Polarization Shearing Interferometer (PSI) Based Wavefront Sensor for Adaptive Optics Application

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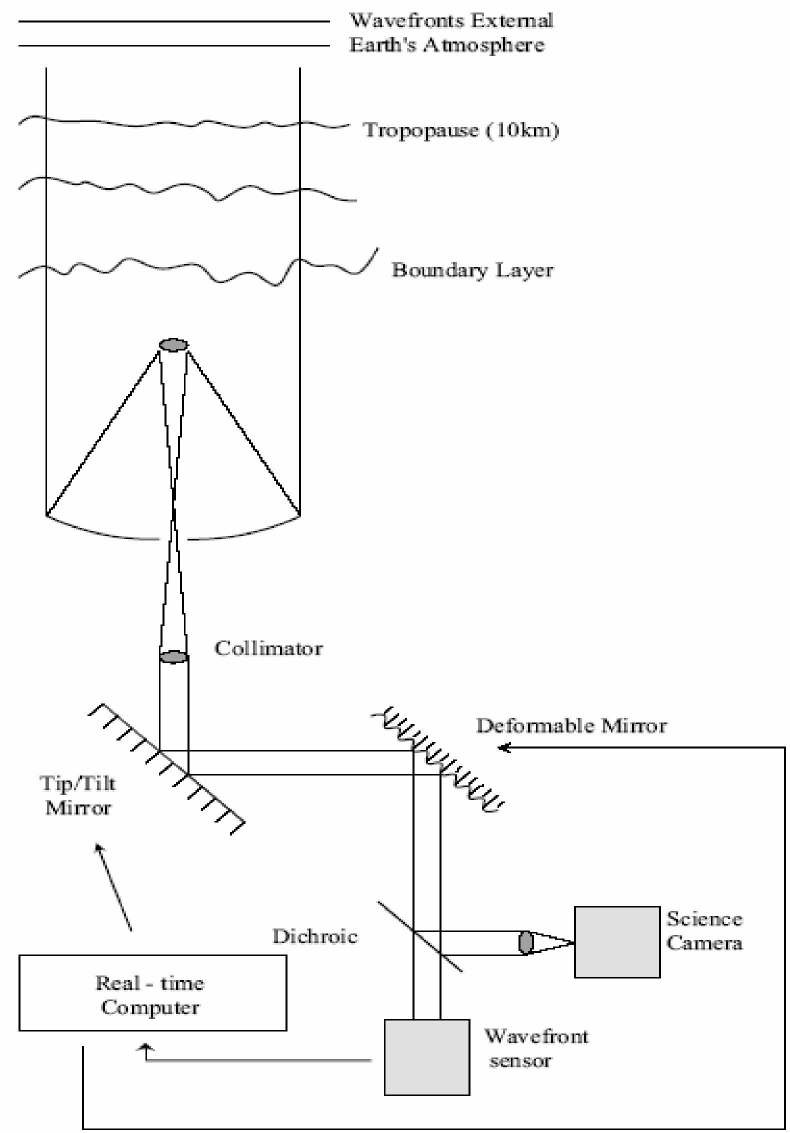
Adaptive Optics

A Closed loop Optical system
to compensate
atmospheric turbulence



Schematic of an AO system

The Main components of an AO system are:

1. Tip/Tilt Mirror
2. Deformable Mirror
3. Wavefront Sensor
4. Wavefront controller
5. Detector

Wavefront Sensor:

A wavefront sensor measures the characteristics of the Wavefront, distorted by the atmospheric turbulence at the pupil plane.

Requirements of a wavefront sensor:

