m_0$, and

$$\mathbf{v}_{\rm lab} = \frac{d\,r}{dt}\,\hat{e}_r + r\,\frac{d\,\theta'}{dt}\,\hat{e}_\theta + \ r\sin\theta'\frac{d\,\phi_p}{dt}\,\hat{e}_\phi\ ,$$

- $$\gamma = \left[1 - \frac{\dot{r}^2}{c^2} - \left(\frac{r}{c}\frac{d\theta'}{dt}\right)^2 - \left(\frac{r\,\sin\theta'}{c}\frac{d\phi_p}{dt}\right)^2\right]^{-1/2} , \tag{5}$$

$$\left[\frac{d}{dt}\left(m\frac{dr}{dt}\right) - m\,r\,\left(\frac{d\theta'}{dt}\right)^2\right]\hat{e}_r = \vec{F}_{\mathrm{Br}} + (q\vec{E}_r + \vec{F}_{\mathrm{cr}}) \qquad (6)$$

- Then the component of centrifugal force in the $\hat{e}_r$ direction is given by

$$\vec{F}_{cr} = (\vec{F}_c \cdot \hat{e}_r)\,\hat{e}_{\mathrm{r}}\,.$$

and $\vec{F}_c$ is given by

$$\vec{F}_c = m\left(\frac{d\phi_p}{dt}\right)^2 \hat{\Omega} \times (\vec{r} \times \hat{\Omega}) = m\,r\,\sin\theta'\left(\frac{d\phi_p}{dt}\right)^2 \hat{e}_p\,, \qquad (7)$$

- The equation of motion in the radial direction reduces to

$$\frac{d}{dt}\left(m\frac{dr}{dt}\right) - m\,r\,\left(\frac{d\theta'}{dt}\right)^2 - m\,r\,\sin^2\theta'\left(\frac{d\phi_p}{dt}\right)^2 = 0\,. \qquad (8)$$

SOLUTION OF EQUATION OF MOTION

- The equation of motion is split into zeroeth and first order equations and the resultant solutions are added appropriately

- Perturbative solution

$$r = r_0 + \epsilon\, r_1 + \epsilon^2\, r_2 ..., \tag{9}$$

and

$$\dot{r} = \dot{r}_0 + \epsilon\, \dot{r}_1 + \epsilon^2\, \dot{r}_2 .... \tag{10}$$

- $\epsilon = r_{\mathrm{L}}/r_{\mathrm{e}}$

Thus we expand

$$\Omega_m^2 = \Omega_{m0}^2 + \epsilon\, \Omega_{m1}^2 + \epsilon^2\, \Omega_{m2}^2 ..... \tag{11}$$

$$\frac{d\Omega_m^2}{dt} = \dot{\Omega}^2{}_{m0} + \epsilon\, \dot{\Omega}^2{}_{m1} + \epsilon^2\, \dot{\Omega}^2{}_{m2} .... \tag{12}$$

In our previous work we have found out a solution to the zeroth order equation:

$$r_0 = \frac{c}{\Omega_{m0}} \mathrm{cn}(\lambda - \Omega_{\mathrm{m0}}\mathrm{t}) \ , \tag{13}$$

and to the first order equation, the solution is

$$r_1 = -y_1 \int \frac{y_2\, \kappa}{w} dt + y_2 \int \frac{y_1\, \kappa}{w} dt \ , \tag{14}$$

and

$$\begin{aligned} y_1 &= \exp\left[-Q_1 t^2 + \sqrt{Q_1} t\right], \\ y_2 &= -\exp\left[-Q_1 t^2 - \sqrt{Q_1} t\right], \end{aligned} \tag{15}$$

RESULTS OF 3D MODEL

•

$$\vec{r} = r(t)\{\sin\theta' \cos\phi_p \,\hat{e}_x + \sin\theta' \sin\phi_p \,\hat{e}_y + \cos\theta' \,\hat{e}_z\} \ , \tag{16}$$

