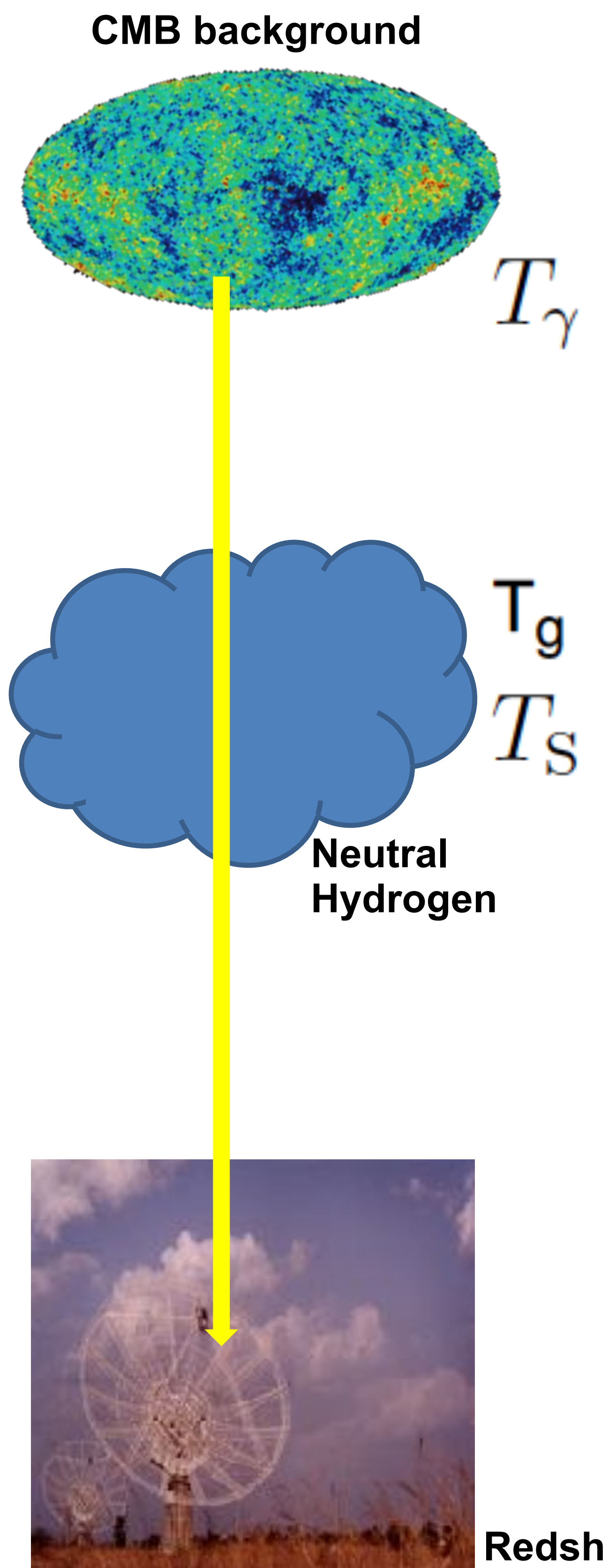
Statistics of the cosmological 21-cm signal



Redshifted HI 21-cm signal

Basics of redshifted 21-cm radiations

Differential brightness temperature

$$\delta T_b(\mathbf{n}, z) = \frac{(T_s - T_\gamma)\tau}{1+z}$$

Spin temperature :

$$n_1/n_0 = 3\exp(-T_\star/T_S)$$

where

$$T_\star \equiv hc/k\lambda_{21\mathrm{cm}} = 0.0628\,\mathrm{K}$$

$$\delta T_b(\mathbf{n}, z) = \bar{T}(z) \times \eta_{\mathrm{HI}}(\mathbf{n}, z)$$

Where

$$\bar{T}(z) = 4.0\,\mathrm{mK}(1+\mathrm{z})^2 \left(\frac{\Omega_b \mathrm{h}^2}{0.02}\right)\left(\frac{0.7}{\mathrm{h}}\right)\frac{\mathrm{H}_0}{\mathrm{H(z)}}$$

and

$$\eta_{\mathrm{HI}}(\mathbf{n}, z) = \frac{\rho_{\mathrm{HI}}}{\bar{\rho}_{\mathrm{H}}}\left(1 - \frac{T_\gamma}{T_s}\right)\left[1 - \frac{(1+z)}{H(z)}\frac{\partial v}{\partial r}\right]$$