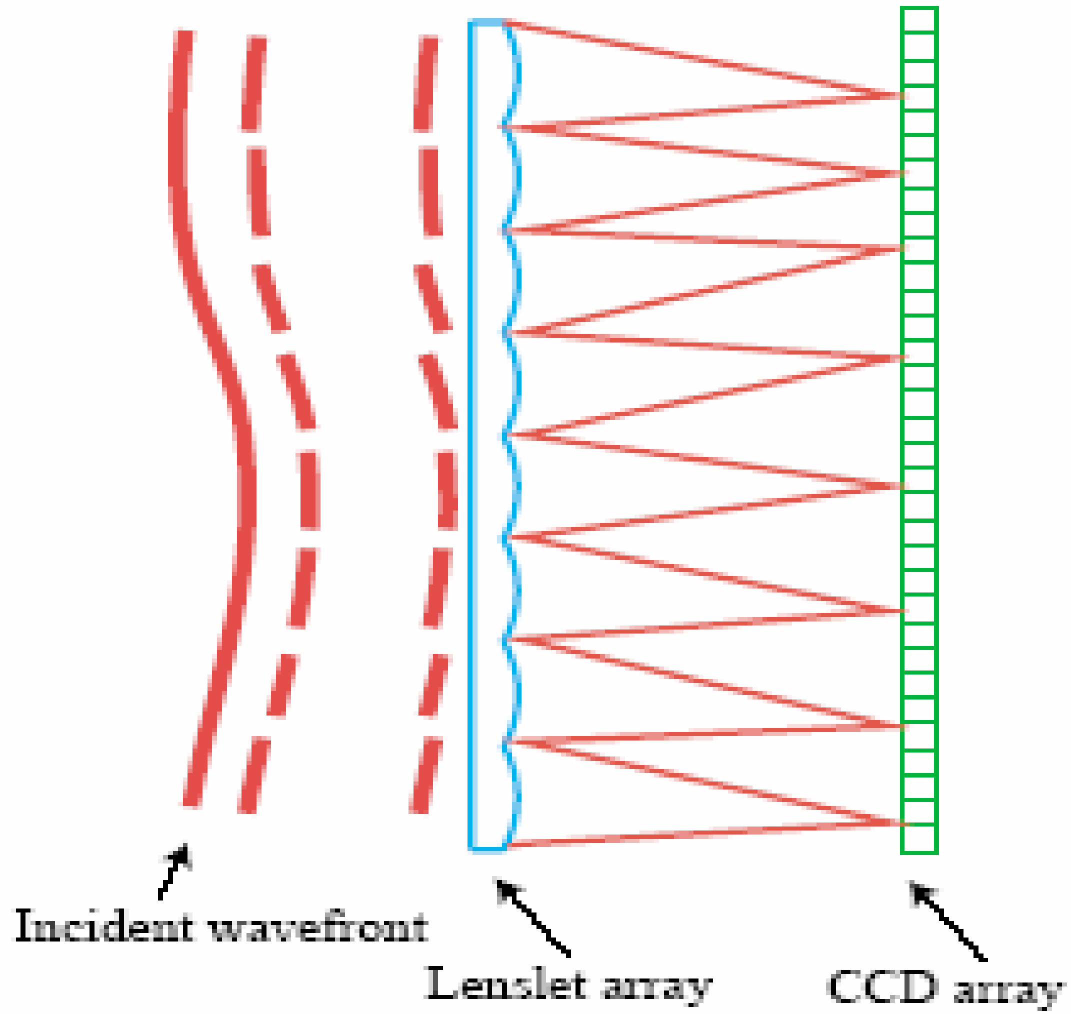
To sense the Wavefront with enough spatial resolution and enough speed for real time compensation for atmospheric seeing.

Three types of wavefront sensors

1. Shack Hartmann Wavefront Sensor
2. Shearing Interferometer
3. Curvature sensor

1. Shack Hartmann (SH) Sensor:

The most commonly used sensor in AO system



Basic Principle of SH sensor

$$g_x = \frac{\sum_{l,m} x_{l,m} I_{l,m}}{\sum_{l,m} I_{l,m}},$$

X - Centroid

$$g_y = \frac{\sum_{l,m} y_{l,m} I_{l,m}}{\sum_{l,m} I_{l,m}},$$

Y - Centroid

Displacements of image centroids in two orthogonal directions *x, y* are proportional to the average wave-front slopes in *x* , *y* over the sub-apertures. Thus, a Shack-Hartmann (S-H) WFS *measures the wave-front slopes.*

x slope $C_x = \Delta_{\mathrm{x}} / \mathrm{f\,M}$
y slope $C_y = \Delta_{\mathrm{y}} / \mathrm{f\,M}$

where f is the focal length of the lenslet array and M is the magnification between the microlens plane and the pupil plane.

Zernike Polynomials form a set of orthogonal polynomials on the unit circle and are most conveniently expressed in polar coordinates ρ and θ

$$Z_{j_{even}} = \sqrt{n+1}\, R_n^m(\rho)\, \sqrt{2} \cos m\theta$$

$$Z_{j_{odd}} = \sqrt{n+1}\, R_n^m(\rho)\, \sqrt{2} \sin m\theta$$

$$Z_j = \sqrt{n+1}\, R_n^0(\rho)$$

where

$$R_n^m(\rho) = \sum_{s=0}^{n-m/2} \frac{(-1)^s (n-s)!}{S!\left(\frac{n+m}{2}-s\right)!\left(\frac{n-m}{2}-s\right)!} \rho^{n-2s}$$

The values of n and m satisfy m < n and n-m is even.

The phase function over a unit aperture can be expanded as

$$\phi (\rho,\theta) = \sum_{0}^{\infty} a_j Z_j (\rho , \theta)$$ where a_j are the Zernike co-efficients

The Derivatives of the Zernike polynomials are may be expressed as linear combination of Zernike Polynomial (Noll 1976)

Therefore

$$A'_{\mathrm{nx}} = \sum_{n=2}^{N} \varepsilon_{mn} A_n$$

$$A'_{\mathrm{ny}} = \sum_{n=2}^{N} \gamma_{mn} A_n$$

are the matrix elements.

Equating to the derivative of the Zernike Polynomial

$$C_x = \sum_j A_{nx}\, \partial Z_j / dZ_x$$

$$C_y = \sum_j A_{ny}\, \partial Z_j / dZ_y$$

Using Least squares method The zernike coefficients are evaluated.

The wavefront is reconstructed using the Zernike co-efficients.