• The curvature radius of particle trajectory can be estimated using the following expression

$$\rho = \frac{|\vec{v}|^3}{|\vec{v} \times \vec{a}|} \ , \tag{17}$$

• **The maximum value that radius of curvature can reach is**

$$r_{\rm L}/(2 \ \sin\alpha)$$

•

$$\rho_m \approx \frac{c}{\Omega_{\rm m0}} \left[4 + (\csc^2\alpha - 1)\sin^2(\Omega_{\rm m0}\, t)\right]^{-1/2} \ .$$

$$\rho_i \approx \frac{4}{3}\sqrt{r\, r_e} \ . \tag{18}$$

- Estimation of power emitted for a pulsar with $\alpha = 90^\circ$ at an altitude of $r = 0.002\, r_L$ (a presumed emission height for core component in normal pulsars) and for a field line with $r_e \sim 1000\, r_L$, we get $\rho_i \approx 1.9\, r_L$, and $\rho_m \approx r_L/2$.

$$P \propto \frac{1}{\rho^2}$$

Thus more than an order of magnitude difference between ρ_i and $\rho_{\rm m}$.

- Emission from the field lines close to magnetic axis can be explained only if rotation is taken into account

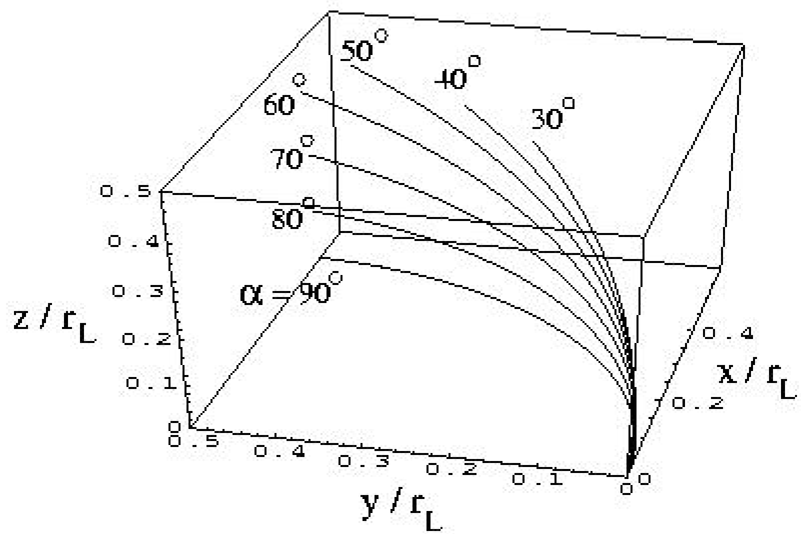


Figure 4: The trajectories of particle moving along magnetic axis at different values of inclination angle α. The azimuthal distortion due to the magnetic field sweep back is neglected in this plot. Used pulsar period $P = 1$ sec and $\phi = 0^\circ$. The curve labled with $\alpha = 90^\circ$ lies in the xy-plane, and all other lie above it.

