Statistics of the cosmological 21-cm signal

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References

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For Wouthuysen-Field effect (Ly-alpha coupling):

Field, 1958, Proc. of the IRE
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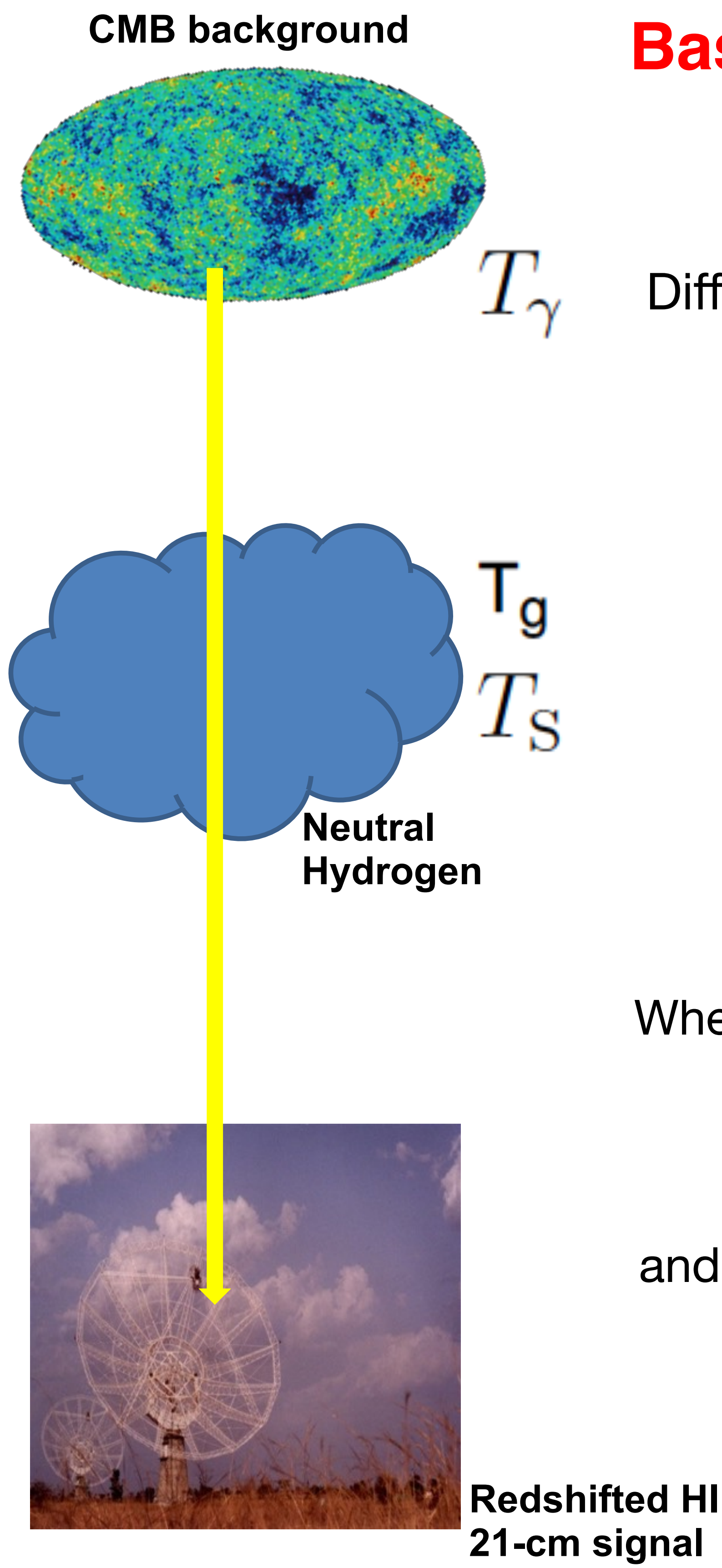


Basics of redshifted 21-cm radiations

Differential brightness temperature

$$\delta T_b(\mathbf{n}, z) = \frac{(T_s - T_\gamma)\tau}{1+z}$$

Spin temperature :

$$n_1/n_0 = 3\exp(-T_\star/T_S)$$

where

$$T_\star \equiv hc/k\lambda_{21\mathrm{cm}} = 0.0628\,\mathrm{K}$$

$$\delta T_b(\mathbf{n}, z) = \bar{T}(z) \times \eta_{\mathrm{HI}}(\mathbf{n}, z)$$

