Lecture 1
Astrophysical Magnetic Fields
Magnetic Universe, Basics of MHD, Detecting magnetic fields and more

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Agenda

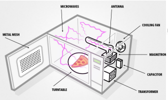
1. **Books and References**
2. **The Magnetic Universe**
3. **Detecting magnetic fields**
   - Methods for detecting B fields
4. **Elements of MHD**
   - Introduction
   - Conservation Laws
   - Induction Equation, Flux Freezing, Lorentz force
   - MHD equations
5. **Turbulence & Dynamos**
   - Turbulence in Astrophysics
   - Generation of Magnetic fields - Dynamos
   - Fluctuation dynamos
   - Mean-field dynamos
6. **Why think about magnetic fields?**

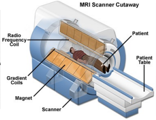
Books and References

- Books and References on Magnetohydrodynamics
  - Shukurov & Subramanian (2022) - *Astrophysical Magnetic fields : From Galaxies to the Early Universe*, CUP
  - Heald et. al., (2020) - *Magnetism Science with the Square Kilometre Array*, Galaxies, Vol. 8
  - R. Beck (2015) - *Magnetic fields in spiral galaxies*, Astronomy & Astrophysics Reviews
  - *Dynamos : Lecture notes of Les Houches Summer School, Session 88*, 2007
  - Brandenburg & Subramanian (2005) - *Astrophysical magnetic fields and nonlinear dynamo theory*, Physics Reports, Vol. 417. arXiv : astro-ph/0405052
  - Russell Kulsrud (2005) - *Plasma Physics for Astrophysics*, PUP
  - Arnab Rai Choudhuri (1999) - *The Physics of Fluids & Plasmas - An Introduction for Astrophysicists*, CUP
  - Frank H. Shu (1991) - *The Physics of Astrophysics (Vol. 2) - Gas Dynamics*, University Science Books

Introduction

- Magnets are ubiquitous in our lives

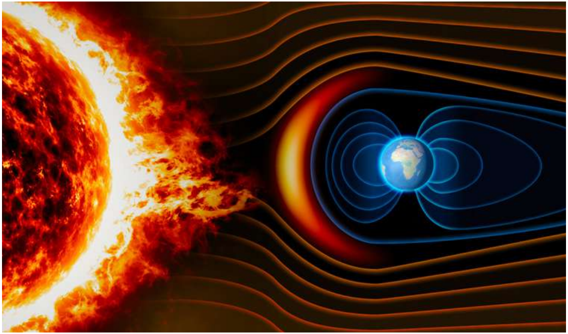





- From credit cards, microwave ovens, speakers to medical instruments
- Until recently, mariners relied on magnetic compass for navigation
- Some birds also seem to have a Magnetic Sixth sense - Magneto-reception
- In fact, magnetism seems to be everywhere in the Cosmos!!

The Magnetic Universe

- Earth has a Dipolar magnetic field structure

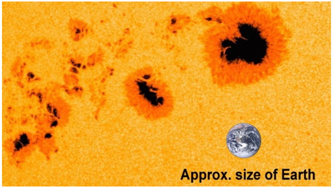


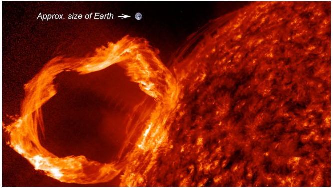
Credit : The Week, April 2019

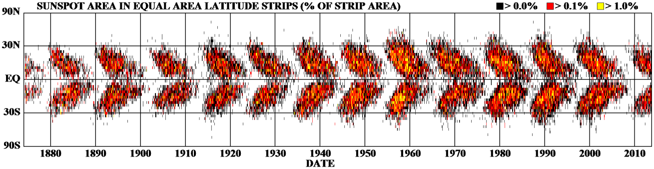
- Field strengths $\approx 1\,\mathrm{G}$, with irregular reversals over a million years!
  - It is not just a handy navigation aid!
  - The Earth's magnetic field is vital for the existence of life
- Shields us from high energy particles arriving from the Sun
- Particles approach near the Earth's surface only at the poles