Drawbacks:

Low spatial resolution
Errors due to large tilt results in cross talk

Polarization Shearing Interferometer

Advantages:

High Spatial Resolution

Simple set up and easy alignment

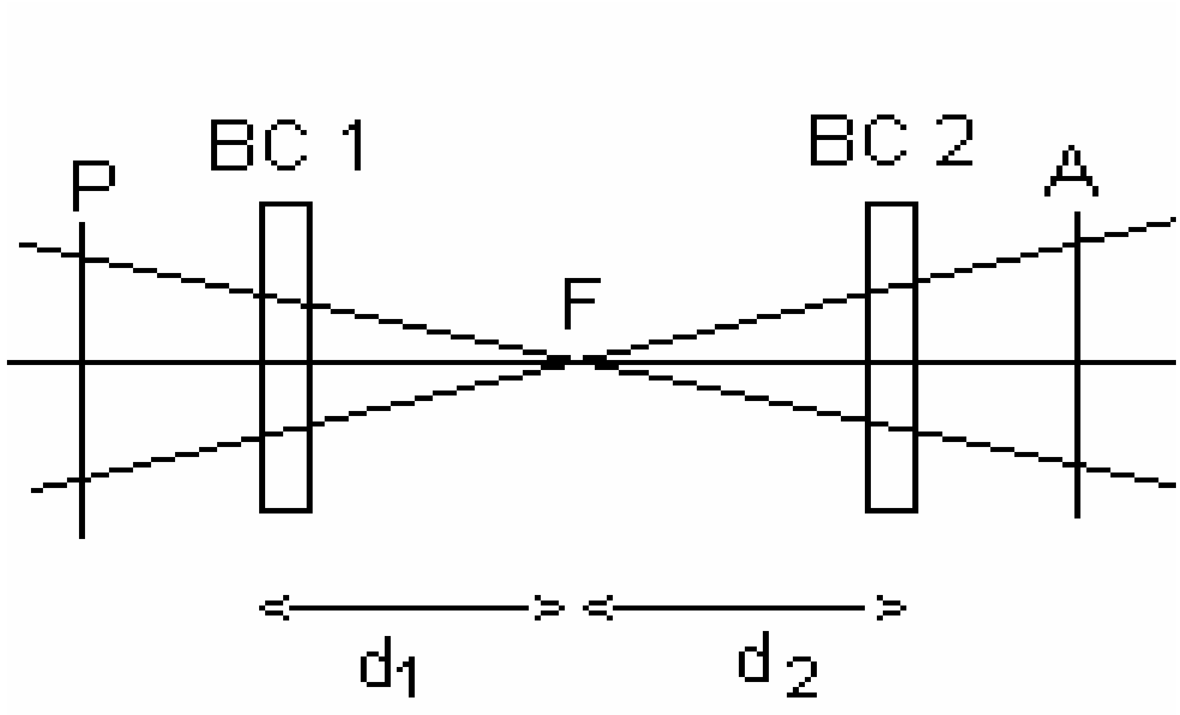
Single record

No reference required

Excellent linearity

Measurement accuracy better than 0.1 arc sec.

Theory:



Two crossed Babinet Compensators BC1 & BC2

$$\text{Shear} = 2\,(n_e - n_o)\tan\alpha\ F$$

α is the wedge angle

PSI - Device

The intensity distribution at the detector plane can be written as

$$I\ (x, y)=a\ (x, y)+b\ (x, y)\cos[2\pi f_o.r - \phi(r)]$$

Where a(x, y) & b(x, y) - Intensity modulations due to imperfections in the polarizer and analyzer and the birefringent material transmission or reflection.

f_{o}- - reference spatial frequency corresponding to the No. of Fringes

$\phi(r)$ is the local fringe phase with respect to f_o

The phase $\phi(r) = 2\pi/\lambda \quad \Delta w(x,y)$

$\Delta w\ (x, y)=w\ (x+S/2,y+T/2) - w\ (x-S/2,y-T/2),$

S & T - shears in the x and y directions respectively

For small shear approximation

$$\Delta W(x, y) = \frac{\partial W(x, y)}{\partial x} S + \frac{\partial W(x, y)}{\partial x} T$$

Phase Estimation

$$I(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 . r - \phi(r)]$$

Fourier Transform Techniques

$$I(r) = a(r) + \tfrac{1}{2} b(r)\, e^{i\phi(r) + fo.r} + \tfrac{1}{2} b(r)\, e^{-i\phi(r) + fo.r}$$

$$I(r) = a(r) + c(r)\, e^{ifo.r} + c^*(r)\, e^{-ifo.r}$$

Where $c(r) = \tfrac{1}{2} b(r)\, e^{i\phi(r)}$

Taking FT of the interferogram

$$\hat{I}(\nu) = A(\nu) + C(\nu - \nu_o) + C^*(\nu + \nu_o)$$

The required phase information is contained in $C(\nu - \nu_o)$ or $C^*(\nu + \nu_o)$

The prior knowledge of the spatial frequency facilitates the separation of either $C(\nu - \nu_o)$ or $C^*(\nu + \nu_o)$ from the other components of the spatial spectrum.

Translation of $C(\nu - \nu_o)$ or $C^*(\nu + \nu_o)$ by either ν_o or $-\nu_o$ toward the center and making zero all the other frequency components left with

$$\hat{I}(\nu) = C(\nu_o)$$

Inverse FT leads to $F^{-1}\{\hat{I}(\nu)\} = c(r) = ½\, b(r)\, e^{i\phi(r)}$

$$\phi(r) = \tan^{-1}[\mathrm{Im}\{c(r)\} / \mathrm{Re}\{c(r)\}]$$

Phase jumps larger than π is corrected by the method suggested by Takeda et.al. the Phase is estimated.