Where

$$\bar{T}(z) = 4.0\,\mathrm{mK}(1+\mathrm{z})^2 \left(\frac{\Omega_\mathrm{b}\mathrm{h}^2}{0.02}\right)\left(\frac{0.7}{\mathrm{h}}\right)\frac{\mathrm{H}_0}{\mathrm{H(z)}}$$

and

$$\eta_{\mathrm{HI}}(\mathbf{n}, z) = \frac{\rho_{\mathrm{HI}}}{\bar{\rho}_{\mathrm{H}}}\left(1 - \frac{T_\gamma}{T_s}\right)\left[1 - \frac{(1+z)}{H(z)}\frac{\partial v}{\partial r}\right]$$

First order perturbations

Hydrogen density

$$\rho_{HI}(n,z) = \bar{\rho}_{HI}(z)(1+\delta_H(n,z))$$

Spin temperature

$$T_s(n,z) = \bar{T}_s(z)(1+s(z)\delta_H(n,z))$$

Gas kinetic temperature

$$T_g(n,z) = \bar{T}_g(z)(1+g(z)\delta_H(n,z))$$

Ionization fraction

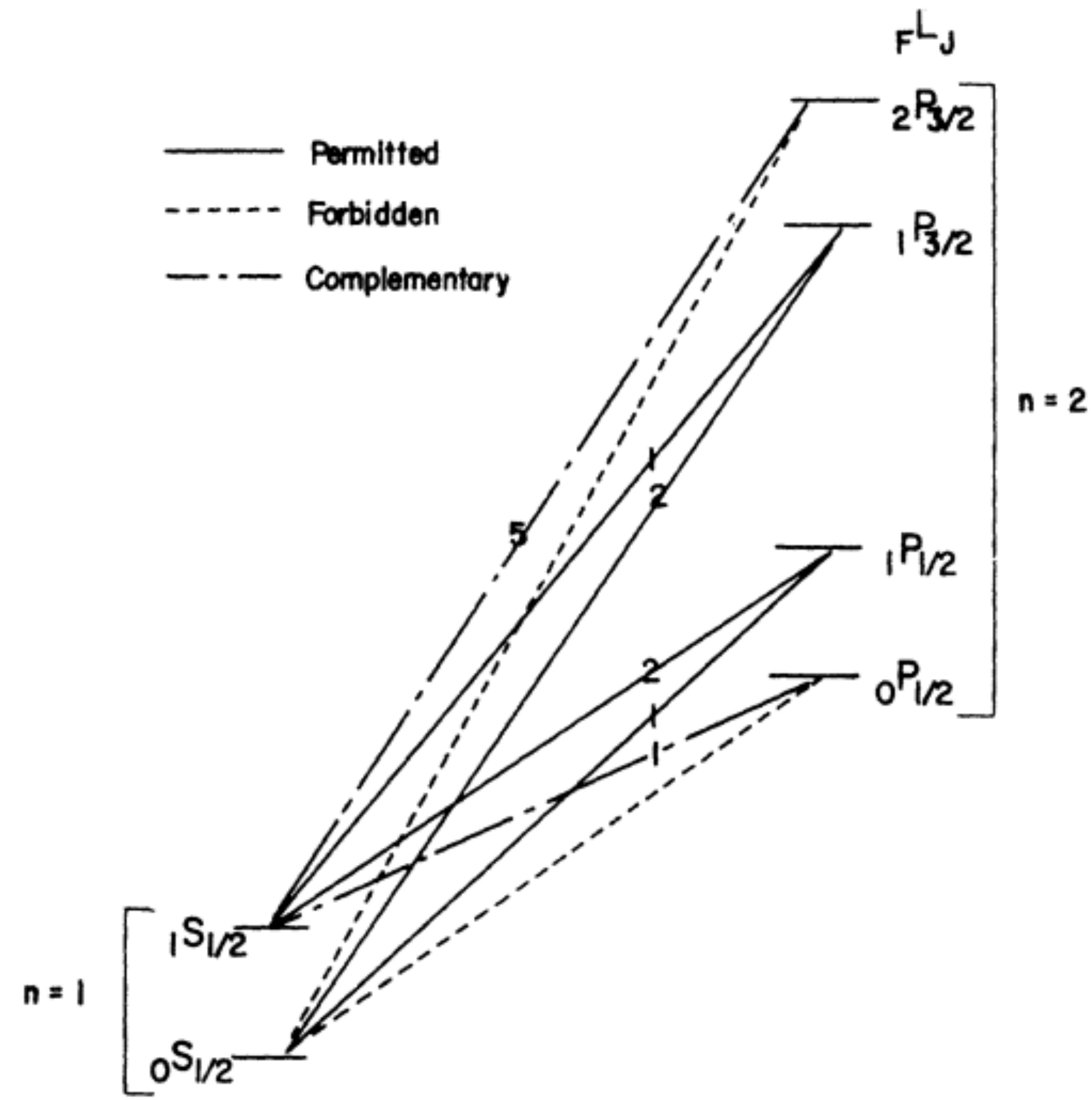
$$x(n,z) = \bar{x}(z)(1+m(z)\delta_H(n,z))$$

