ELECTRIC DIPOLE MOMENT OF THE ELECTRON AND ITS COSMOLOGICAL IMPLICATIONS

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OUTLINE OF THE TALK

- INTRODUCTION
- ATOMIC EDM IN THE NON-RELATIVISTIC CASE
- ATOMIC EDM IN THE RELATIVISTIC CASE
- ATOMIC EDM EXPERIMENTS
- PRELIMINARY RESULTS
- COSMOLOGICAL IMPLICATIONS OF EDM
- FUTURE WORK

INTRODUCTION

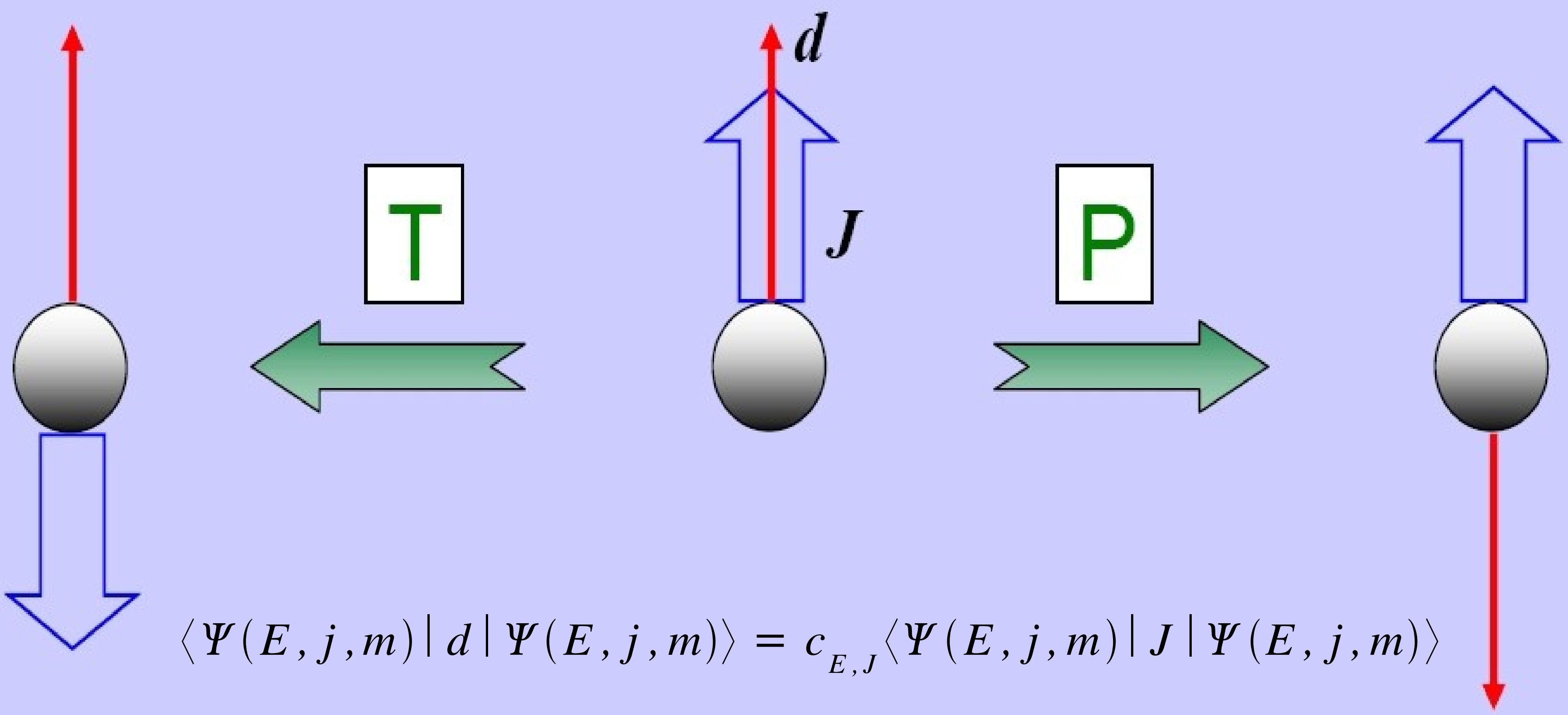
- The Electric Dipole Moment for a pair of equal and opposite charges is defined as the magnitude of the charge times the distance between them.