Wavefront Reconstruction:

A direct method of retrieving the wavefront is to express the Wavefront in terms of Zernike Polynomial.

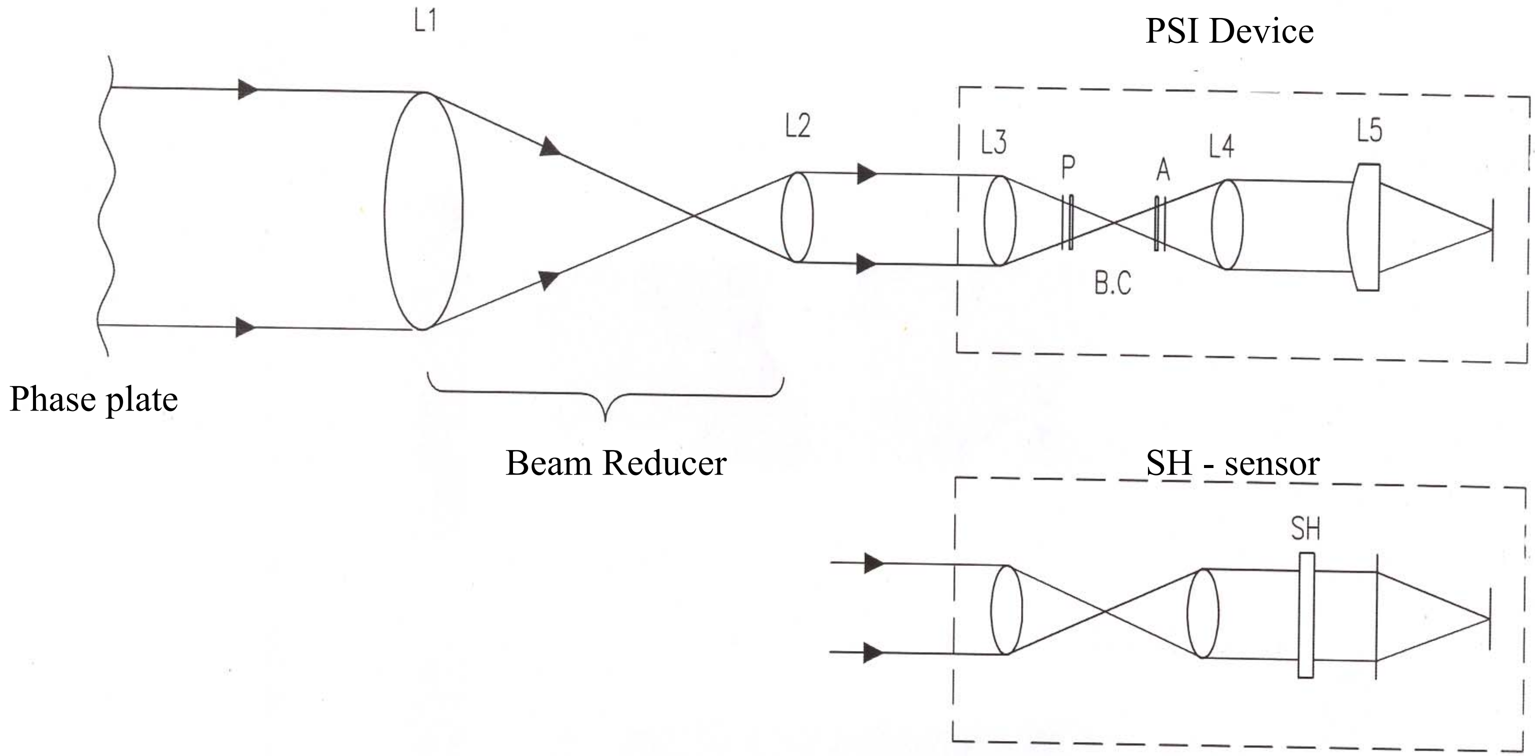
The derivative of the Zernike Polynomial in terms of Zernike Polynomials has been given by Noll.

