Speckle Interferometry at IIA

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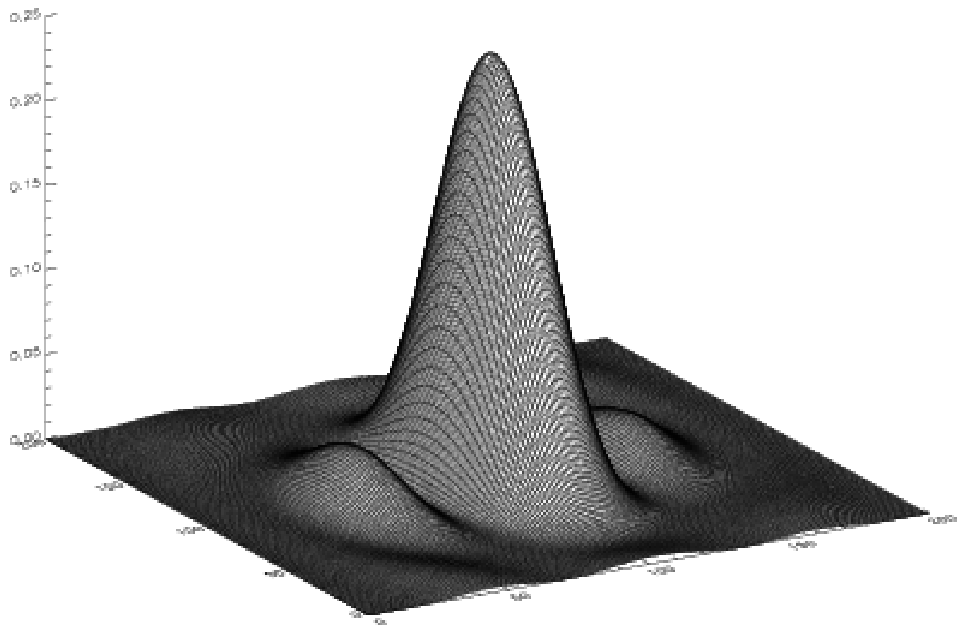
Diffraction-limit of VBT (0.05 arcseconds)



Effect of turbulence

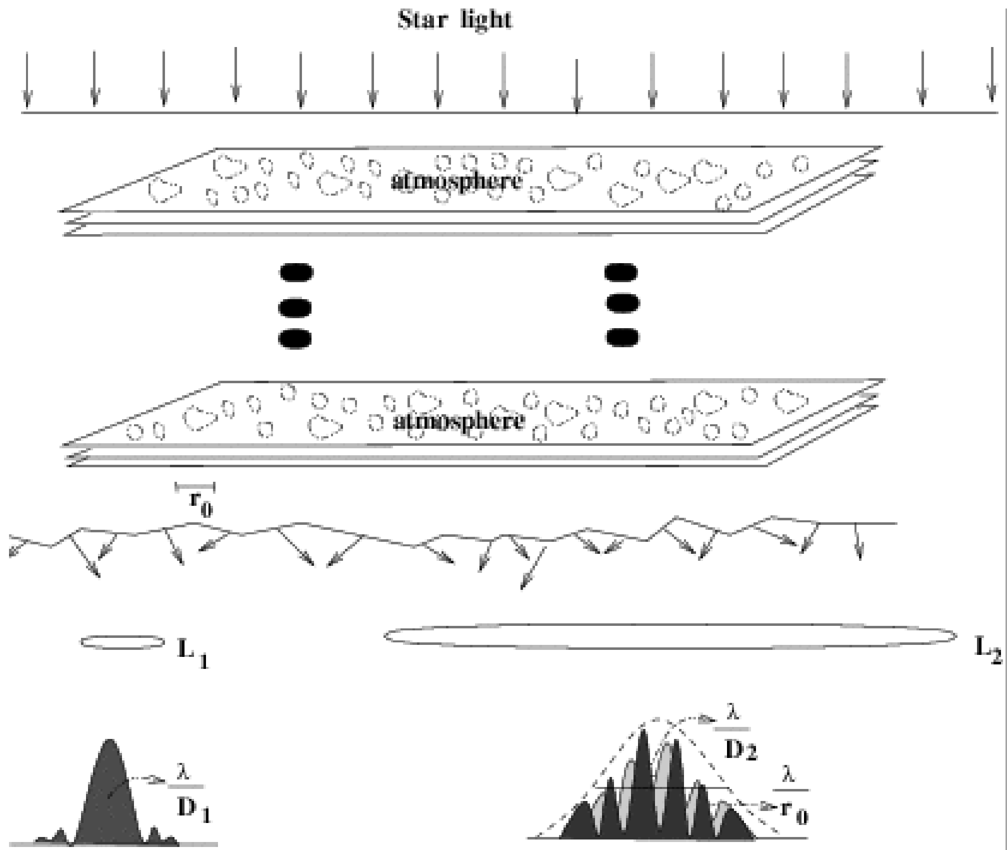
- With no turbulence FWHM is diffraction limit of telescope, $\theta \sim \lambda/D$. For a 2 meter class telescope, $\theta = 0.05''$ at $\lambda = 0.5\mu$m.

- With turbulence, image is spread out into speckles. The long exposure image resembles a blob rather than a point. The size of the image gets much larger (0.5-2$''$).

- The image motion and blurring together are referred to as seeing. It is the total effect of distortion in the path of the light via different contributing layers of the atmosphere to the detector.

- The sources of image degradation predominantly comes from the surface layer, as well as from the aero dynamical disturbances surrounding the telescope and its enclosure, such as,

- (i) thermal distortion of primary and secondary mirrors when they get heated up, (ii) dissipation of heat by the secondary, (iii) rise in temperature at the primary cell, (iv) at focal point causing temperature gradient close to detector.