- The electron being a point charge can have intrinsic EDM, as in reality, it is surrounded by a virtual charge cloud.

- For an electron, the EDM is given by , $\vec{d} = d_e \vec{\sigma}$ and it aligns either parallel or anti-parallel to the total angular momentum.

- The atomic EDM can arise due to any/all of the following:

  - Electron EDM
  - Nuclear EDM
  - P and T violating Electron – Nucleus Interactions



$$\langle \Psi(E,j,m) | d | \Psi(E,j,m) \rangle = c_{E,J} \langle \Psi(E,j,m) | J | \Psi(E,j,m) \rangle$$

$$\rightarrow d = 0$$

Permanent EDM of a particle contradicts both P - & T - invariance.

ATOMIC EDM IN THE NON-RELATIVISTIC CASE

The interaction between the electron spin and internal electric field exerted by the nucleus and the other electrons gives,

$$-d_e \vec{\sigma} \cdot \vec{E}_i$$

The total atomic Hamiltonian is then,

$$H = \frac{p^2}{2m} + V(r) - d_e \vec{\sigma} \cdot \vec{E}_i$$

Using perturbation theory, we can write,

$$H = H_O + \lambda H^{/}$$

where,

$$H_O = \frac{p^2}{2m} + V(r); \qquad \lambda H^{/} = -d_e \vec{\sigma} \cdot \vec{E}_i; \qquad H_O |\Psi_\alpha^O\rangle = E_O |\Psi_\alpha^O\rangle$$

$$H \xrightarrow{\Pi} \Pi H \Pi^{-1} = \frac{p^2}{2m} + V(r) + d_e \vec{\sigma} \cdot \vec{E}_i$$

$\Rightarrow$ **H is not invariant under parity. i.e., [H, Π] ≠ 0**

$$H \xrightarrow{T} T H T^{-1} = \frac{p^2}{2m} + V(r) + d_e \vec{\sigma} \cdot \vec{E}_i$$

$\Rightarrow$ **H is not invariant under time reversal. i.e., [H, T] ≠ 0**

Hence a net electric dipole moment of an atom could exist if the electron were to have an EDM.

When there is an external electric field, induced electric dipole moment arises.

$$e\vec{r}$$

Total electric dipole moment of an atom is then given by,

$$\vec{D} = d_e\vec{\sigma} + e\vec{r}$$

The expectation value of atomic EDM is,

$$\langle D \rangle = \langle \Psi | D | \Psi \rangle$$

Using perturbation theory,

$$|\Psi_\alpha\rangle = |\Psi_\alpha^O\rangle + \lambda|\Psi_\alpha^1\rangle + \lambda^2|\Psi_\alpha^2\rangle + \cdots$$

As λ is small, λ^2 term can be neglected.

Assume, the applied field is in the ***z*** direction. $d_e \vec{\sigma}_z$ term is even under parity, where as, $e\vec{z}$ term is odd under parity. The wavefunctions should be chosen accordingly.

If $|\Psi_\alpha^O\rangle$ and $|\Psi_\alpha^1\rangle$ are of opposite parity, then the non-vanishing terms of the Dipole moment are:

$$\langle D\rangle = \underbrace{\langle \Psi_\alpha^O | d_e \vec{\sigma}_z | \Psi_\alpha^O\rangle}_{D^O} + \underbrace{\lambda\langle \Psi_\alpha^O | e\vec{z} | \Psi_\alpha^1\rangle + \lambda\langle \Psi_\alpha^1 | e\vec{z} | \Psi_\alpha^0\rangle}_{D^1}$$