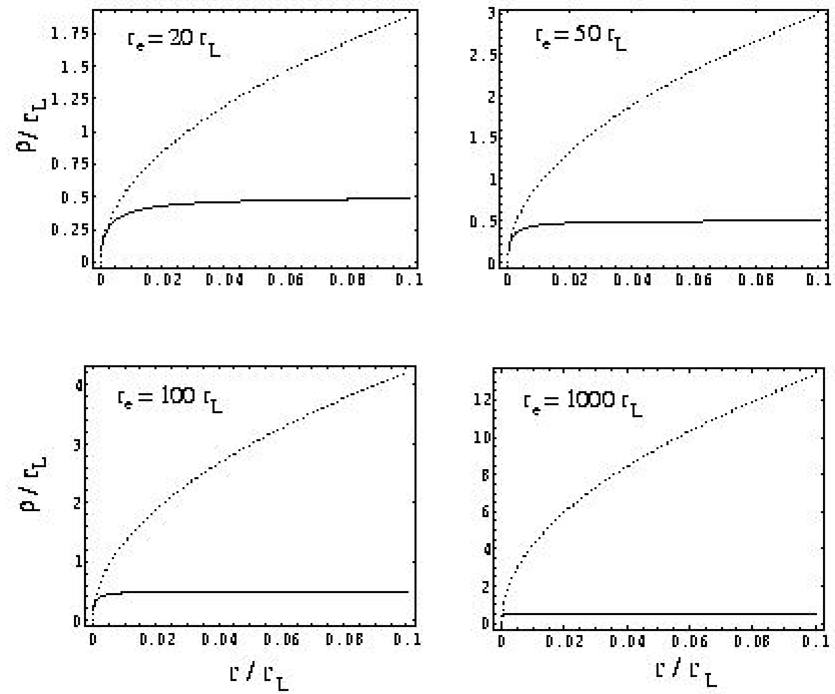


Figure 5: The curvature radius of particle trajectory at different r_e of the field lines which lie in the meridional plane ($\phi = 0^\circ$). In each panel, the dotted curve represent the stationary case while the continuous curve represent the rotating. Chosen $\alpha = 90^\circ$ and $P = 1$ sec.

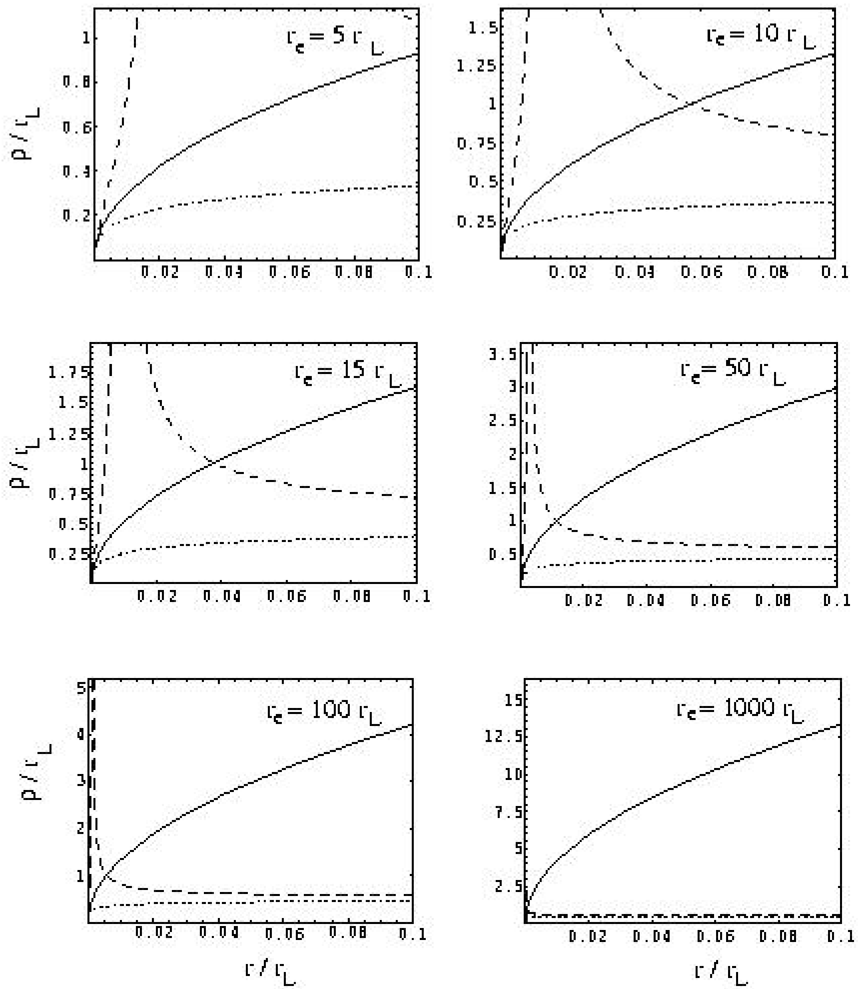


Figure 6: The curvature radius of particle trajectory at different r_e of the field lines which lie in the leading ($\phi = -90^\circ$) and trailing sides ($\phi = 90^\circ$). In each panel, the continuous curve represent the stationary case ($\phi = \pm 90^\circ$), while in the rotating case the dashed line curve represent the trailing ($\phi = -90^\circ$) side and the dotted one the leading side ($\phi = 90^\circ$) . Chosen $\alpha = 90^\circ$ and $P = 1$ sec.