Star light
atmosphere
atmosphere
r0
L1
L2
λ
D1
λ
D2
λ
r0

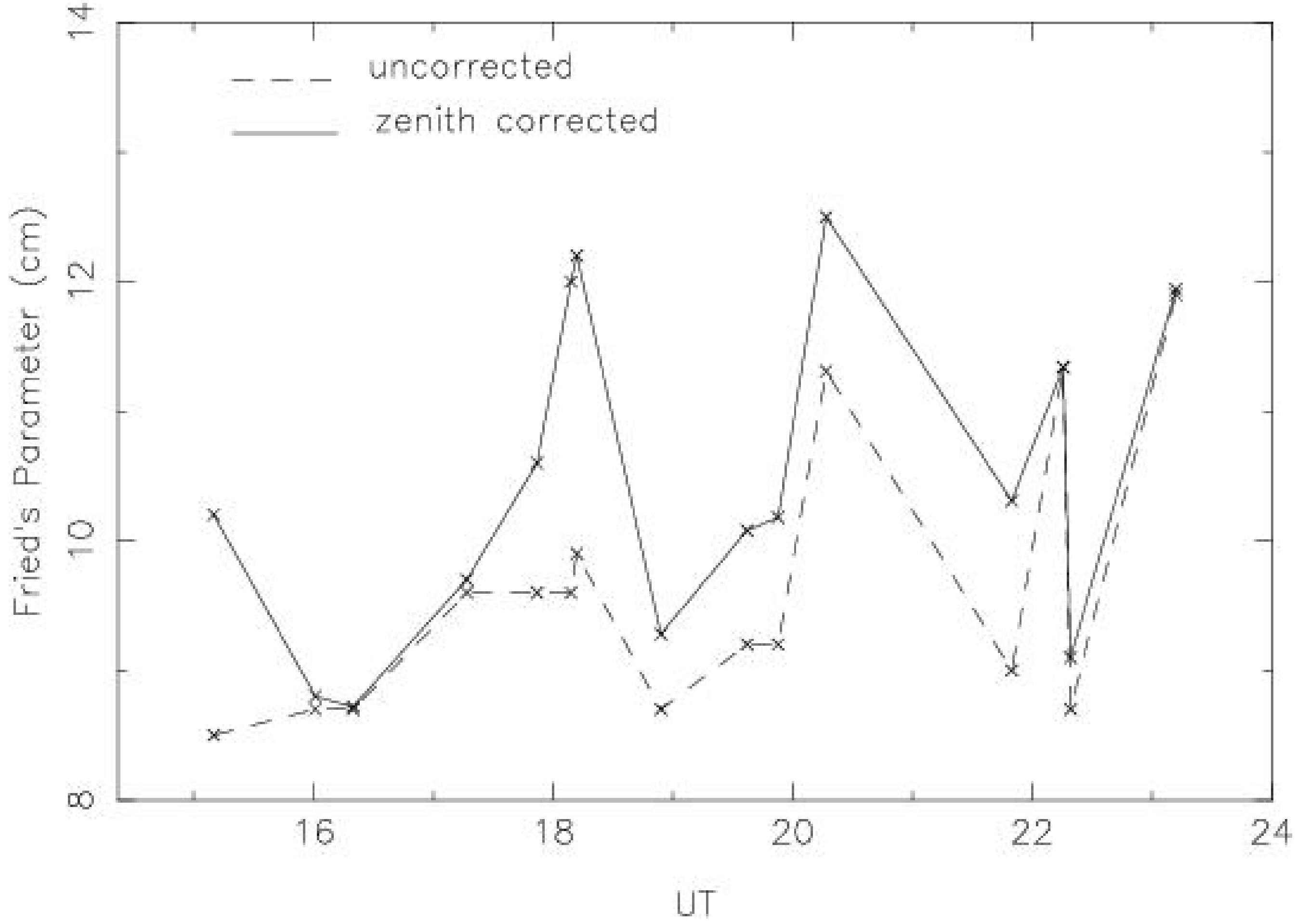
Fried's parameter (r_0)

• The Fried's parameter, r_0 is the distance over which optical phase distortion has mean square value of 1 rad^2. For a good site, the typical value of $r_0 \sim$ 20 cm.

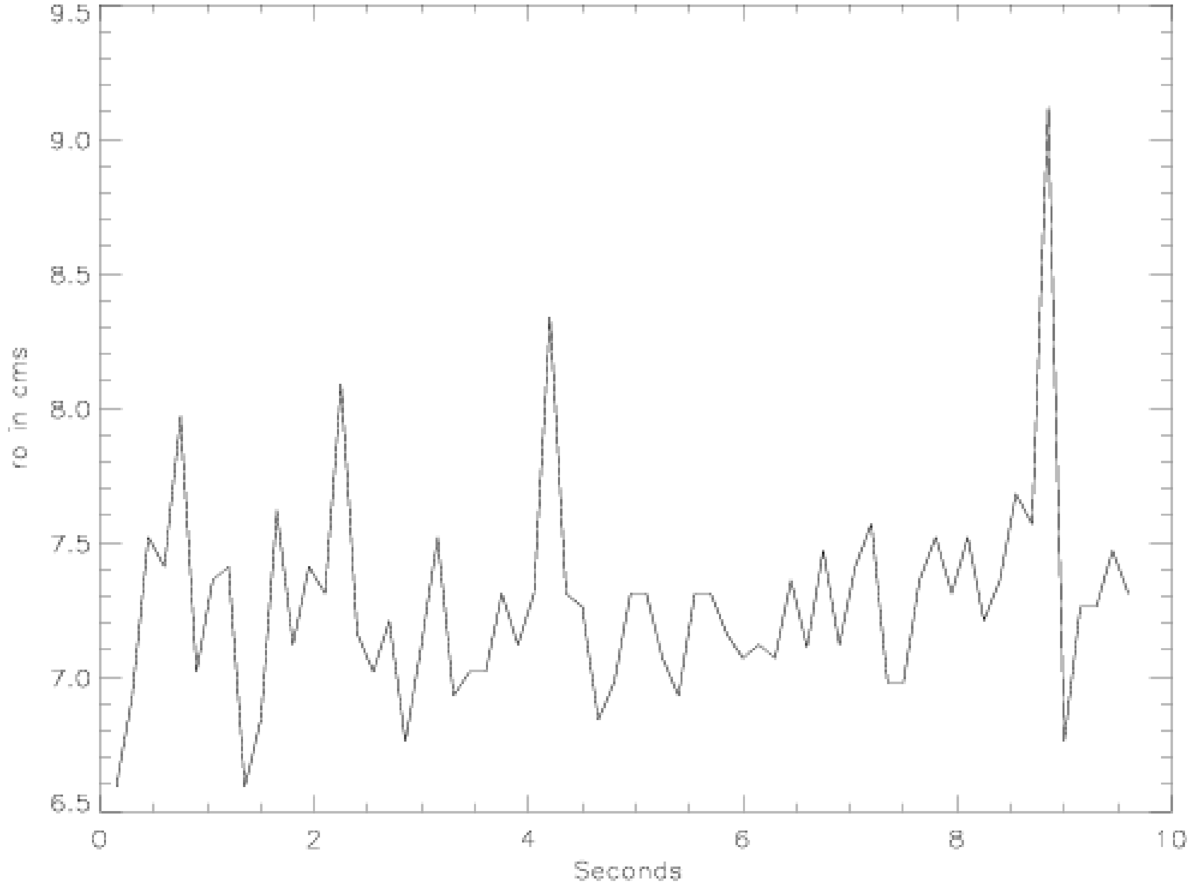
• The resolution at the image plane of a telescope is dictated by the width of the PSF of both the atmosphere and the telescope, which is of the order of $(1.22\lambda/r_0)$.