The Magnetic Universe

- Magnetism of the Sun responsible for a whole range of phenomena



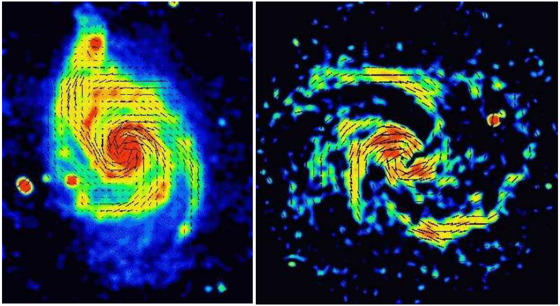




- Strong fields $\sim 3 \times 10^3\,\mathrm{G}$ in sunspots
- Butterfly Diagram : Variation of sunspot number
- Exhibits plethora of features - Solar prominences, Coronal mass ejections; test bed for MHD theories

The Magnetic Universe

- Magnetic fields in spiral galaxies(Left : M31, Right : NGC 6946)



  - **Spiral galaxies** : Thin rotating discs of $\sim 10^{10}$ stars and interstellar gas, multiphase interstellar medium (ISM)
  - Interstellar gas : $\langle n \rangle \sim 1\,\mathrm{cm}^{-3}$, $10^{-3} < n < 10^{6}\,\mathrm{cm}^{-3}$, $10 < T < 10^{7}\,\mathrm{K}$
- $\langle \mathbf{B}_{\mathrm{tot}} \rangle = 9\,\mu\mathrm{G}$ in a sample of 74 spirals; large-scale fields about half the value of the random field
  - Large-scale fields correlated on $10\,\mathrm{kpc}$ scales
  - Similar field strengths found in interacting galaxies

The Magnetic Universe

- Magnetic fields in gaseous halos of galaxies
  - Hot, ionized, quasi-spherical envelopes of galactic disks

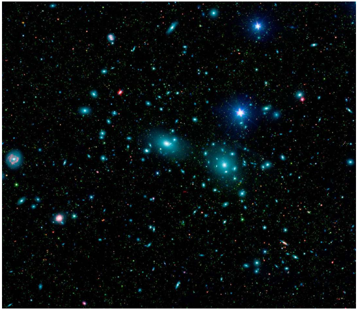
NGC 891, Credit : NASA & MPIfR, Bonn

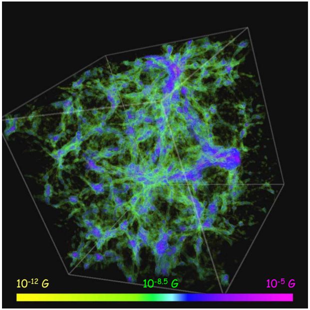
- Numbers for galaxy halos :
  - Number densities : $n \simeq 10^{-3}\,\mathrm{cm}^{-3}$
  - $T \simeq 10^6\,\mathrm{K}$, $c_s \simeq 10^2\,\mathrm{km\,s}^{-1}$, $L \simeq 10\,\mathrm{kpc}$

- Field runs parallel to the plane near the disk, vertical components above and below the plane forms an X shaped structure in the halo
  - Field strength in the halos similar to those in the disks
  - How to explain the occurrence of these fields?

The Magnetic Universe

- Galaxy Clusters : Largest gravitationally bound system in our Universe; $M \sim 10^{14} - 10^{15}\, M_\odot$, size of several Mpc
  - Baryonic matter contained in hot X-ray emitting ICM ($T \sim 10^7 - 10^8\,\mathrm{K}$, $n \sim 10^{-2} - 10^{-4}\,\mathrm{cm}^{-3}$)
  - Field strengths $\approx \mu\mathrm{G}$ ordered on several kpc scales; fields detected with the help of Faraday rotation measure





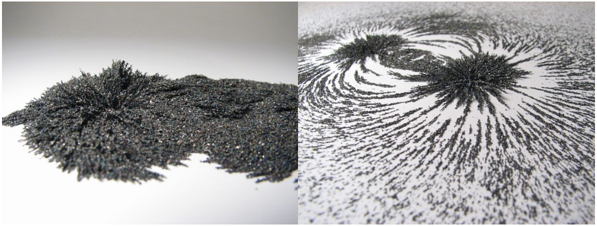
- Left : Coma Cluster (SDSS/SST), Right : Ryu et al., 2008, Science
- No overall rotation; What generates and maintains the magnetic field?