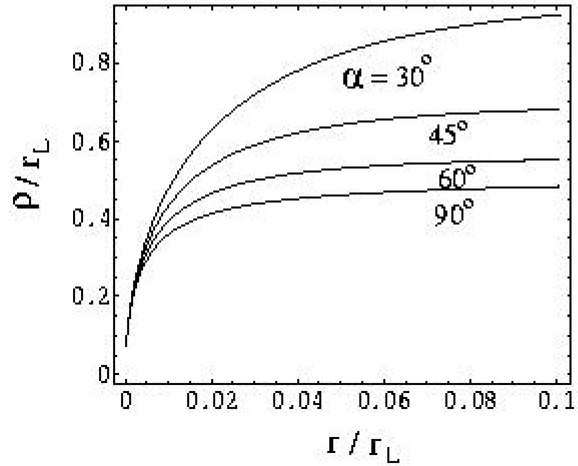


Figure 7: The curvature radius vs r/r_L at different angles of inclination α for particles following the field line with $r_e = 15$. Chosen $\phi = 0°$ and $P = 1$ sec.

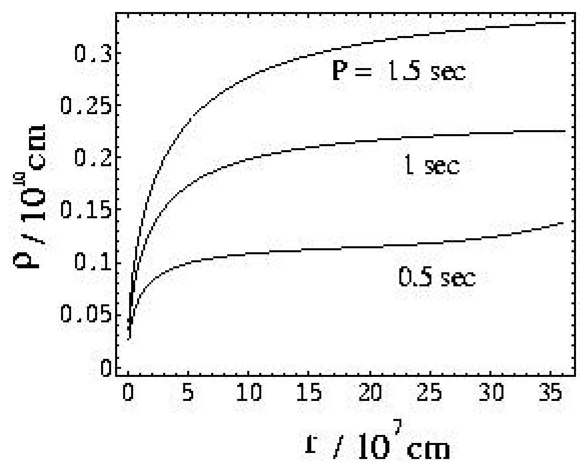


Figure 8: The curvature radius vs r/r_L at different pulsar rotation periods P based on Eq. (17). Chosen $\alpha = 90°$; $\phi = 0°$ and $r_e = 15$.

CORE EMISSION HEIGHT OF PSR B2111+46

- Devised a new method to estimate the core emission heights from the intensity and polarization data.
- Analyzed the multifrequency data for Intensity and polarization
- Radius to Frequency mapping for the core heights found
- The α and β have been found out from the χ^2 fitting of BCW curve (Bleaskeiwicz, Cordes, Wasserman :1991). Absolute emission heights of the conal components estimated.

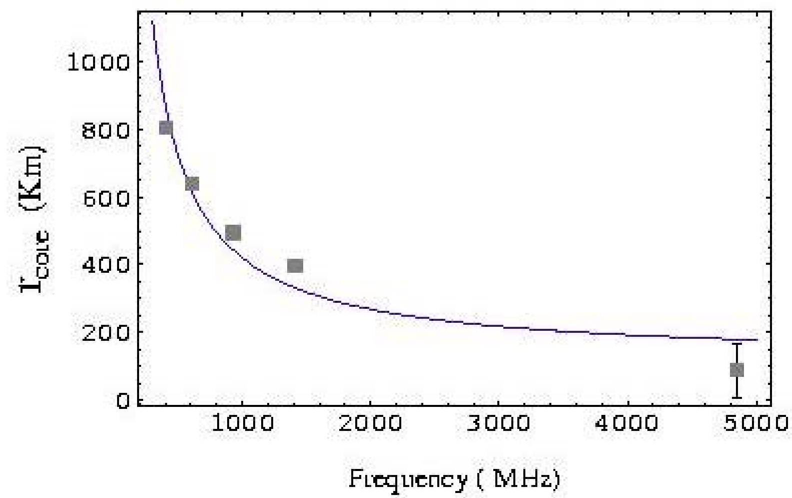


Figure 9: The frequency ν in MHz on X-axis and the core emission height r_{core} on the Y-Axis. the points are fitted with a curve of the form $r = \frac{C}{\nu} + d$ The fit parameters are C=3.008 $\times$ 10^5., d= 117.8