• If the telescope diameter $D \gg r_0$, the image size of a point source is $(\lambda/r_0) \gg (\lambda/D)$.

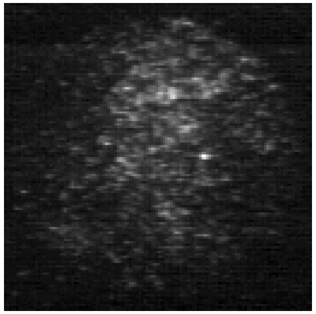
• The form of transfer function, $<\mid \hat{S}(\mathbf{u}) \mid^2>$ is obtained by calculating Wiener spectrum of the instantaneous intensity distribution from an unresolved source (Saha and Chinnappan, 1999).


uncorrected
zenith corrected
Fried's Parameter (cm)
UT
8
10
12
14
16
18
20
22
24

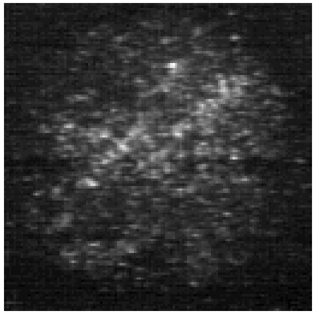
Fluctuations of r_0 (Saha et al. 2004)



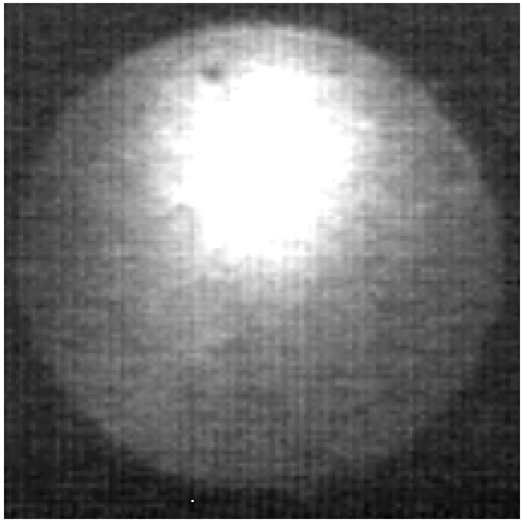
Real time speckles of HR4689 taken at 2.34 meter VBT, Kavalur



Instantaneous specklegram of HR4689 at the 2.34 meter VBT, Kavalur



Average of 128 specklegrams of HR4689



Conventional image

• The intensity distribution of the image, $I(\mathbf{x})$, is the convolution of intensity distribution of the object $O(\mathbf{x})$ and the **PSF**, $S(\mathbf{x})$, i.e.,

$$I(\mathbf{x}) = O(\mathbf{x}) \star S(\mathbf{x}),$$

where $\mathbf{x} = (x, y)$ is two dimensional (2-D) space vector, and $\star$ stands for the convolution.

• For long exposure, the PSF is defined by its ensemble average, $\langle S(\mathbf{x})\rangle$, and therefore, the average illumination, $\langle I(\mathbf{x})\rangle$ is given by,

$$\langle I(\mathbf{x})\rangle = O(\mathbf{x}) \star \langle S(\mathbf{x})\rangle,$$

where $\langle\rangle$ indicates the ensemble average.

• 2-D Fourier transform (FT) provides,

$$\left\langle \hat{I}(\mathbf{u})\right\rangle = \widehat{O}(\mathbf{u}).\left\langle \hat{S}(\mathbf{u})\right\rangle,$$

where $\mathbf{u} = (u, v)$ the 2-D spatial frequency vector, $\widehat{O}(\mathbf{u})$ is the object spectrum, $\left\langle \hat{S}(\mathbf{u})\right\rangle$ the transfer function of the atmosphere and the telescope for long exposure images.

Restoration

Restoration attempts to reconstruct or recover an image that has been degraded by using prior knowledge of degraded phenomenon

- Model re-degradation
- Apply the inverse process
- Spatial domain, Frequency domain

  g (x,y) = H [f x,y) +η(x,y)]