Fundamental Questions

- How to detect the presence of magnetic fields?
- How did such magnetic fields arise and how are they maintained?
- How do we describe the motion of a conducting fluid?
  - MHD - study of the magnetic properties of electrically conducting fluids
- Why is it even necessary to think about magnetic fields?

Detecting the presence of magnetic fields

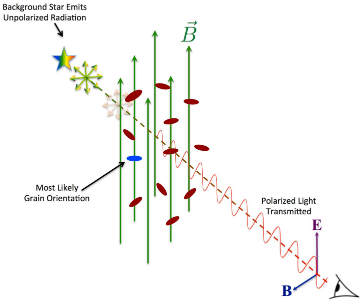
- Magnetism is invisible!! Is there a way to detect their presence?
  - Recall the simple experiment with a bar magnet and iron filings



- **How do we observe magnetic fields in the sky?**
  - Zeeman splitting of spectral lines - mostly used for the Sun
  - Light polarization by interstellar dust
  - Faraday rotation of polarised emission and synchrotron emission

Detecting the presence of magnetic fields

- 1949 : John Hall & Albert Hiltner independently showed that star light is polarised



- Refers to the orientation of the oscillation of light waves
- Star light is expected to be unpolarised!
- What then causes star light to become polarized?

- Interstellar space is dusty; Dust particles act like tiny compasses in the presence of magnetic field
- Polarised starlight reveals the presence of magnetic fields

Detecting the presence of magnetic fields

- Faraday Rotation : Magneto - Optical Phenomenon

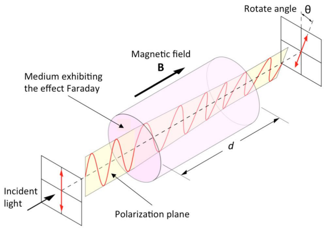


Image Credit : Wikipedia

- Discovered by Michael Faraday in 1845
- Angle of polarisation rotates as the light passes through a foreground magnetized region
  - Information about the line-of-sight component of the magnetic field and its direction
  - Stronger the field, the more rotation is produced

- Amount of rotation : $\psi = \psi_0 + \mathrm{RM}\lambda^2; \mathrm{RM} \propto \int_0^L n_e B_{||}\, ds$

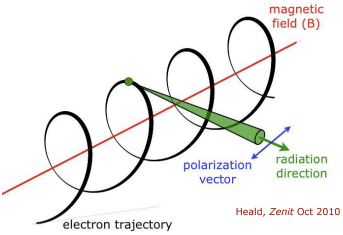
Faraday rotation measure in galaxies

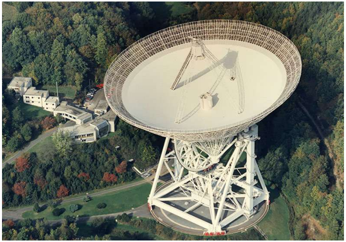
$$\mathrm{RM} = 0.81 \frac{\mathrm{rad}}{\mathrm{m}^2} \int_0^L \frac{n_e}{1\ \mathrm{cm}^{-3}} \frac{B_{||}}{1\ \mu\mathrm{G}} \frac{dl}{1\ \mathrm{pc}}$$

- Normalizations need to be appropriately adjusted for galaxy clusters

Detecting the presence of magnetic fields

- Polarized Synchrotron Emission





- Left : Synchrotron emission, Right : 100m Effelsburg radio telescope
- Produced by relativistic electrons spiraling around magnetic field lines
  - Information about the total strength of the magnetic field
  - The Effelsburg radio telescope played a pioneering role in inferring about galactic magnetic fields way back in the 70-80's

Detecting the presence of magnetic fields

- Total Intensity of the synchrotron emission

$$I_\nu = \int_0^L \epsilon_\nu \, ds \propto \int_0^L n_{\rm cr} \, B_\perp^2 \, ds,$$

  - ϵ_ν is the synchrotron emissivity and $n_{\rm cr}$ is the number density of cosmic ray electrons, $B_\perp$ is the magnetic field in the sky plane