From the Time-independent Non-degenerate perturbation theory, we have,

$$|\Psi_\alpha^1\rangle = \sum_{I\neq\alpha} |\Psi_I^O\rangle \frac{\langle \Psi_I^O | H' | \Psi_\alpha^O\rangle}{E_\alpha^O - E_I^O}$$

$$H' = -\vec{\sigma}\cdot\vec{E}_i$$

After little algebra we can show that, $[\,\vec{\sigma}\cdot\vec{\nabla}\,,\,H_O\,] = e\,H^{\prime}$

$$|\,\Psi^{1}_{\alpha}\,\rangle = \frac{1}{e}\sum_{I\neq\alpha}|\,\Psi^{O}_{I}\rangle\,\frac{\langle\Psi^{O}_{I}\,|\,[\,\vec{\sigma}\cdot\vec{\nabla}\,,\,H_O\,]\,|\,\Psi^{O}_{\alpha}\,\rangle}{E^{O}_{\alpha}-E^{O}_{I}}$$

. . . .

Finally we get, $\langle D^{1}\rangle = -d_e\langle\Psi^{O}_{\alpha}\,|\,\vec{\sigma}_z\,|\,\Psi^{O}_{\alpha}\rangle$

$\langle D^{O}\rangle = d_e\langle\Psi^{O}_{\alpha}\,|\,\vec{\sigma}_z\,|\,\Psi^{O}_{\alpha}\rangle$ $\quad\langle\,D\,\rangle = \langle\,D^{O}\,\rangle + \langle\,D^{1}\,\rangle$ $\quad\boxed{\langle\,D\,\rangle = 0}$

Hence, in the non-relativistic scenario, even though the electron is assumed to have a tiny EDM, when all the interactions in the atom are considered, the total atomic EDM becomes zero.

ATOMIC EDM IN THE RELATIVISTIC CASE

The total atomic Hamiltonian, including intrinsic electron EDM, is given by,

$$H = \underbrace{c\vec{\alpha}\cdot\vec{p} + \beta m c^2 + V_N(r)}_{H^O} \underbrace{- d_e \beta \vec{\sigma}\cdot\vec{E}_i}_{\lambda H^/}$$

The Expectation value of atomic EDM in presence of applied electric field,

$$\langle D \rangle = \underbrace{\langle \Psi^O_\alpha | d_e \beta \vec{\sigma}_z | \Psi^O_\alpha \rangle}_{D^O} + \underbrace{\lambda \langle \Psi^O_\alpha | e\vec{z} | \Psi^1_\alpha \rangle + \lambda \langle \Psi^1_\alpha | e\vec{z} | \Psi^0_\alpha \rangle}_{D^1}$$

After little algebra we get,

$$e H^/ = [\, \beta\vec{\sigma}\cdot\vec{\nabla} , H_O \,] - [\, \beta\vec{\sigma}\cdot\vec{\nabla} , c\vec{\alpha}\cdot\vec{p} \,]$$

$$|\Psi_\alpha^1\rangle = \frac{1}{e}\sum_{I\neq\alpha} |\Psi_I^O\rangle \frac{\langle\Psi_I^O|[\beta\vec{\sigma}\cdot\vec{\nabla}, H_O] - [\beta\vec{\sigma}\cdot\vec{\nabla}, c\vec{\alpha}\cdot\vec{p}]|\Psi_\alpha^O\rangle}{E_\alpha^O - E_I^O}$$

$$\langle D^1\rangle = d_e\langle\Psi_\alpha^O|e\vec{z}|\Psi_\alpha^1\rangle + d_e\langle\Psi_\alpha^1|e\vec{z}|\Psi_\alpha^0\rangle$$

$$\langle D^1\rangle = \langle D^2\rangle + \langle D^3\rangle$$

$$\langle D^2\rangle = \frac{d_e}{e}\sum_{I\neq\alpha}\frac{\langle\Psi_\alpha^O|e\vec{z}|\Psi_I^O\rangle\langle\Psi_I^O|[\beta\vec{\sigma}\cdot\vec{\nabla}, H_O]|\Psi_\alpha^O\rangle}{E_\alpha^O - E_I^O} + c.c.$$