  Degraded image - H:degraded function

  η(x,y):noise function

  $g(x,y)=h(x,y) * f(x,y)+ \eta(x,y)$

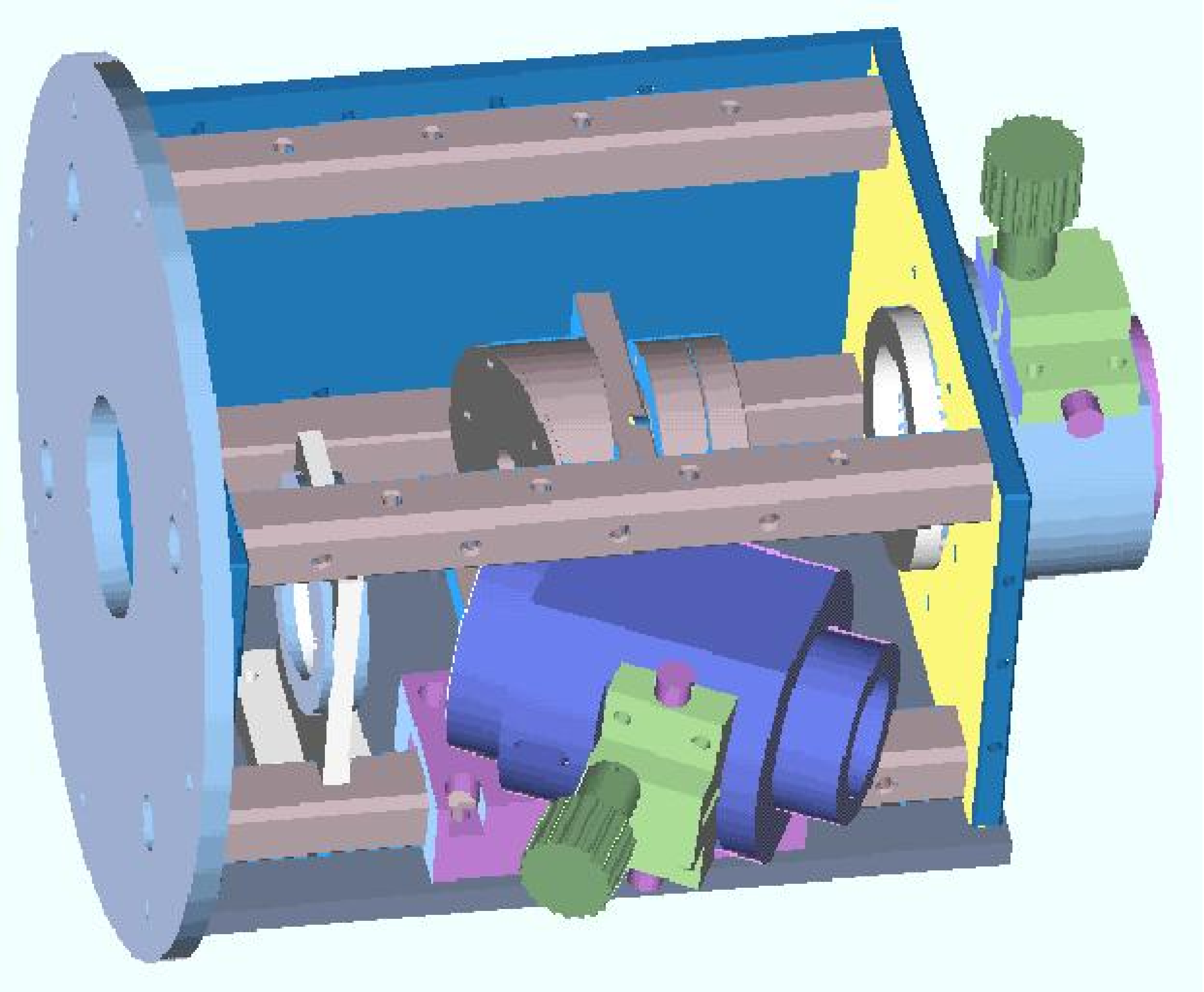
  PSF * convolution

- Convolution in spatial domain and multiplication in frequency domain constitute a fourier transform pair

- Freq. Domain representation: G(u,v)= H(u,v)+ f(u,v)+ N(u,v)

  (capital letters are fourier transforms)
- Simulate no behaviour and effects of noise is central to image restoration
- When no information is available about PSF, we can resort to blind documentation

Speckle Camera system for 2.34 m VBT, Kavalur (Saha et al. Expt. Astron., 9, 39)



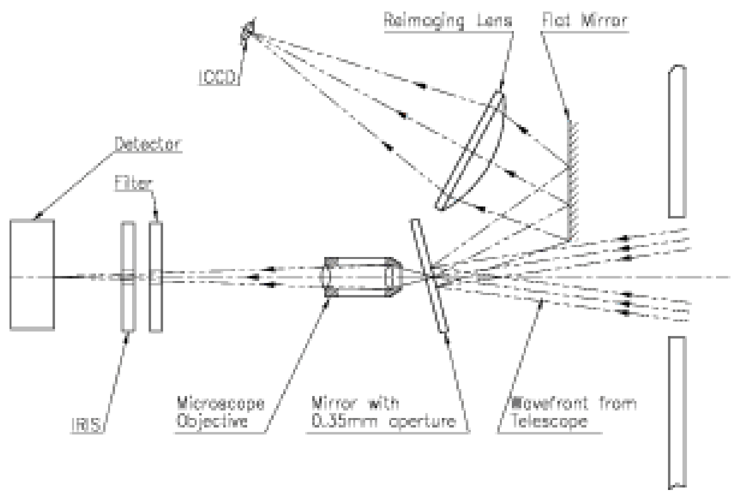


Figure 9: Optical ray diagram of the Speckle camera system (Saha et al., [32, 33] for the use at VBT, Kavalur, India.

Speckle interferometry

• For each of the short exposure instantaneous records, the intensity, $I(\mathbf{x})$ is given by,