- Polarized intensity : PI_ν and Fractional polarisation : (p_ν)

$$PI_\nu = \sqrt{Q_\nu^2 + U_\nu^2}, \quad p_\nu = PI_\nu / I_\nu,$$

  where Q_ν, U_ν are the Stokes parameters

- In galaxies : $PI_\nu \propto \int_0^L n_{\rm cr} \overline{\mathbf{B}}_\perp^2 \, ds$; $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$, $\langle \mathbf{B} \rangle = \overline{\mathbf{B}}$, $\langle \mathbf{b} \rangle = 0$
  - Traces the regular magnetic field whereas $I - P$ traces the turbulent magnetic field

- Together with Faraday Rotation measure, synchrotron emission provides the most important observational methods for galactic and extragalactic magnetic fields

MHD : Introduction and Objective

- **MHD = Equations of fluid dynamics + Maxwell's equations**
- Plasma treated as a continuous medium, described by a single temperature, density and bulk velocity
- For a pure hydrodynamical fluid, description completely specified by
  - $\rho \rightarrow$ Mass Density, $\mathbf{v} \rightarrow$ Flow velocity, $p \rightarrow$ Pressure
  - Governing equations derived from conservation laws
- However, description of a conducting fluid requires additional variables
  - $\rho_e \rightarrow$ Charge density, $\mathbf{J} \rightarrow$ Current density
  - $\mathbf{E} \rightarrow$ Electric field, $\mathbf{B} \rightarrow$ Magnetic field
- Objective : To find a set of closed equations describing the evolution of these variables

Conservation Laws

- Conservation laws can be used to derive evolution equations
- Conservation of Mass $\Rightarrow$ **Continuity Equation**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

  - Need an evolution equation for the fluid velocity
- Conservation of Momentum $\Rightarrow$ **Navier-Stokes Equation**

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \rho \nu \nabla^2 \mathbf{v} + \rho \mathbf{F} + \text{Extra terms}$$

where ν is the viscosity of the fluid

  - Reynolds number : $\mathrm{Re} = |(\mathbf{v} \cdot \nabla)\mathbf{v}| / |\nu \nabla^2 \mathbf{v}| = v\, L/\nu$
  - Additional terms when the fluid is conducting
  - **Check :** Fluid Mechanics by Landau & Lifshitz

MHD Basics

Maxwell's Equations (Gaussian CGS units)

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \;\; \nabla \cdot \mathbf{B} = 0$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{J}, \;\; \nabla \cdot \mathbf{E} = 4\pi\rho_e$$

- Need a relation between the current density and the fields
- If the fluid is moving, what fields should we use?
  - Fields in the fluid's local rest frame : $\{\mathbf{J}', \mathbf{E}', \mathbf{B}'\}$
  - Fields in the laboratory frame : $\{\mathbf{J}, \mathbf{E}, \mathbf{B}\}$
- Ohm's Law in the fluid's local rest frame $\mathbf{J}' = \sigma \mathbf{E}'$, where σ is the conductivity
- Relate $\mathbf{E}'$ and $\mathbf{B}'$ to $\mathbf{E}$ and $\mathbf{B}$ in the lab frame
  - Carry out Lorentz transformation between frames

Lorentz transformation between frames

$$E'_{\parallel} = E_{\parallel}, \quad B'_{\parallel} = B_{\parallel}$$

$$\mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{\perp}\right), \ \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{\mathbf{v}}{c} \times \mathbf{E}_{\perp}\right),$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor

- Can be simplified further if velocities are assumed to be non-relativistic, $\Rightarrow$ neglect terms of order v^2/c^2
  - Lorentz factor $\gamma \approx 1$
  - The electric and magnetic fields are related by

$$\mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}, \quad \mathbf{B}' = \mathbf{B},$$

  - **Exercise :** Show that to order $|v|/c$, $\mathbf{J}' = \mathbf{J}$
  - $\mathbf{J}/\sigma = \lambda\mathbf{J} = \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}/c$, where $\lambda = \sigma^{-1}$ is the resistivity of the fluid

MHD Basics

- Ohm's Law for a conducting fluid

$$\mathbf{J} = \sigma\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right), \quad |E| \approx \frac{|v|}{c}|B|$$