$$\langle D^3\rangle = -c\frac{d_e}{e}\sum_{I\neq\alpha}\frac{\langle\Psi_\alpha^O|e\vec{z}|\Psi_I^O\rangle\langle\Psi_I^O|[\beta\vec{\sigma}\cdot\vec{\nabla}, \vec{\alpha}\cdot\vec{p}]|\Psi_\alpha^O\rangle}{E_\alpha^O - E_I^O} + c.c.$$

$$\langle D^O \rangle = d_e \langle \Psi_\alpha^O | \beta \vec{\sigma}_z | \Psi_\alpha^O \rangle \qquad \langle D^1 \rangle = \langle D^2 \rangle + \langle D^3 \rangle$$

$$\langle D^2 \rangle = -d_e \langle \Psi_\alpha^O | \beta \vec{\sigma}_z | \Psi_\alpha^O \rangle$$

$$\boxed{\langle D^3 \rangle = \frac{4 c d_e}{\hbar} \sum_{I \neq \alpha} \frac{\langle \Psi_I^O | \vec{z} | \Psi_\alpha^O \rangle \langle \Psi_\alpha^O | i \beta \gamma_5 p^2 | \Psi_I^O \rangle}{E_\alpha^O - E_I^O}}$$

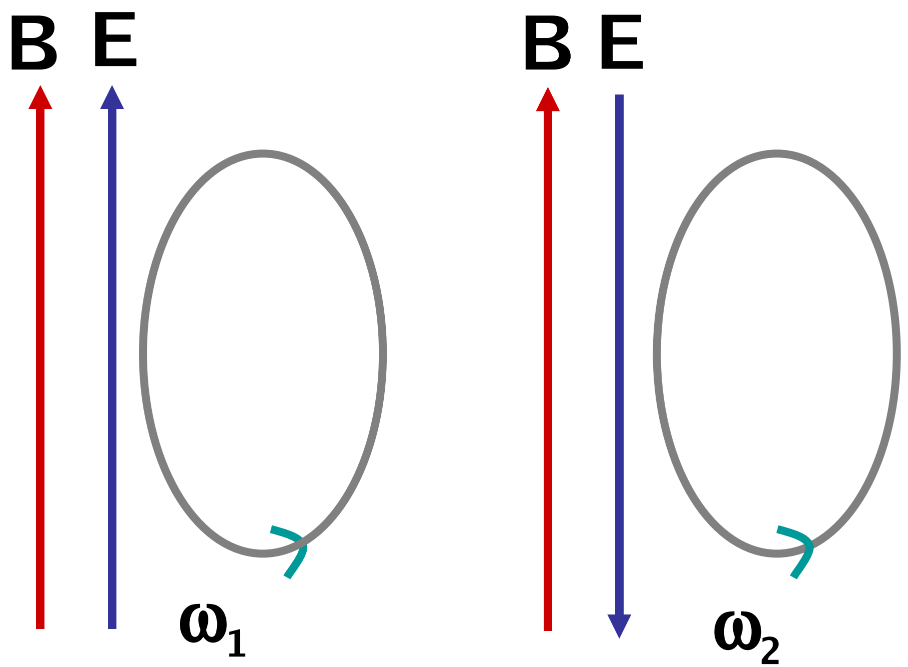
$$\langle D \rangle = \langle D^O \rangle + \langle D^2 \rangle + \langle D^3 \rangle \Rightarrow \langle D \rangle \neq 0$$

Hence, in the relativistic scenario, the total atomic EDM is non-zero.