Equating to the derivative of the Zernike Polynomial

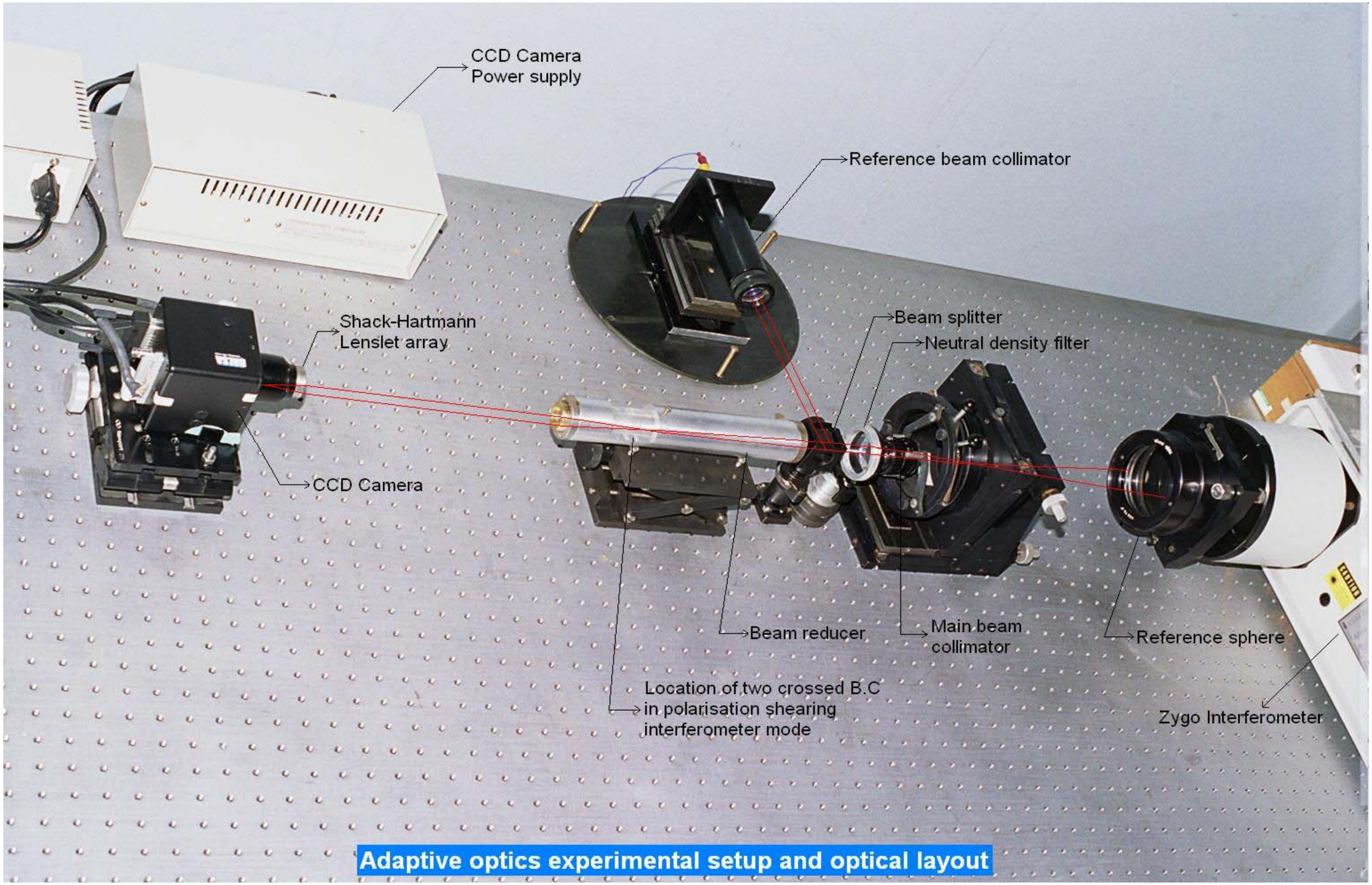
$$(\partial W/ \partial x) S = S \sum_{j} A_{nx} \, \partial