- Solve for **E**, substitute in the Faraday equation and neglect displacement current ⇒ Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}\times\mathbf{B} - \eta\nabla\times\mathbf{B}),$$

where, $\eta = c^2/4\pi\sigma$ is the magnetic diffusivity

Simple consequences

- $\mathbf{v} = 0 \Rightarrow$ Pure difusion and decay
- $\eta \to 0$, the flux $\Phi = \int_S \mathbf{B}\cdot\mathbf{dS}$ is frozen, $d\Phi/dt \to 0$
- Magnetic Reynolds number : $\mathrm{Rm} = v\,L/\eta$, for astrophysical systems $\mathrm{Rm} \gg 1$

MHD Basics

- Instructive to clarify the role of $\nabla \times (\mathbf{v} \times \mathbf{B})$ term

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = -(\mathbf{v} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}(\nabla \cdot \mathbf{v})$$

  - Advection (1st term), Stretching (2nd term) and Compression (3rd term)

- **Exercise :** Find the solution of the induction equation when $\mathbf{v} = (0, Sx, 0), \mathbf{B}_0 = (B_0, 0, 0)$ for $\eta = 0$.

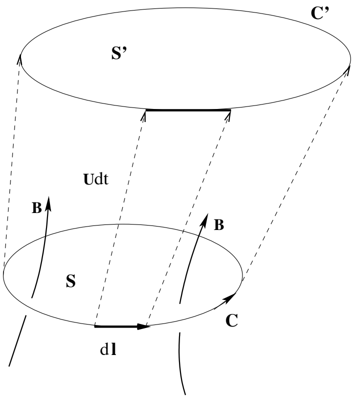
- Important parameters in different astrophysical settings

	T [K]	ρ [g cm^{-3}]	P_m	u_rms [cm s^{-1}]	L [cm]	R_m
Solar CZ (upper part)	10^4	10^{-6}	10^{-7}	10^6	10^8	10^6
Solar CZ (lower part)	10^6	10^{-1}	10^{-4}	10^4	10^{10}	10^9
Protostellar discs	10^3	10^{-10}	10^{-8}	10^5	10^{12}	10
CV discs and similar	10^4	10^{-7}	10^{-6}	10^5	10^7	10^4
AGN discs	10^7	10^{-5}	10^4	10^5	10^9	10^{11}
Galaxy	10^4	10^{-24}	(10^{11})	10^6	10^{20}	(10^{18})
Galaxy clusters	10^8	10^{-26}	(10^{29})	10^8	10^{23}	(10^{29})

  - Magnetic Prandtl number : $\mathrm{Pr}_M = \mathrm{Rm}/\mathrm{Re} = \nu/\eta$
  - Galaxies and clusters have very large Pr_M due to very low densities and much higher temperatures

- Reference : Brandenburg & Subramanian, Physics Reports, 2005

Flux Freezing



- Magnetic flux

$$\Phi = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}$$

- Interested to know the time rate of change of Φ

- Change in the flux :

$$d\Phi = \int_{S'} \mathbf{B}(t + dt) \cdot \mathrm{d}\mathbf{S} - \int_S \mathbf{B}(t) \cdot \mathrm{d}\mathbf{S}$$

Flux Freezing

- Using the fact that $\nabla \cdot \mathbf{B} = 0$ at time $t + dt \Rightarrow$

$$\int_{S'} \mathbf{B}(t+dt) \cdot d\mathbf{S} = \int_S \mathbf{B}(t+dt) \cdot d\mathbf{S} - \oint_C \mathbf{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{U}dt),$$

- Therefore,

$$d\Phi = \int_S [\mathbf{B}(t+dt) - \mathbf{B}(t)] \cdot d\mathbf{S} - \oint_C \mathbf{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{U}dt).$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} - \oint_C (\mathbf{U} \times \mathbf{B}) \cdot d\mathbf{l}$$

- Using $\oint_C (\mathbf{U} \times \mathbf{B}) \cdot d\mathbf{l} = \int_S \nabla \times (\mathbf{U} \times \mathbf{B}) \cdot d\mathbf{S}$

$$\frac{d\Phi}{dt} = \int_S \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{U} \times \mathbf{B})\right] \cdot d\mathbf{S} = \eta \int_S (\nabla^2 \mathbf{B}) \cdot d\mathbf{S}$$