$$I(\mathbf{x}) = O(\mathbf{x}) \star S(\mathbf{x}),$$

• In the Fourier plane,

$$\widehat{I}(\mathbf{u}) = \widehat{O}(\mathbf{u}).\widehat{S}(\mathbf{u}).$$

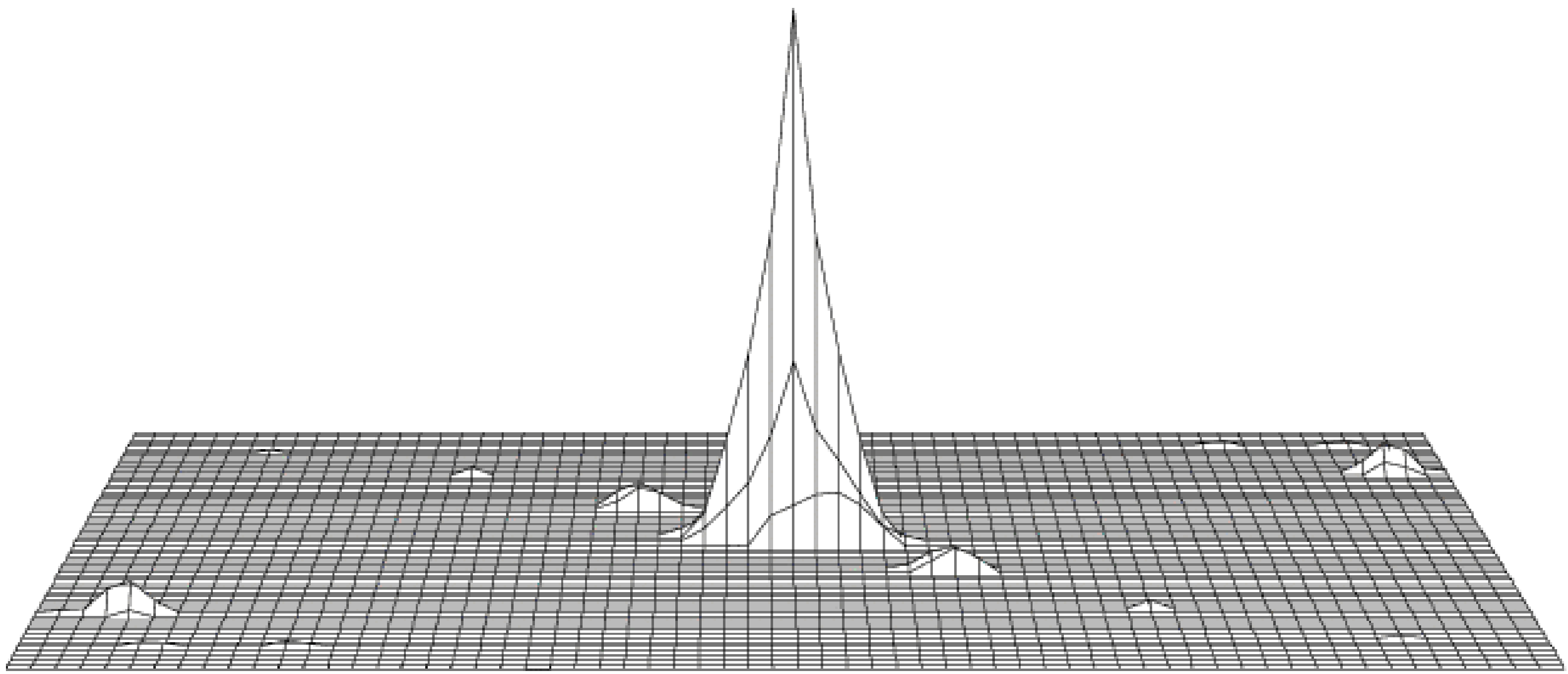
• The ensemble average of the power spectrum is,

$$\left\langle\left|\widehat{I}(\mathbf{u})\right|^2\right\rangle = \left|\widehat{O}(\mathbf{u})\right|^2 . \left\langle\left|\widehat{S}(\mathbf{u})\right|^2\right\rangle + \left\langle\left|\widehat{N}(\mathbf{u})\right|^2\right\rangle .$$

• Saha and Maitra (2001) developed an algorithm, where a Wiener parameter, w_1, is added to PSF power spectrum which helps reconstruction with a few realizations.

$$\left|\widehat{O}(\mathbf{u})\right|^2 = \frac{\left\langle\left|\widehat{I}(\mathbf{u})\right|^2\right\rangle}{\left[\left\langle\left|\widehat{S}(\mathbf{u})\right|^2\right\rangle + w_1\right]}.$$

HR4689ac:



Blind Iterative Deconvolution Technique

- In this technique, the iterative loop is repeated enforcing image-domain and Fourier-domain constraints until two images are found that produce the input image when convolved together.

$$\widetilde{O}(\mathbf{u}) = \hat{I}(\mathbf{u})\frac{\widetilde{O}_f(\mathbf{u})}{\hat{S}(\mathbf{u})}.$$

- The Wiener filter, $\widetilde{O}_f(\mathbf{u})$,

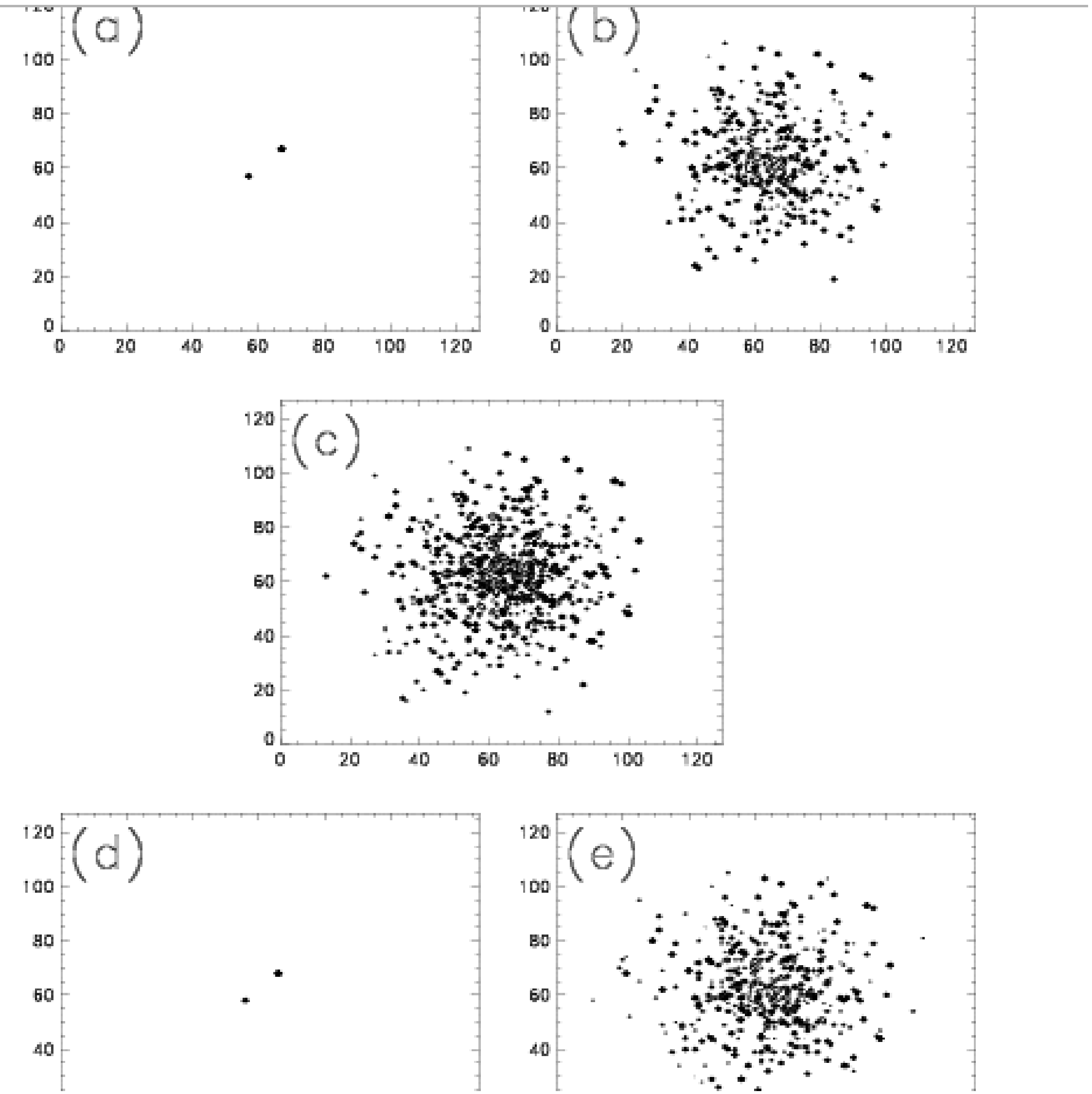
$$\widetilde{O}_f(\mathbf{u}) = \frac{\hat{S}(\mathbf{u})\hat{S}^*(\mathbf{u})}{|\hat{S}(\mathbf{u})|^2 + |\widetilde{N}(\mathbf{u})|^2}.$$