Differential brightness temperature

$$\delta T_b(n,z) = \delta\bar{T}_b(z) + \Delta_{\delta T_b}(n,z)$$

$$\Delta_{\delta T_b} = \bar{T}(z)\left[\left(1-\frac{T_\gamma}{\bar{T}_s}+\frac{T_\gamma}{\bar{T}_s}s\right)\delta_{\rm H}+\mu^2\left(1-\frac{T_\gamma}{\bar{T}_s}\right)\delta_{\rm DM}\right]$$

Wouthuysen–Field coupling



Spin temperature

Connection with other quantities

$$T_s^{-1} = \frac{T_\gamma^{-1} + x_\alpha T_g^{-1} + x_c T_g^{-1}}{1 + x_\alpha + x_c}$$

Collisional coupling co-efficient

$$x_c = \frac{T_* C_{10}}{T_\gamma A_{10}},\ C_{10} - \ \text{Collisional de} - \text{excitation rate}$$

Lyman α coupling co-efficient

$$x_\alpha = \frac{T_* P_{10}^\alpha}{T_\gamma A_{10}}$$

where P_{10}^α is the transition rate from triplet to singlet due to WF coupling

P_{10}^α can be connected to the total Lyman alpha scattering rate P_α as

$$P_{10}^\alpha = \frac{4}{27} P_\alpha$$

Finally,

$$x_\alpha = \frac{16\pi^2 T_\star e^2 f_\alpha}{27 A_{10} T_\gamma m_e c} S_\alpha J_\alpha$$

where, J_α is the background Lyman α background flux

Wouthuysen–Field coupling

Source emissivity

$$\epsilon(\nu, z) = \epsilon_b(\nu)\frac{\Omega_b}{m_p\Omega_m}\frac{d}{dt}\int f_\star(M,t)Mn(M,t)dM$$

$\epsilon(\nu, z)$ - number of photons per unit comoving volume per unit proper time per unit frequency range at redshift z’ and frequency nu’

M - mass of dark matter halo.

n(M, t) - comoving number density of halos at proper time t per unit mass

$f_*(M, t)$ - fraction of baryons that have been converted to stars

m_p - proton mass

$\epsilon_b(\nu)$ - number of photons emitted per baryon

Wouthuysen–Field coupling

Ly alpha flux

$$J_\alpha = \frac{(1+z)^2}{4\pi} \sum_{n=2}^{\infty} f_{recycle}(n) \int_z^{z_{max}} \frac{c\epsilon(\nu', z')}{H(z')} dz'$$

Fluctuations in spin temperature s(z)

$$s = \frac{x_c}{1+x_c}\left[g\frac{\bar{T}_s}{\bar{T}_g} - \left(\frac{\bar{T}_s}{\bar{T}_g} - 1\right)\left\{1 + g\frac{d(\ln K_{10}^{HH})}{d(\ln T_g)}\right\}\right]$$

Gas kinetic temperature evolution with redshift

$$\frac{\partial T_{\rm g}}{\partial z} = \frac{2T_{\rm g}}{1+z} - \frac{32\sigma_{\rm T} T_o^4 \sigma_{\rm SB}}{3m_e c^2 H_o\sqrt{\Omega_{\rm m0}}}(T_\gamma - T_{\rm g})(1+z)^{3/2}\frac{x}{1+x}$$

Evolution of fluctuations (1st order) in the gas kinetic temperature with redshift

$$\frac{\partial g}{\partial z} = \left(\frac{2}{3} - g\right)\frac{1}{\delta_{\rm H}}\frac{\partial \delta_{\rm H}}{\partial z} + \frac{32\sigma_T T_o^4 \sigma_{\rm SB}}{3m_e c^2 H_o\sqrt{\Omega_{m0}}}(1+z)^{3/2}$$
$$\times\frac{\bar{x}}{1+\bar{x}}\left[\frac{T_\gamma}{\bar{T}_g}g - m\left(\frac{T_\gamma}{\bar{T}_g} - 1\right)\right]$$