  - As $\eta \to 0, d\Phi/dt \to 0$. Therefore Φ is a constant
  - Flows in a conducting fluid can amplify magnetic fields

The Lorentz force - Influence of magnetic field on velocity

- Magnetic field influences the velocity through the Lorentz force

$$\mathbf{F}_L = q\left[\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c}\right]$$

- Consider a conducting fluid with n_i ions and n_e electrons per unit volume
- The Lorentz force density is given by,

$$\begin{aligned}\mathbf{f}_L &= +en_i\left[\mathbf{E} + \frac{\mathbf{u}_i\times\mathbf{B}}{c}\right] - en_e\left[\mathbf{E} + \frac{\mathbf{u}_e\times\mathbf{B}}{c}\right] \\ &= \rho_e\mathbf{E} + [+\mathbf{en_iu_i} - \mathbf{en_eu_e}]\times\mathbf{B}/\mathbf{c} \sim \frac{\mathbf{J}\times\mathbf{B}}{\mathbf{c}}\end{aligned}$$

  - $|\rho_e\mathbf{E}|/|(\mathbf{J}\times\mathbf{B})/c| \sim v^2/c^2 \ll 1$, $\Rightarrow \mathbf{F}_L$ due to $\mathbf{E}$ negligible

- Using $\nabla\times\mathbf{B} = (4\pi/c)\mathbf{J}$

$$\mathbf{F}_L = \frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{4\pi} = -\nabla\left(\frac{\mathbf{B}^2}{8\pi}\right) + \frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{4\pi}$$

  - For straight field lines, $(\mathbf{B}\cdot\nabla)\mathbf{B} = 0$
  - Magnetic pressure can still be non zero

The Lorentz force - Influence of magnetic field on velocity

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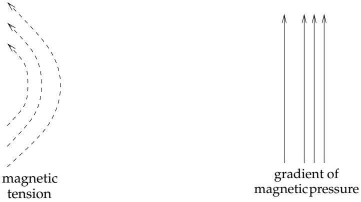
- $|\rho_e\mathbf{E}|/|(\mathbf{J}\times\mathbf{B})/c| \sim v^2/c^2 \ll 1$, $\Rightarrow$ **$\mathbf{F}_L$ due to $\mathbf{E}$ negligible**



MHD equations

Full set of MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\nabla p - \rho \nabla \phi + \rho \nu \nabla^2 \mathbf{v} + \frac{\mathbf{J} \times \mathbf{B}}{c},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),$$

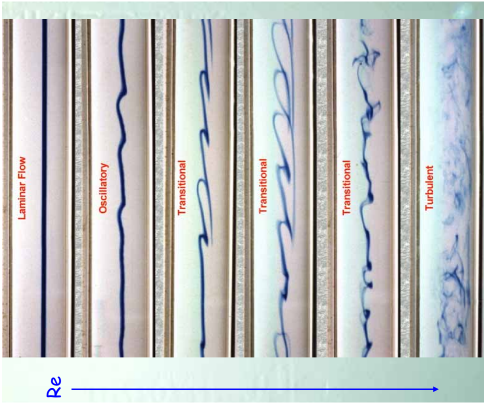
$$\begin{aligned} \frac{\partial \rho E}{\partial t} &+ \nabla \cdot [\mathbf{v}(\rho E + p_*) - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})] \\ &= \rho \mathbf{g} \cdot \mathbf{v} + \nabla \cdot (\mathbf{v} \cdot \tau + \sigma \nabla T) \\ &+ \nabla \cdot [\mathbf{B} \times (\eta(\nabla \times \mathbf{B}))], \end{aligned}$$

$$\nabla^2 \phi = 4\pi G \rho.$$

- $p_* = p + \mathbf{B}^2/8\pi$
- $E = \mathbf{v}^2/2 + \epsilon + \mathbf{B}^2/8\pi\rho$

Turbulence

- What kind of velocities are we talking about?
  - In astrophysical systems, velocities are turbulent
- Turbulence - a flow regime characterized by random variations of pressure and velocity in space and time
  - Onset of turbulence is determined by the Reynolds number with $\mathrm{Re} > 1000 \Rightarrow$ turbulent regime