- The Wiener filtering spectrum, $\widetilde{O}(\mathbf{u})$, takes the form:

$$\widetilde{O}(\mathbf{u}) = \hat{I}(\mathbf{u})\frac{\hat{S}^*(\mathbf{u})}{\hat{S}(\mathbf{u})\hat{S}^*(\mathbf{u}) + \widetilde{N}(\mathbf{u})\widetilde{N}^*(\mathbf{u})}. \tag{1}$$

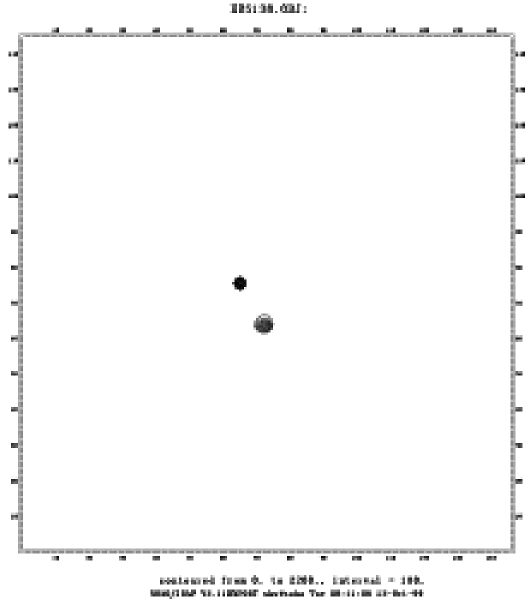
- This result $\widetilde{O}(\mathbf{u})$ is transformed to image space, the negatives in the image are set to zero, and the positives outside a prescribed domain (called object support) are set to zero.

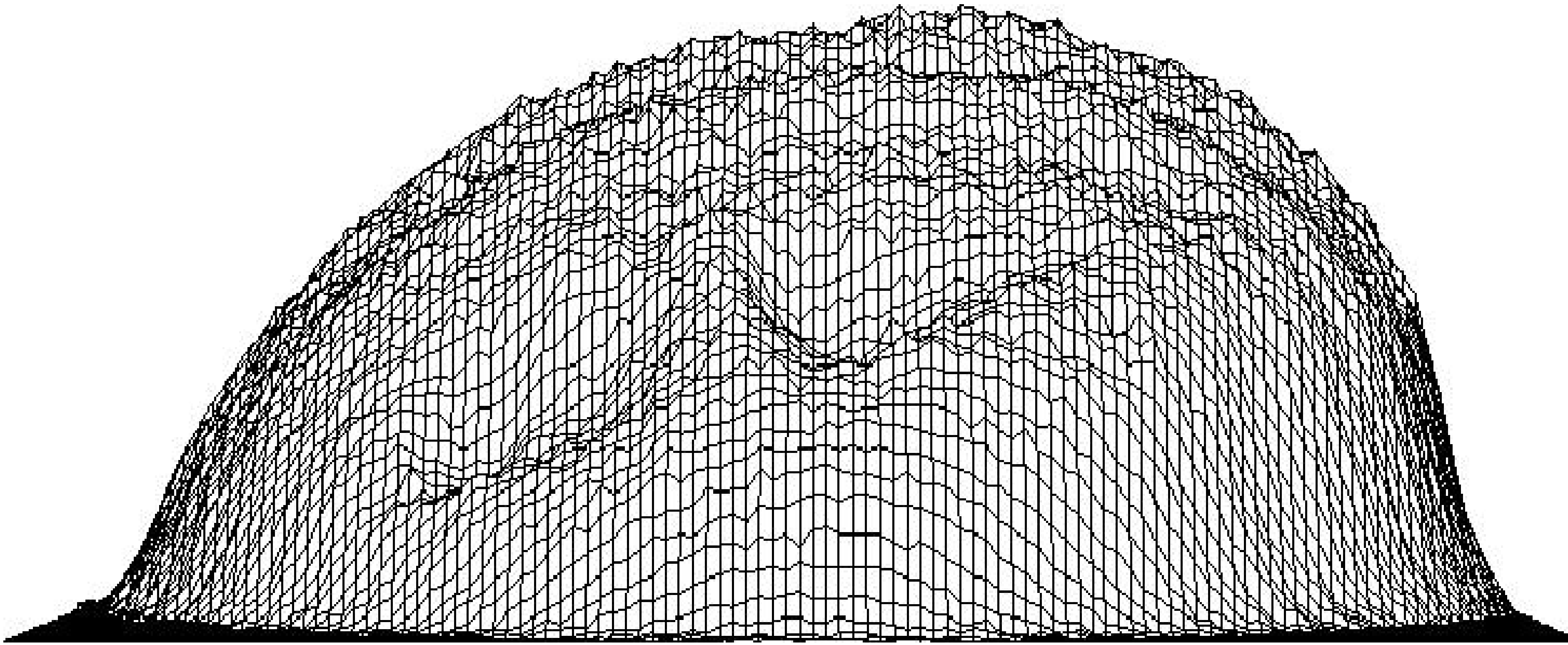
- A new estimate of the PSF is next obtained by Wiener filtering the original image $I(\mathbf{x})$ with a filter constructed from the constrained object $O(\mathbf{x})$. This completes one iteration.