EXPERIMENTS ON ATOMIC EDM

. . . Principle of Measurement

$$H_I = -\vec{D}\cdot\vec{E} - \vec{\mu}\cdot\vec{B}$$



$$\omega_1 = \frac{2\,\mu\cdot B + 2\,D\cdot E}{\hbar}$$

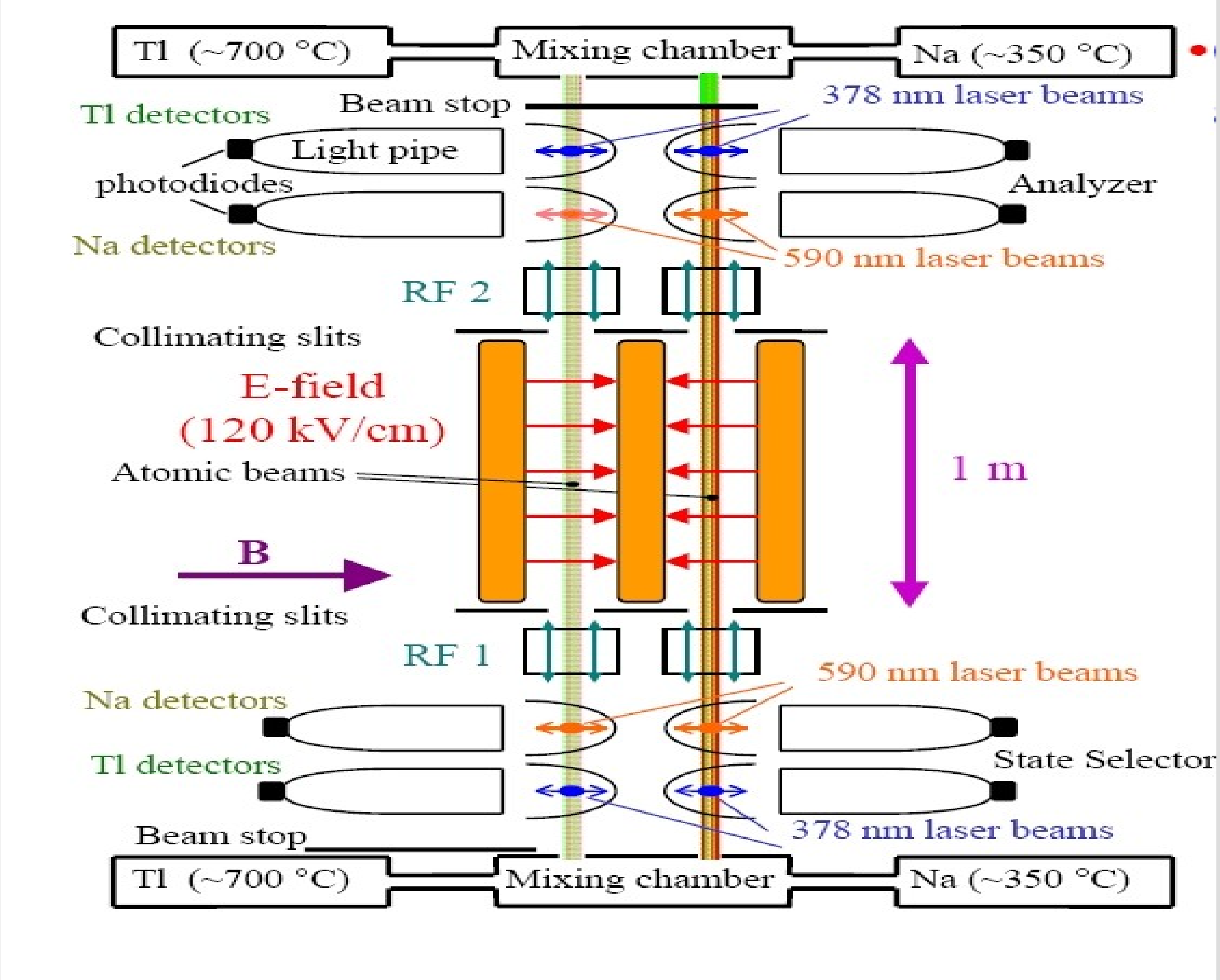
$$\omega_2 = \frac{2\,\mu\cdot B - 2\,D\cdot E}{\hbar}$$

$$\Delta\omega = \omega_1 - \omega_2 = \frac{4\,D\cdot E}{\hbar}$$

If the atomic EDM, D ~ 10^{-26} *e-cm* and E = 10^5 V/cm; $\Delta\omega \sim 10^{-5}$ Hz