Table 1: The parameters in relevance to the emission geometry of pulsar B2111+46 for the core components

Frequency (MHz)	ϕ_C (deg)	ϕ_{in} (deg)	$\delta\phi_{core}$ (deg)	r_{em} (Km)	ρ/r_L	γ	S/S_{lof}
4850	-0.23±0.15	0.23±0.17	0.11±0.06	91±23	0.14 ±0.03	762±58	0.430±0.054
1408	-0.99±0.04	0.99±0.04	0.50±0.02	395±06	0.58 ±0.01	762±04	0.206±0.002
925	-1.24±0.07	1.24±0.17	0.62±0.04	492±18	0.73 ±0.03	822±09	0.185±0.003
610	-1.60±0.05	1.60±0.05	0.80±0.02	635±07	0.94 ±0.01	769±03	0.163±0.001
408	-2.02±0.10	2.02±0.20	1.01±0.06	804±22	1.19± 0.03	729±06	0.145±0.002
333	-2.46±0.10	2.46±0.20	1.23±0.06	978±20	1.45± 0.03	690±05	0.131±0.001

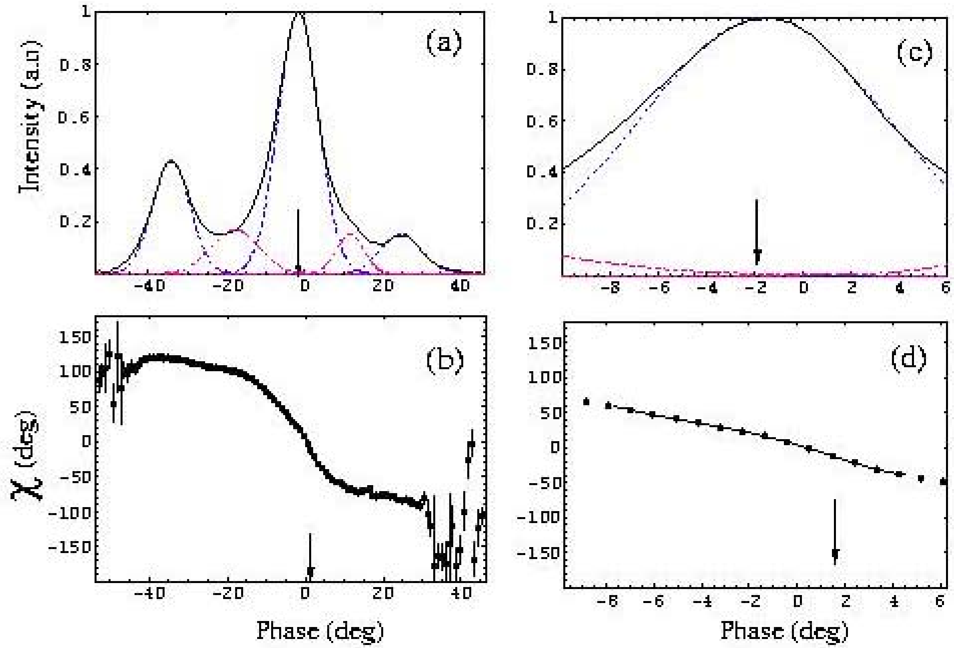


Figure 10: (a)The Intensity profile fitted with the gaussians (dotted lines) corresponding to the components at the respective frequency 610 MHz. The arrow points to the core peak phase. (b) The corresponding **PPA** profile is fitted with a polynomial. The arrow points to the phase of the point of inflexion. Figs. (c) and (d) are the zoomed out versions of figs. (a) and (b) respectively.

CONCLUSION

- We have developed a model for the charged particle acceleration in pulsar magnetospheres including the effects of rotation, valid for the radio emission region.
- The pulse profile of PSR B2111+46 is analyzed. Our model is found to match with observational results.
- Based on the 3D model, a detailed simulation of Intensity and polarization polarization profile is being currently worked on.