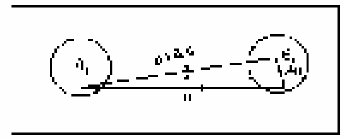


(a)
(b)
(c)
(d)
(e)
0
20
40
60
80
100
120

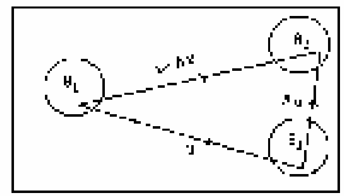
BID image reconstruction of HR5138 (Saha and Venkatakrishnan, 1997, 25, 329)







(a)



(b)

Knox-Thomson Technique

- **The KT correlation may be defined in Fourier space as products of $\hat{I}(\mathbf{u})$.**

$$\hat{I}(\mathbf{u}, \mathbf{\Delta u}) = < \hat{I}(\mathbf{u})\hat{I}^*(\mathbf{u} + \mathbf{\Delta u}) > .$$

- **This provides us the product,**

$$\left\langle \hat{I}(\mathbf{u})\hat{I}^*(\mathbf{u} + \mathbf{\Delta u})\right\rangle = \widehat{O}(\mathbf{u})\widehat{O}^*(\mathbf{u} + \mathbf{\Delta u}) \left\langle \hat{S}(\mathbf{u})\hat{S}^*(\mathbf{u} + \mathbf{\Delta u})\right\rangle .$$

-

$$|\hat{I}(\mathbf{u})||\hat{I}(\mathbf{u}+\mathbf{\Delta u})|e^{i\psi_I} = |\widehat{O}(\mathbf{u})||\widehat{O}(\mathbf{u}+\mathbf{\Delta u})|e^{i\psi_O}|\hat{S}(\mathbf{u})||\hat{S}(\mathbf{u}+\mathbf{\Delta u})|e^{i\psi_S}.$$

- **$(\mathbf{\Delta}\psi_\mathbf{S})$ turns out to be zero with large no. of frames.**

Triple correlation Method

• Triple correlation technique is a generalization of is insensitive to (i) the atmospherically induced random phase errors, (ii) the random motion of the image centroid, and (iii) the permanent phase errors introduced by telescope aberrations.

• A triple correlation is obtained by multiplying a shifted object, $I(\mathbf{x}+\mathbf{x}_1)$ is multiplied with the original object, $I(\mathbf{x})$, followed by cross correlating the product mask, $I(\mathbf{x})I(\mathbf{x}+\mathbf{x}_1)$ with the original one.

• The triple correlation of a specklegram is given by,

$$I(\mathbf{x}_1,\mathbf{x}_2) = \int_{-\infty}^{\infty} I(\mathbf{x})I(\mathbf{x}+\mathbf{x}_1)I(\mathbf{x}+\mathbf{x}_2)d\mathbf{x}, \tag{10}$$

where $\mathbf{x}_j = \mathbf{x}_{jx} + \mathbf{x}_{jy}$ are 2-D spatial coordinate vectors.

• The ensemble averaged bispectrum is expressed as,

$$\begin{aligned} \hat{I}(\mathbf{u}_1,\mathbf{u}_2) &= \left\langle \hat{I}(\mathbf{u}_1)\hat{I}^*(\mathbf{u}_1+\mathbf{u}_2)\hat{I}(\mathbf{u}_2)\right\rangle, & (11)\\ &= \widehat{O}(\mathbf{u}_1)\widehat{O}^*(\mathbf{u}_1+\mathbf{u}_2)\widehat{O}(\mathbf{u}_2) & \\ &\quad \times \left\langle \hat{S}(\mathbf{u}_1)\hat{S}^*(\mathbf{u}_1+\mathbf{u}_2)\hat{S}(\mathbf{u}_2)\right\rangle, & (12) \end{aligned}$$

where $\mathbf{u}_j = \mathbf{u}_{jx} + \mathbf{x}_{jy}$;

$$\hat{I}(\mathbf{u}_j) = \int_{-\infty}^{\infty} I(\mathbf{x})e^{-i2\pi\mathbf{u}_j.\mathbf{x}}d\mathbf{x},$$

$$\hat{I}^*(\mathbf{u}_1+\mathbf{u}_2) = \int_{-\infty}^{\infty} I(\mathbf{x})e^{-i2\pi(\mathbf{u}_1+\mathbf{u}_2).\mathbf{x}}d\mathbf{x}$$

- **The object bispectrum is given by,**

$$\begin{aligned}\hat{I}_O(\mathbf{u}_1,\mathbf{u}_2) &= \widehat{O}(\mathbf{u}_1)\widehat{O}^*(\mathbf{u}_1+\mathbf{u}_2)\widehat{O}(\mathbf{u}_2) \\ &= \frac{\langle \hat{I}(\mathbf{u}_1)\hat{I}^*(\mathbf{u}_1+\mathbf{u}_2)\hat{I}(\mathbf{u}_2)\rangle}{\langle \hat{S}(\mathbf{u}_1)\hat{S}^*(\mathbf{u}_1+\mathbf{u}_2)\hat{S}(\mathbf{u}_2)\rangle}. \qquad (11)\end{aligned}$$

- **The modulus $|\widehat{O}(\mathbf{u})|$ can be evaluated from the object bispectrum $\hat{I}_O(\mathbf{u}_1,\mathbf{u}_2)$. The argument of equation (9) provides the phase-difference and is expressed as,**

$$arg|\hat{I}(\mathbf{u}_1,\mathbf{u}_2)| = \psi(\mathbf{u}_1)+\psi(\mathbf{u}_2)-\psi(\mathbf{u}_1+\mathbf{u}_2). \qquad (12)$$

- **The object phase-spectrum is encoded in the term,**

$$e^{i\theta_O(\mathbf{u}_1,\mathbf{u}_2)}.$$

- **Let $\theta_O(\mathbf{u}_1,\mathbf{u}_2)$ be the phase of the object bispectrum;**

$$\begin{aligned}\widehat{O}(\mathbf{u}) &= |\widehat{O}(\mathbf{u})|e^{i\psi(\mathbf{u})}, \qquad (13)\\ \hat{I}_O(\mathbf{u}_1,\mathbf{u}_2) &= |\hat{I}_O(\mathbf{u}_1,\mathbf{u}_2)|e^{i\theta_O(\mathbf{u}_1,\mathbf{u}_2)}. \qquad (14)\end{aligned}$$