Turbulence in Astrophysics

- Turbulence requires a continuous supply of energy

Sources

- Instabilities in a flow - Shear instability
- Solar convection
- Cosmological structure formation shocks, merger events
- Supernovae explosion in the ISM
- From subsonic (in cluster cores) to supersonic (in the ISM)

Significance

- Energy transfer from large scales of motion
- Jupiter's great Red spot
- Augments molecular transport - causing mixing of the fluid
- Large/Small -scale field generation via turbulent dynamo

- **Check out :** Kolmogorov's hypothesis

The Bicycle Dynamo

- Consider the simple example of a Bicycle Dynamo

- Mechanical energy transformed to electrical energy
  - Peddling action rotates the magnet; changing magnetic field induces electric currents ⇒ illuminates the light bulb
- In astrophysical objects there are no external magnets, wires, frames etc.
  - Kinetic energy in fluid motions tapped to amplify magnetic energy

Generation of Magnetic fields

- Magnetic fields evolve as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

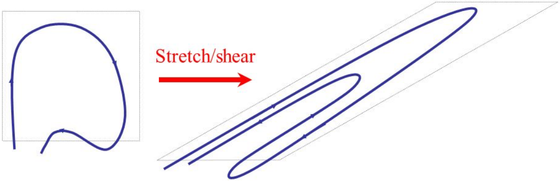
where $\mathbf{v}$ is a solution of the momentum equation

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\nabla p + \rho \nu \nabla^2 \mathbf{v} + \frac{\mathbf{J} \times \mathbf{B}}{c}$$

  - **Dynamos :** class of velocity fields that allow a weak seed magnetic field to grow
- **What if we start with a very weak magnetic field?**
  - Ratio of $|(\mathbf{J} \times \mathbf{B})/c| / |\rho(\mathbf{v} \cdot \nabla)\mathbf{v}| \approx (B^2/8\pi)/(\rho v^2/2) \ll 1$
  - Either the field decays or it grows such that $(B^2/8\pi)/(\rho v^2/2) \sim 1$
- Galaxies and Galaxy clusters are turbulent systems
  - If $\langle B^2 \rangle$ grow? How? $\rightarrow$ **Turbulent Dynamos**
  - Fluctuation dynamos & Mean-field dynamos
  - When and how do dynamos saturate - active area of research

Fluctuation dynamos

- Ideally suited for amplifying fields in the ICM, may also operate in galaxies
  - Growth by random stretching by turbulent eddies
  - Field grows exponentially at first and then saturates
  - Saturation achieved on a scale-by-scale basis with smaller scales saturating the field at that scale first



Batchelor 1950, Ruzmaikin & Zeldovich 1990, Childress & Gilbert 1995

- Average magnetic energy evolution governed by stretching, compression and dissipation terms; $\nabla \cdot \mathbf{u}$ term is negligible in subsonic flows

$$\frac{\delta}{\delta t}\langle \mathbf{b}^2/2\rangle = \langle \mathbf{s}_{ij}\mathbf{b}_i\mathbf{b}_j\rangle - \langle \frac{1}{2}|\mathbf{b}^2|(\nabla \cdot \mathbf{u})\rangle - \langle \eta |\mathbf{j}^2|\rangle$$

Fluctuation dynamos

- Ideally suited for amplifying fields in the ICM, may also operate in galaxies
  - Growth by random stretching by turbulent eddies
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Characteristics of Fluctuation dynamos

- Generic in any random/turbulent flow for $\mathrm{Rm} > \mathrm{Rm}_{\mathrm{cr}} \sim 200$
- Field is amplified on the eddy-turnover time-scale; $\tau_l \sim l/v_l \propto l^{2/3}$
- Growth time $\sim 0.3\,\mathrm{Gyr}$ in galaxy clusters
- Fields correlated on scales at most on the scale of turbulence
- Field structure appears to be highly intermittent; long tails in the PDF or increased values of the kurtosis