Ionization equation at redshift z<700

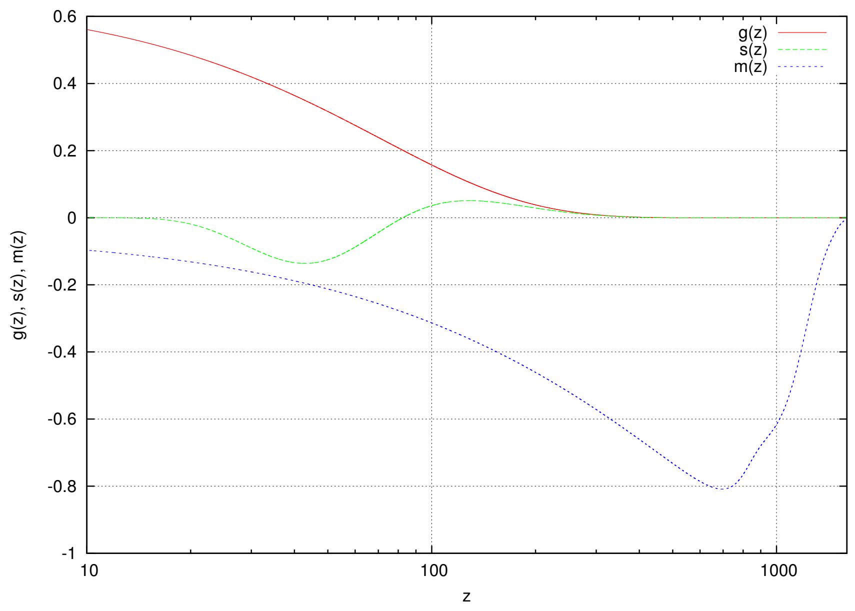
$$\frac{dx}{dz} = \frac{\alpha_e x^2 n_H}{(1+z)H(z)}$$

Evolution of fluctuations (1st order) in the ionisation fraction with redshift

$$\frac{dm}{dz} = -m\frac{dln\delta_H}{dz} + \frac{\alpha_e n_H x}{(1+z)H(z)}\left[\frac{\delta_\alpha}{\delta_H} + 1 + m\right]$$

$$\delta_\alpha = \frac{\partial\alpha_e}{\partial T_g}\frac{\bar{T}_g}{\alpha_e(\bar{T}_g)}\delta_g$$

Evolution of perturbations



HI power spectrum

$$\eta_{\mathrm{HI}}(\mathbf{n}, z) = \int \frac{d^3k}{(2\pi)^3} e^{-i\,\mathbf{k}\cdot r_\nu\,\mathbf{n}} \tilde{\eta}_{\mathrm{HI}}(\mathbf{k}, z)$$

HI power spectrum during dark ages

$$\delta T_b(\mathbf{n}, z) = \bar{T}(z) \times \eta_{\mathrm{HI}}(\mathbf{n}, z)$$

Where

$$\bar{T}(z) = 4.0\,\mathrm{mK}(1+\mathrm{z})^2 \left(\frac{\Omega_\mathrm{b}\mathrm{h}^2}{0.02}\right)\left(\frac{0.7}{\mathrm{h}}\right)\frac{\mathrm{H}_0}{\mathrm{H(z)}}$$

and

$$\eta_{\mathrm{HI}}(\mathbf{n}, z) = \frac{\rho_{\mathrm{HI}}}{\bar{\rho}_{\mathrm{H}}}\left(1 - \frac{T_\gamma}{T_s}\right)\left[1 - \frac{(1+z)}{H(z)}\frac{\partial v}{\partial r}\right]$$

Fourier transform

$$\eta_{\mathrm{HI}}(\mathbf{n}, z) = \int \frac{d^3k}{(2\pi)^3} e^{-i\,\mathbf{k}\cdot r_\nu\,\mathbf{n}} \tilde{\eta}_{\mathrm{HI}}(\mathbf{k}, z)$$

3D power spectrum $P_{\mathrm{HI}}(\mathbf{k}, z)$ is defined through

$$\langle \tilde{\eta}_{\mathrm{HI}}(\mathbf{k}, z)\, \tilde{\eta}^*_{\mathrm{HI}}(\mathbf{k}^{'}, z)\rangle = (2\pi)^3\, \delta^3_D(\mathbf{k} - \mathbf{k}^{'})\, P_{\mathrm{HI}}(\mathbf{k}, z)$$

HI power spectrum during dark ages

$$\Delta_{\delta T_b} = \bar{T}(z)\left[\left(1 - \frac{T_\gamma}{\bar{T}_s} + \frac{T_\gamma}{\bar{T}_s}s\right)\delta_{\mathrm{H}} + \mu^2\left(1 - \frac{T_\gamma}{\bar{T}_s}\right)\delta_{\mathrm{DM}}\right]$$

HI 21-cm power spectrum during dark ages

$$P_{T_b}(k,z) = \bar{T}^2(z)\left[\left(\frac{T_\gamma}{\bar{T}_s} - s\frac{T_\gamma}{\bar{T}_s} - 1\right)b(k,z) + \mu^2\left(\frac{T_\gamma}{\bar{T}_s} - 1\right)\right]^2 P(k,z),$$

where

$$\delta_{\mathrm{H}} = b(k,z)\delta_{\mathrm{DM}}$$

P(k,z) is the dark matter power spectrum at redshift z

HI 21-cm power spectrum during dark ages