Z_j / \partial Z_x$$

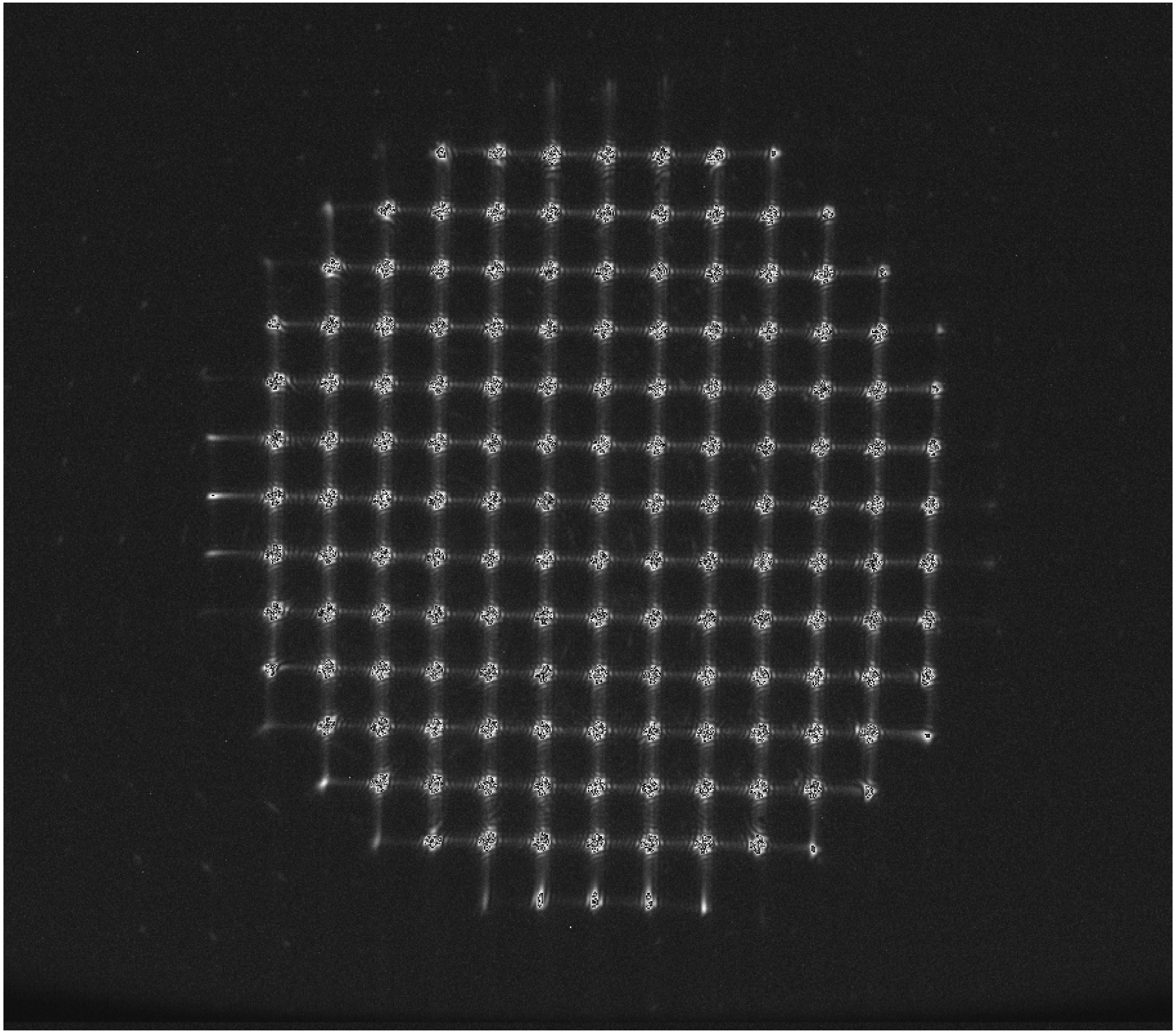
$$(\partial W/ \partial y) T = T \sum_{j} A_{ny} \, \partial Z_j / \partial Z_y$$