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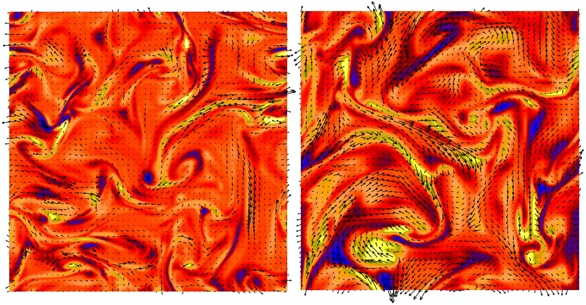
- 2D snapshots of $B_z/B_{\rm rms}$ in the kinematic (left) and saturated phase (right)



Sur 2019, Sur, Basu & Subramanian 2021

Mean-field dynamos

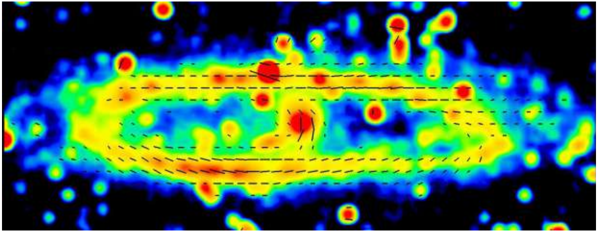
- Theoretical ansatz : $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}, \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}, \quad \overline{\mathbf{u}} = \mathbf{0}, \quad \overline{\mathbf{b}} = \mathbf{0}$
- The mean-field satisfies the dynamo equation

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}}) + \eta \nabla^2 \overline{\mathbf{B}}$$

  - $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}} \approx \alpha \overline{\mathbf{B}} - \eta_t \overline{\mathbf{J}}$
  - $\alpha = (-\tau/3)\langle \omega \cdot \mathbf{u} \rangle$ is the mean helicity of turbulence; also known as the $\alpha -$ *effect*
  - $\eta_t = (\tau/3)\langle \mathbf{u}^2 \rangle$ is the turbulent magnetic diffusivity
- Large-scale magnetic field generation in galaxies : Interstellar medium in spiral galaxies are :
  - Rotating, stratified, contains electrically conducting fluid
  - Randomly stirred by supernovae and stellar winds
- Perfect conditions for the Galactic Dynamo to operate and generate large-scale fields

Mean-field dynamos

- Large-scale magnetic fields in M31



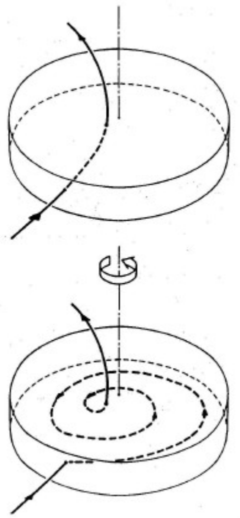
Credit : MPIfR, Bonn

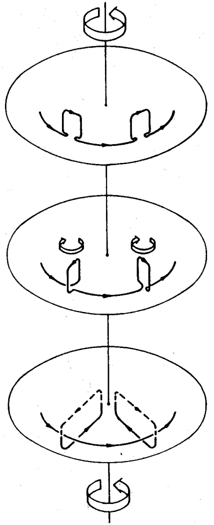
- Galactic shear generates $\overline{B}_\phi$ from $\overline{B}_r$
- Supernovae drive helical turbulence in the disk
- Helical motions generate $\overline{B}_r$ from $\overline{B}_\phi$ through the α-effect
- Growth time of the magnetic field $\sim 10^9$ yr

Mean-field dynamos

- Magnetic field generation in disk galaxies

  Ruzmaikin, Sokoloff & Shukurov, 1988





Thinking about magnetic fields

- Pursuit of one's curiosity! $\Rightarrow$ Hallmark of a civilized society
- May lead to technology spin-offs; Potential to improve the material quality
- Implications for **Space Weather** - A variety of technologies rely heavily on near-Earth space conditions
- Astrophysical context :
  - Effects on Star-formation - collapse and fragmentation of clouds
  - Can affect mixing properties of fluids
  - Fields generated in the **First Stars** provides seed fields for the First Galaxies
- A new Magnetic Era to usher in with the **Square Kilometre Array (SKA)**

Quote from astronomer Lo Woltjer

The larger the one's ignorance, the stronger the magnetic field!

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