Fluctuations in the differential brightness temperature

First order perturbations

Hydrogen density

$$\rho_{HI}(n,z) = \bar{\rho}_{HI}(z)(1+\delta_H(n,z))$$

Spin temperature

$$T_s(n,z) = \bar{T}_s(z)(1+s(z)\delta_H(n,z))$$

Gas kinetic temperature

$$T_g(n,z) = \bar{T}_g(z)(1+g(z)\delta_H(n,z))$$

Ionization fraction

$$x(n,z) = \bar{x}(z)(1+m(z)\delta_H(n,z))$$

Differential brightness temperature

$$\delta T_b(n,z) = \delta\bar{T}_b(z) + \Delta_{\delta T_b}(n,z)$$

$$\Delta_{\delta T_b} = \bar{T}(z)\left[\left(1-\frac{T_\gamma}{\bar{T}_s}+\frac{T_\gamma}{\bar{T}_s}s\right)\delta_{\mathrm{H}} + \mu^2\left(1-\frac{T_\gamma}{\bar{T}_s}\right)\delta_{\mathrm{DM}}\right]$$

Spin temperature

Connection with other quantities

$$T_s^{-1} = \frac{T_\gamma^{-1} + x_\alpha T_g^{-1} + x_c T_g^{-1}}{1 + x_\alpha + x_c}$$

Collisional coupling co-efficient

$$x_c = \frac{T_* C_{10}}{T_\gamma A_{10}}, \; C_{10} - \text{ Collisional de} - \text{excitation rate}$$

Lyman α coupling co-efficient

$$x_\alpha = \frac{T_* P_{10}^\alpha}{T_\gamma A_{10}}$$

where P_{10}^α is the transition rate from triplet to singlet due to WF coupling

P_{10}^α can be connected to the total Lyman alpha scattering rate P_α as

$$P_{10}^\alpha = \frac{4}{27} P_\alpha$$

Finally,

$$x_\alpha = \frac{16\pi^2 T_\star e^2 f_\alpha}{27 A_{10} T_\gamma m_e c} S_\alpha J_\alpha$$

where, J_α is the background Lyman α background flux

Fluctuations in spin temperature s(z)

$$s = \frac{x_c}{1+x_c}\left[g\frac{\bar{T}_s}{\bar{T}_g} - \left(\frac{\bar{T}_s}{\bar{T}_g} - 1\right)\left\{1 + g\frac{d(\ln K_{10}^{HH})}{d(\ln T_g)}\right\}\right]$$

Gas kinetic temperature evolution with redshift

$$\frac{\partial T_{\mathrm{g}}}{\partial z} = \frac{2T_{\mathrm{g}}}{1+z} - \frac{32\sigma_{\mathrm{T}} T_o^4 \sigma_{\mathrm{SB}}}{3m_e c^2 H_o \sqrt{\Omega_{\mathrm{m0}}}}(T_\gamma - T_{\mathrm{g}})(1+z)^{3/2}\frac{x}{1+x}$$

Evolution of fluctuations (1st order) in the gas kinetic temperature with redshift

$$\frac{\partial g}{\partial z} = \left(\frac{2}{3} - g\right)\frac{1}{\delta_{\mathrm{H}}}\frac{\partial \delta_{\mathrm{H}}}{\partial z} + \frac{32\sigma_T T_o^4 \sigma_{\mathrm{SB}}}{3m_e c^2 H_o \sqrt{\Omega_{m0}}}(1+z)^{3/2}$$
$$\times \frac{\bar{x}}{1+\bar{x}}\left[\frac{T_\gamma}{\bar{T}_g} g - m\left(\frac{T_\gamma}{\bar{T}_g} - 1\right)\right]$$

Ionization equation at redshift z<700

$$\frac{dx}{dz} = \frac{\alpha_e x^2 n_H}{(1+z)H(z)}$$

Evolution of fluctuations (1st order) in the ionisation fraction with redshift

$$\frac{dm}{dz} = -m\frac{dln\delta_H}{dz} + \frac{\alpha_e n_H x}{(1+z)H(z)}\left[\frac{\delta_\alpha}{\delta_H} + 1 + m\right]$$