Using Least squares method the zernike coefficients are evaluated. The wavefront is reconstructed knowing the Zernike Co-efficients.

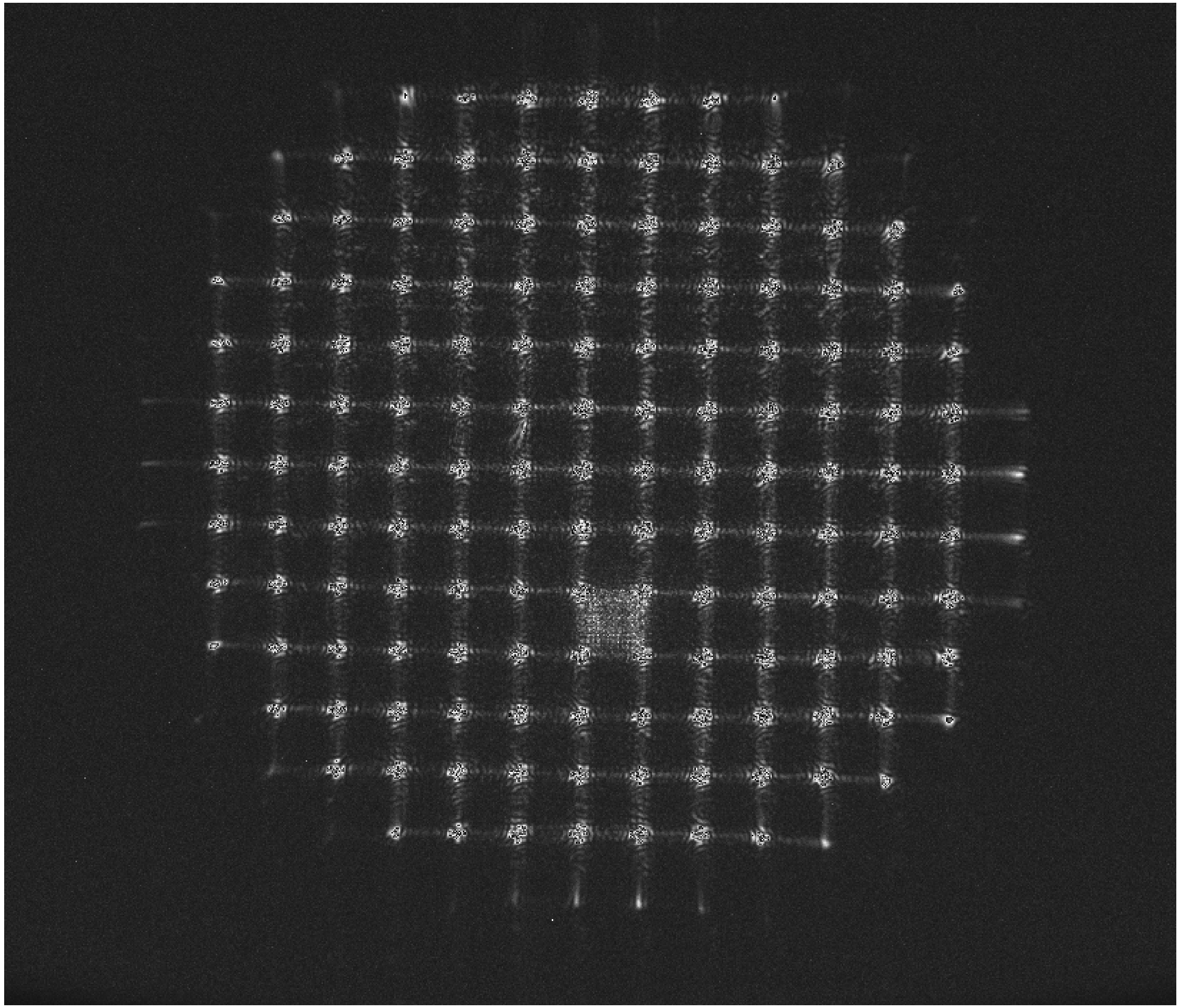


Schematic of the optical layout for PSI and SH sensors

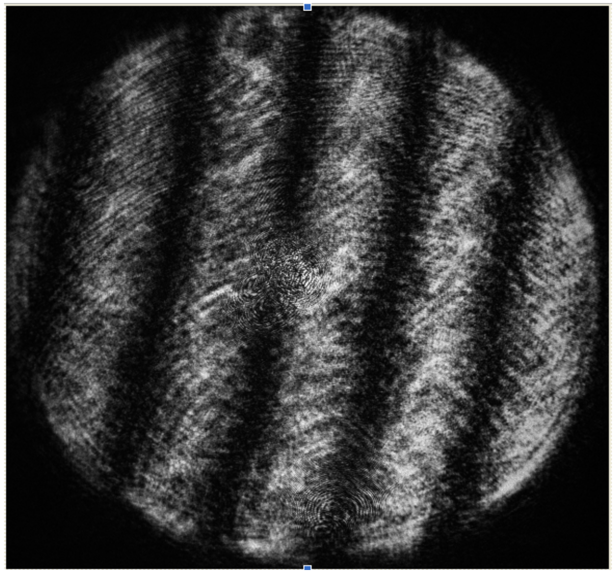

CCD Camera
Power supply
Reference beam collimator
Shack-Hartmann
Lenslet array
Beam splitter
Neutral density filter
CCD Camera
Beam reducer
Main beam
collimator
Reference sphere
Location of two crossed B.C
in polarisation shearing
interferometer mode
Zygo Interferometer
Adaptive optics experimental setup and optical layout



Reference Shack Hartmann Frame



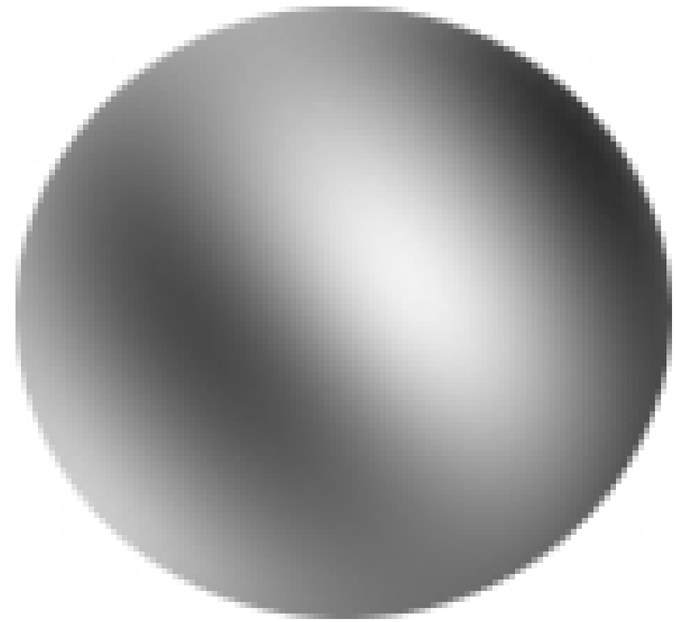
Shack Hartmann Frame for the Phase plate