- **Equations (13) and (14) may be inserted into equation (11), yielding the relations,**

$$\begin{aligned}\hat{I}_O(\mathbf{u}_1,\mathbf{u}_2) &= |\widehat{O}(\mathbf{u}_1)||\widehat{O}(\mathbf{u}_2)||\widehat{O}(\mathbf{u}_1+\mathbf{u}_2)| \\ &\quad e^{i[\psi_O(\mathbf{u}_1)-\psi_O(\mathbf{u}_1+\mathbf{u}_2)+\psi_O(\mathbf{u}_2)]} \rightarrow, \qquad (15)\\ \theta_O(\mathbf{u}_1,\mathbf{u}_2) &= \psi_O(\mathbf{u}_1)-\psi_O(\mathbf{u}_1+\mathbf{u}_2)+\psi_O(\mathbf{u}_2). \qquad (16)\end{aligned}$$

Equation (16) is a recursive one for evaluating the phase of the object FT at coordinate $\mathbf{u}=\mathbf{u}_1+\mathbf{u}_2$.

• The reconstruction of the object phase-spectrum from the phase of the bispectrum is recursive in nature. If the object spectrum at $\mathbf{u}_1$ and $\mathbf{u}_2$ is known, the object phase-spectrum at $(\mathbf{u}_1 + \mathbf{u}_2)$ can be computed.

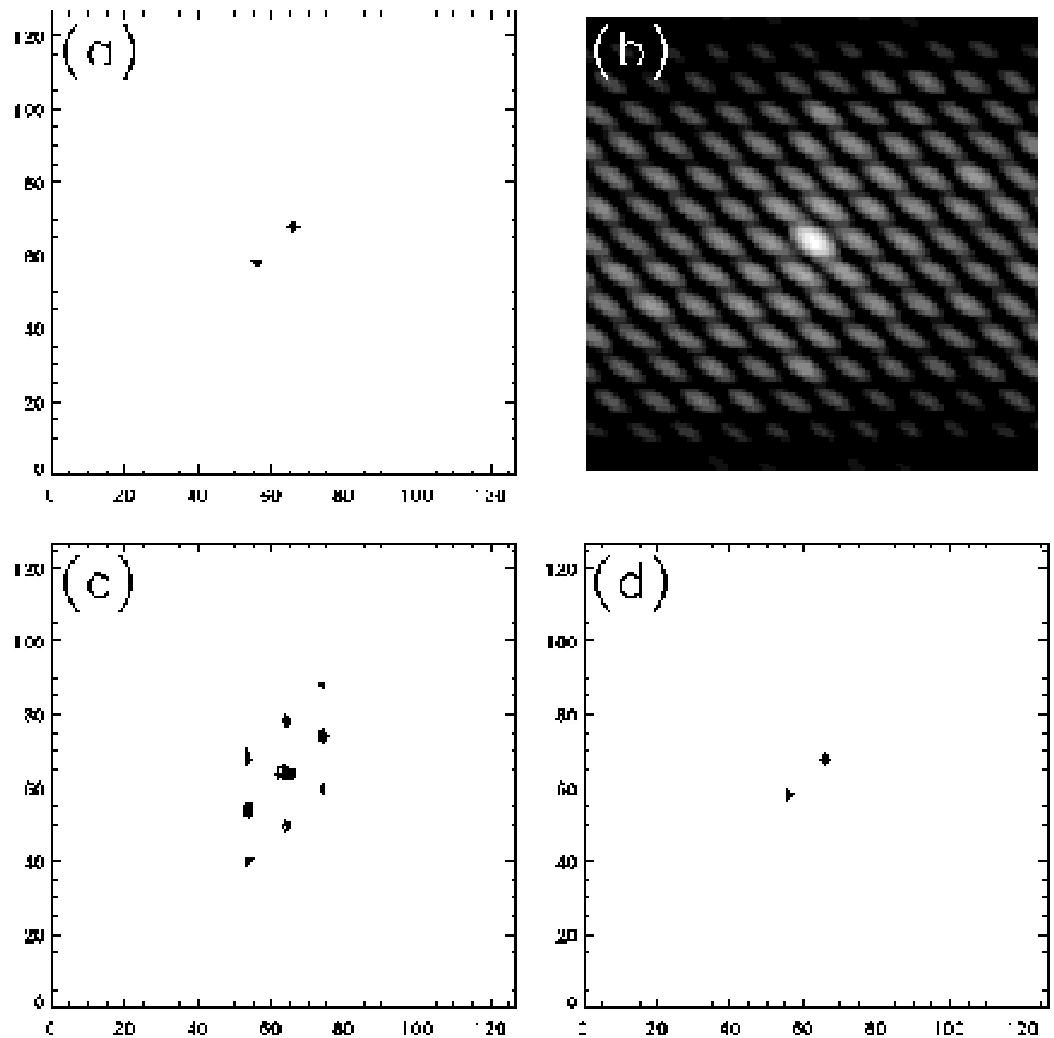
• The bispectrum phases are mod 2π, therefore, the recursive reconstruction in equation (10) may lead to π phase mismatches between the computed phase-spectrum values along different paths to the same point in frequency space. The phases from different paths to the same point cannot be averaged to reduce noise under this condition.

• A variation of the nature of computing argument of the term, $e^{i\psi_O(\mathbf{u}_1+\mathbf{u}_2)}$, is needed to obtain the object phase-spectrum and the equation (17) translates into,

$$e^{i\psi_O(\mathbf{u}_1+\mathbf{u}_2)} = e^{i[\psi_O(\mathbf{u}_1)+\psi_O(\mathbf{u}_2)-\theta_O(\mathbf{u}_1,\mathbf{u}_2)]}. \tag{18}$$

• The values obtained using the unit amplitude phasor recursive re-constructor are insensitive to the π phase ambiguities.

• Assuming $\psi(0,0) = 0, \psi(0,\pm 1) = 0$ and $\psi(\pm 1, 0) = 0$, the phases are calculated by the unitary amplitude method (Saha et al. 1999b). Since the bispectrum is a 4-D function, the calculated values are stored in 1-D array and used them later to calculate the phase by keeping track of the component frequencies.



(a)
(b)
(c)
(d)

References

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- 2. Ayers & Dainty, 1988, Opt. Lett, 13, 547.
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