The heavy paramagnetic atoms like Cesium, Thallium, Francium are normally used to calculate electron EDMs as for these atoms, the atomic EDM will actually be several hundred times larger than the EDM of a bare electron because,

$$D \;\alpha\; d_e \alpha^2 Z^3$$



Berkley Tl EDM experiment

The present best limit on electron EDM is 1.67×10^{-27} e-cm comes from Thallium Expt.

PRELIMINARY RESULTS

We have,

$$\langle D \rangle = \frac{4 d_e}{\alpha} \sum_{I \neq a} \frac{\langle \phi_a | \vec{z} | \phi_I \rangle \langle \phi_I | i \beta \gamma_5 p^2 | \phi_a \rangle}{E_a - E_I}$$

We compute the ratio of atomic EDM and the electron EDM, (D / d_e) called Enhancement factor (EF).

Dirac - Fock results of EF for **Cesium** (z = 55, [Xe] 6s)

ORBITAL	EDM	E 1	EDM * E1	ENG. DIFF	EF	TOTAL EF
2P	89.59	0.00022	0.02	199.34	0.00021	0.00021
3P	-42.49	-0.0017	0.07	40.34	0.0037	0.0039
4P	-19.11	-0.014	0.27	7.33	0.078	0.082
5P	6.49	0.24	1.56	0.79	4.06	4.14
6P	1.61	-1.76	-2.83	-0.058	98.22	102.36
7P	3.86	0.06	0.24	-0.440	-1.08	101.28
8P	-9.04	-0.011	0.10	-2.11	-0.099	101.18

Cesium

EF - Core : 04.5
EF - Virtuals : 95.2
EF - Total : 99.7

Reference :
B.P.Das
Aspects of Many-body effects...
LNC, 50, 411 (1988)

Dirac - Fock results of EF for **Thallium** (z = 81, [Hg] 6p)

ORBITAL	EDM	E 1	EDM * E1	ENG. DIFF	EF	TOTAL EF
1 S	-3553.63	-0.00006	0.23	3166.39	0.00014	0.00014
2 S	-1278.05	-0.00035	0.44	569.63	0.0016	0.0017
3 S	601.31	0.0013	0.78	138.79	0.011	0.013
4 S	300.99	0.0048	1.46	32.53	0.090	0.10
5 S	133.61	0.017	2.22	5.73	0.77	0.87
6 S	44.61	-0.84	-37.47	0.49	-151.36	-150.48
7 S	10.41	0.84	8.74	-0.10	-172.05	-322.54
8 S	-7.97	-0.33	2.63	-0.15	-33.88	-356.43
9 S	-19.14	-0.35	6.70	-0.28	-47.85	-404.28
10 S	-44.82	-0.15	6.72	-0.91	-15.11	-419.39
11 S	-103.37	-0.043	4.46	-3.93	-2.26	-421.66

Thallium

EF - Core : - 153.2
EF - Virtuals : - 267.3
EF - Total : - 420.7

Reference :
Z. W. Liu and H. P. Kelly
PRA, 45, R4210 (1992)

COSMOLOGICAL IMPLICATIONS OF EDM

EDM of elementary particle $\Rightarrow$ T - violation $\Rightarrow$ CP - violation

The observed baryon asymmetry is, $\eta = \dfrac{n_B - n_{\bar{B}}}{n_\gamma} = (2.6 - 6.2) \times 10^{-10}$

The Sakharov's conditions for the generation of Matter – Antimatter asymmetry in the Universe :

- Baryon Number violating interactions in the Early Universe

  Otherwise, it only reflects the asymmetric initial conditions.