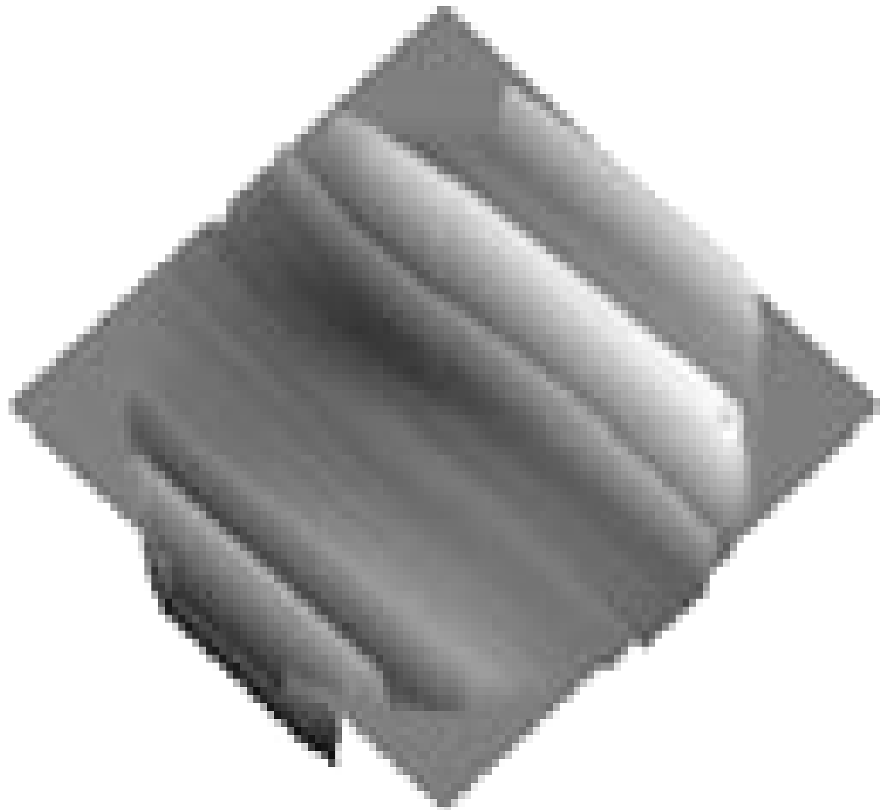
PSI Inteferogram using above setup

Simulated Interferogram

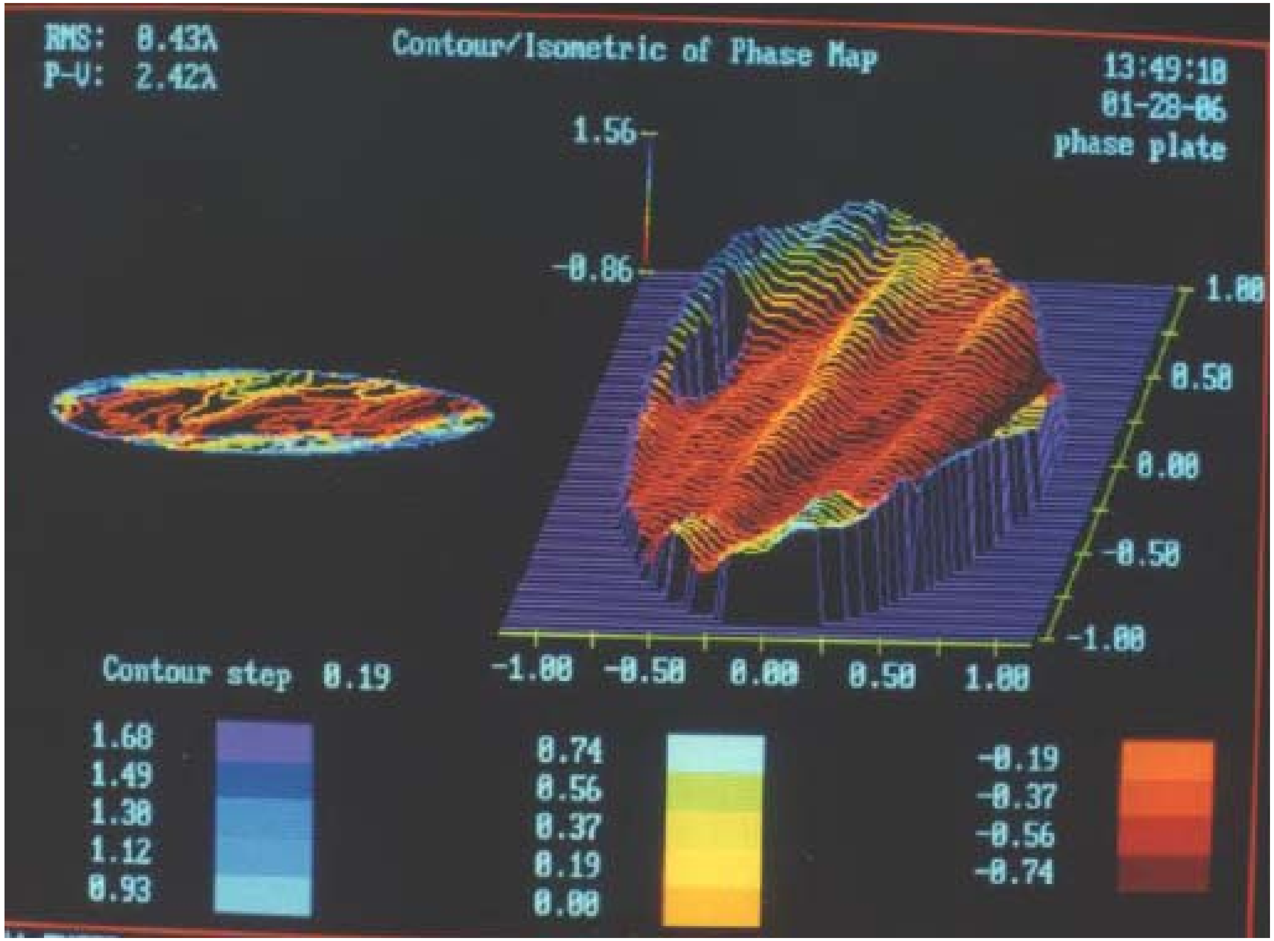
Phasemap from PSI



Phasemap from SH sensor



Surface map obtained from PSI



Surface map of the phase plate measured from Zygo

Comparative Results:

Method	RMS
PSI Method	0.48 λ
OPD method	0.43 λ
Shack Hartmann	0.38 λ

Conclusion:

The Polarization Shearing Interferometer (PSI) can be used as an effective Wavefront sensor for Adaptive Optics Application.

Thank You