- Both C and CP violation

  To supply a preferential arrow either for matter/antimatter.

- Departure from Thermal Equilibrium

  Otherwise, the number densities of both would be equal.

CP VIOLATION IS THE COMMON THEME

Many theories like GUTs, Left-Right symmetric model, Multi-Higgs model, SUSY, ... including Electro-Weak Model have explained the baryogenesis scenario, with different sources of CP violation:

- GUT baryogenesis:
- Electro - Weak baryogenesis:
- Lepto - baryogenesis:

- Lepton asymmetry is generated in decays of heavy Majorana neutrino, N. Evidently, N decay will not conserve leptonic charge.

- During the electro-weak stage, the lepton asymmetry is transformed into baryon asymmetry by Sphaleron processes. Sphalerons do not conserve B and L individually but only conserve (B – L) charge. Initial L would thus be redistributed in equilibrium in almost equal shares between B and L.

In Baryo-through-Leptogenesis model, Ref: Markus Luty, Phys. Rev. D, 45, 455 (1992)

$$n_B \;\alpha\; N_{B-L} \;\alpha\; \epsilon$$

CPV parameter

$$\epsilon \;\alpha\; m_{D3}^2 \frac{M_{N1}}{M_{N2,3}} \sin\delta$$

Phase of CPV

In Left-Right Symmetric Model, Ref: Jose F. Nieves, Phys. Rev. D 33, 3324 (1986)

$$L_{CC} = \frac{g}{\sqrt{(2)}} \sum_a \left(W_L^{\mu} \overline{l_{aL}} \gamma^{\mu} \nu_{aL} + W_R^{\mu} \overline{l_{aR}} \gamma_{\mu} N_{aR} \right) + H.C.$$

$$d_e \;\alpha\; \sin 2\xi$$

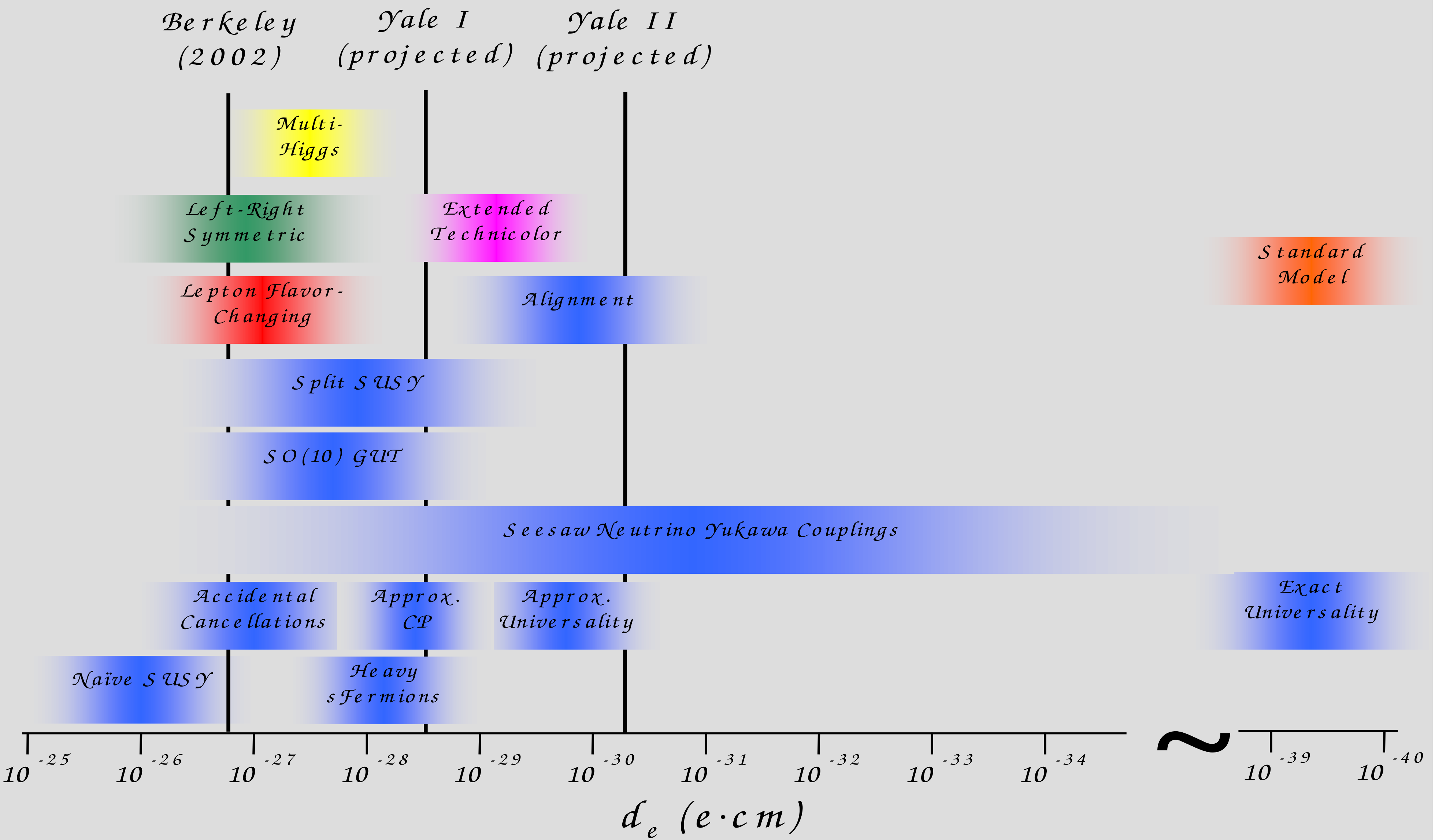
Mixing angle

$$\begin{vmatrix} W_L \\ W_R \end{vmatrix} = \begin{vmatrix} \cos\xi & \sin\xi \\ -e^{i\alpha}\sin\xi & e^{i\alpha}\cos\xi \end{vmatrix} \begin{vmatrix} W_1 \\ W_2 \end{vmatrix}$$

Phase of CPV

. . . Search for Physics beyond the Standard Model

- Despite over 45 years of experiments a permanent EDM still has not been found.
- This fact gives a stringent constraint on how CP symmetry is violated (or not violated) in the Universe.
- Standard Model of Particle Physics predicts EDMs at least 8 orders of magnitude smaller than the present experimental limits, unlike, many Non-Standard Models which predict EDM values much close to the present experimental limits.
- A non-zero EDM is, thus, a background free signal of CP violation beyond the SM. Several Atomic and Molecular EDM experiments are underway.
- Many experiments (eg. BaBar at SLAC) are looking for additional CP violation beyond the Standard Model.


Berkeley (2002)
Yale I (projected)
Yale II (projected)
Multi-Higgs
Left-Right Symmetric
Extended Technicolor
Standard Model
Lepton Flavor-Changing
Alignment
Split SUSY
SO(10) GUT
Seesaw Neutrino Yukawa Couplings
Accidental Cancellations
Approx. CP
Approx. Universality
Exact Universality
Naïve SUSY
Heavy sFermions
10^-25
10^-26
10^-27
10^-28
10^-29
10^-30
10^-31
10^-32
10^-33
10^-34
10^-39
10^-40
d_e (e·cm)

FUTURE WORK

- Extend the electron EDM calculation from Relativistic Mean Field Approximation (Dirac – Fock Method) to Relativistic Correlated Theory (Coupled Cluster Method), including some refinements, to get better limits on electron EDM.

- Apply the knowledge of electron EDM and CP violation in the realm of micro-physics to constrain Matter - Antimatter Asymmetry in the Universe, perhaps, using Left - Right Symmetric Model.

- Also put some limits on the mass and mixing angles in the Neutrino Sector which is another crucial area to know the new physics beyond the Standard Model of Particle Physics.